Kondo Impurities Coupled to a Helical Luttinger Liquid: RKKY-Kondo Physics Revisited

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We show that the paradigmatic Ruderman-Kittel-Kasuya-Yosida (RKKY) description of two local magnetic moments coupled to propagating electrons breaks down in helical Luttinger liquids when the electron interaction is stronger than some critical value. In this novel regime, the Kondo effect overwhelms the RKKY interaction over all macroscopic interimpurity distances. This phenomenon is a direct consequence of the helicity (realized, for instance, at edges of a time-reversal invariant topological insulator) and does not take place in usual (nonhelical) Luttinger liquids.

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The seminal problem of the indirect exchange interaction [Ruderman-Kittel-Kasuya-Yosida (RKKY)] between two spatially localized magnetic moments, i.e., Kondo impurities (KIs), weakly coupled to propagating electrons has a well-known solution [1]. The paradigmatic approach can be reformulated in the contemporary language as follows: one integrates out fermionic degrees of freedom and reduces the resulting nonlocal Lagrangian to the effective spin Hamiltonian. The second step is usually justified by a scale separation; the spin dynamics is slower than the electron one if the electron-spin coupling is weak. RKKY induces perceptible interimpurity correlations if an interimpurity distance R is smaller then the thermal length and the electron coherence length. This RKKY theory is the obvious simplification since it neglects another fundamental phenomenon, namely, the Kondo effect [2]. If the temperature is below the Kondo temperature, $T < T_K$, the antiferromagnetic Kondo coupling drives the single KI to the strong coupling limit where the electrons screen KI. Hence, the Kondo screening is an antagonist of the RKKY interaction.

The RKKY-Kondo interplay has attracted large attention for several decades [3–7] and remains a hot topic of research because of its importance for new systems, such as graphene [8,9], strongly correlated quantum wires, and carbon nanotubes, which are described by the Luttinger liquid model [10,11]. The latter are especially interesting because the Kondo effect can be enhanced by the interactions [12–14]. The common wisdom is that the RKKY physics dominates in a broad macroscopic range of R in three- and low-dimensional systems.

In this Letter, we will demonstrate that, surprisingly, the paradigmatic RKKY approach breaks down in strongly correlated helical systems—helical Luttinger liquids (HLLs). We will show that the reason for this unexpected finding is the nontrivial and unusually increased RKKY-Kondo competition.

Helicity means the lock-in relation between electron spin and momentum: helical electrons propagating in opposite directions have opposite spins. This protects the helical transport against effects of spinless impurities. HLL can appear at edges of time-reversal invariant 2D topological insulators [15–19] and in purely 1D interacting systems [20,21]. The Kondo effect [22–26] and RKKY [27–32] in the topological insulators have been intensively studied for the past several years. This increasing interest is partly related to the hypothesis that Kondo-RKKY effects can be responsible for deviations of the helical conductance from its ideal value; see Refs. [33–37] and discussions therein.

At a simple phenomenological level, one can find "the winner of the RKKY-Kondo competition" by comparing T_K with the characteristic energy of RKKY, E_{RKKY}. The latter has the meaning of the energy gap, which opens after the RKKY correlations lift a degeneracy in the energy of the uncorrelated KIs. In the absence of Coulomb interactions, $T_K^{(0)} \propto \exp(-1/\rho_0 J)$ and $E_{\mathrm{RKKY}}^{(0)} \propto J^2/R^d$, where ρ_0 is the density of states of the electrons at the Fermi surface, J is the Kondo coupling constant, and d is the space dimension. If $\rho_0 J \ll 1$, there is a broad range of macroscopic distances where $E_{\mathrm{RKKY}}^{(0)} \gg T_K^{(0)}$, T. In this case, RKKY is expected to overwhelm the Kondo screening.

The situation drastically changes in HLL with the strong interaction. Let us concentrate on the XXZ Kondo coupling with small constants $J_\perp$, $J_z \ll 1/\rho_0$; see the formal definition in Eqs. (5) and (6) and temporarily neglect J_z. The electron repulsion is reflected by the Luttinger parameter of HLL: $K \leq 1$ [38]; $K = 1$ corresponds to noninteracting fermions. Both E_{RKKY} [see Eq. (13)] and T_K (see Ref. [22]) are modified by the interaction

$$E_{\mathrm{RKKY}} \sim D(\rho_0 J_\perp)^2 (\xi/R)^{2K-1}, \quad 1/2 < K \leq 1; \qquad (1)$$

$$T_K \propto \begin{cases} T_K^{(0)}, & 0 < 1 - K \ll 1; \\ D(\rho_0 J_\perp)^{\frac{1}{1-K}} \gg T_K^{(0)}, & 1 - K \gg \rho_0 J_\perp. \end{cases} \quad (2)$$

Here ξ (D) is the spatial (energy) UV cutoff, i.e., the lattice spacing (bandwidth). A naive formal extension of Eq. (1) to the regime $K < 1/2$ would lead to a paradoxical result: $E_{\rm RKKY}$ seems to grow without bound with the increase of the interimpurity distance. The results presented below ultimately refute any possibility of such an effect.

Based on the above explained phenomenological arguments, we expect that, if $E_{\rm RKKY}(R \sim \xi) > T_K$, there exists a broad range of macroscopic distances where RKKY dominates over the Kondo effect. In the opposite case, $E_{\rm RKKY}(R \sim \xi) < T_K$, the Kondo physics dominates everywhere. The border between these two phases is defined by the condition $E_{\rm RKKY} \sim T_K$. We will show that it corresponds to the critical value of the effective interaction parameter

$$\tilde{K} = K(1 - \rho_0 J_z / 2K)^2, \qquad \tilde{K}_{\rm crit} = 1/2, \quad (3)$$

see the phase diagram in Fig. 1 [39]. The paradigmatic RKKY theory is valid only at $\tilde{K} > 1/2$ and fails at $\tilde{K} < 1/2$. Namely, the spin subsystem cannot be described by an effective (RKKY-like) Hamiltonian at $\tilde{K} < 1/2$. These statements, which are proven below at a more formal level, are our main result.

The rest of this Letter is organized as follows: First, we will rederive the RKKY Hamiltonian by integrating out HLL degrees of freedom and discuss the difference between helical and usual (not helical but spinful) cases. We will combine the microscopic diagrammatic approach with one-loop renormalization group (RG) arguments to

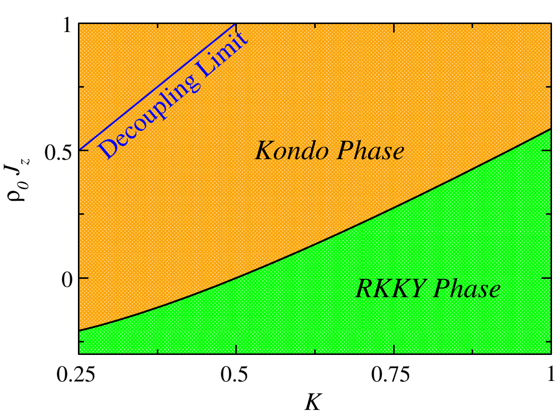

FIG. 1. Phase diagram of the two Kondo impurities coupled to the HLL. Green and orange regions demonstrate the RKKY and the Kondo phases, respectively. Axes show values of the Luttinger parameter $1/4 \le K \le 1$ and the dimensionless coupling constant $|\rho_0 J_z| < 1$; see Eqs. (4) and (5). The critical line, which separates the phases, is defined by the equation $\tilde{K} = 1/2$. The decoupling limit corresponds to $\tilde{K} = 0$ [24].

explain how RKKY stops Kondo renormalizations and why the paradigmatic theory of RKKY is valid in the range $1/2 < \tilde{K}$ and fails at $\tilde{K} < 1/2$. By exploiting the extreme situation close to the decoupling limit [24], we will demonstrate that the strong effective interaction makes the RKKY-induced spin correlations irrelevant. We thus could conclude that the physics is fully dominated by the Kondo effect at $\tilde{K} < 1/2$.

The model.—We use functional integrals in the Matsubara formulation with the imaginary time τ. The bosonized Lagrangian density of HLL [18,22,24,35,40] is

$$\mathcal{L}_{\rm HLL} = [(\partial_\tau \phi)^2 + (u \partial_x \phi)^2]/(2\pi u K); \quad (4)$$

here u is the velocity of bosonic excitations. The electron-KI interaction is described by Lagrangians of the forward or backward scattering,

$$\mathcal{L}_{\rm fs} = i J_z a_{\rm fs}/(\pi u K) \sum_{j=1,2} \delta(x - x_j) S_j^z \partial_\tau \phi; \quad (5)$$

$$\mathcal{L}_{\rm bs} = J_\perp/(2\pi\xi) \sum_{j=1,2} \delta(x - x_j) [S_j^+ e^{-2i a_{\rm bs} \phi} + {\rm c.c.}]. \quad (6)$$

Here x_j are impurity positions with $R = |x_1 - x_2|$, S_j^μ are fields describing KI spin degrees of freedom, and we have introduced auxiliary dimensionless constants $a_{\rm fs,bs}$, which are explained below. Equations (4)–(6) describe the low energy physics; i.e., all fields are smooth on the scale of ξ. In particular, $2k_F$ oscillations (k_F is the Fermi momentum) are eliminated from $\mathcal{L}_{\rm bs}$ by the spin rotation $S_j^\pm e^{\mp 2ik_F x_j} \to S_j^\pm$. We note in passing that, unlike previously studied examples [41–43], features of a single particle density of states and the precise level of the chemical potential are unimportant for the RKKY-Kondo physics, which we explore. This is the peculiarity of the interacting 1D systems described by the bosonization approach [40]. We emphasize that the helicity of our model implies that it has only U(1) spin symmetry, but no SU(2) symmetry. We restrict ourselves to the case of spin-1/2 KIs and choose a parametrization for S fields in terms of Grassmann fields corresponding to Dirac fermions [44,45],

$$S_j^+ = (\bar{d}_j + d_j)\bar{c}_j, \qquad S_j^z = \bar{c}_j c_j - 1/2. \quad (7)$$

Each Grassmann field has the usual dynamical Lagrangian $\mathcal{L}_f = \bar{\psi}_j \partial_\tau \psi_j$, $\psi_j = \{c_j, d_j\}$ [46,47].

In the initial formulation, one chooses $a_{\rm fs,bs} = 1$; however, the gauge transformation of c fermions, $c_j \to c_j \exp[i\lambda\phi(x_j)]$, which is equivalent to the Emery-Kivelson rotation [24,48], allows one to represent the theory in two extreme forms,

$$\text{Representation 1}: \; a_{\rm fs} = 0, \quad a_{bs} = 1 - \kappa; \quad (8)$$

$$\text{Representation 2}: \; a_{\rm fs} = 1 - 1/\kappa, \quad a_{bs} = 0, \tag{9}$$

where $\kappa \equiv \rho_0 J_z/2K$, $\rho_0 = 1/\pi u$.

RKKY phase.—Let us start from Eq. (8) and derive the effective Hamiltonian of KIs from the perturbation theory in $J_\perp$. To this end, we expand $\exp(-\int d\{x,\tau\}\mathcal{L}_{\rm bs})$ up to $O(J_\perp^2)$, integrate over ϕ, and reexponentiate the result. This yields the action which describes spin interactions,

$$\mathcal{S} = -\frac{J_\perp^2}{(2\pi\xi)^2}\sum_{j,j'}\int d\tau_1 d\tau_2 S_j^+(\tau_1)\Pi(\tau_1-\tau_2)S_{j'}^-(\tau_2); \tag{10}$$

Π is governed by the correlation function of the bulk bosons [40,49]

$$\Pi(t) = \left\{\left(\frac{\beta u}{\pi\xi}\right)^2\left[\sin^2(\pi t T) + \sinh^2\left(\frac{x_j - x_{j'}}{L_T}\right)\right]\right\}^{-\tilde{K}}. \tag{11}$$

Here $\beta \equiv 1/T$; $L_T \equiv \beta u/\pi$ is the thermal length. We will consider the macroscopic spatial range $\xi \ll R \ll L_T$. If $1/2 < \tilde{K} \leq 1$, the main contribution to $S_{j,j'}$ results from a small time difference, $T|\tau_1 - \tau_2| \sim |x_j - x_{j'}|/L_T \ll 1$. This allows us to reduce $\mathcal{S}_{j,j'}$ to the local action of RKKY,

$$\mathcal{S}_{\rm RKKY} = -E_{\rm RKKY}\int d\tau[S_1^+(\tau)S_2^-(\tau) + \text{c.c.}]. \tag{12}$$

The terms with $j = j'$ do not contribute to Eq. (12) because $S_j^+(\tau)S_j^-(\tau) \propto (\bar{d}_j + d_j)^2 = 0$. We have introduced in Eq. (12) the RKKY energy

$$E_{\rm RKKY} = \frac{2J_\perp^2}{(2\pi\xi)^2}\int_0^\beta dt\Pi(t). \tag{13}$$

$E_{\rm RKKY}$ can be expressed in terms of the hypergeometric functions. Its asymptotic behavior for $R/L_T \ll 1$ is

$$E_{\rm RKKY} \propto \frac{J_\perp^2}{u\xi}\left(\frac{\Gamma(\frac{1}{2}-\tilde{K})}{\Gamma(1-\tilde{K})}\left(\frac{\xi}{L_T}\right)^\alpha + \frac{\Gamma(\tilde{K}-\frac{1}{2})}{\Gamma(\tilde{K})}\left(\frac{\xi}{R}\right)^\alpha\right); \quad \alpha = 2\tilde{K} - 1. \tag{14}$$

If $1/2 < \tilde{K} < 1$ and $T \to 0$, the first term in Eq. (14) vanishes and the second one reproduces the usual RKKY energy. The failure of the paradigmatic theory starts from $\tilde{K} = 1/2$, where both terms of Eq. (14) are needed to cancel out divergences. Both contributions must be kept also at $\tilde{K} < 1/2$: neglecting the first term leads to nonphysical results, like growth of $E_{\rm RKKY}$ with increasing R (cf. Ref. [31]). However, the first term diverges at $\tilde{K} < 1/2$ in the $T \to 0$ limit. Moreover, the local time approximation used to derive Eq. (12) loses its validity because the UV singularity of Π becomes too weak and the integral is now given by all (not small) time differences $|\tau_1 - \tau_2| < \beta$. All this signals that the physics changes at the point $\tilde{K} = 1/2$ and the RKKY theory cannot be extended to smaller values of $\tilde{K}$.

We emphasize the difference between Eq. (13) and its counterpart for the spinful (almost) SU(2) symmetric Luttinger liquid: in the latter case, Π is a product of the charge and the spin sector contributions $\Pi_{c,s}$ with the Luttinger parameters $K_{c,s}$ [10]. This makes Π more singular at small times. For example, if $J_z = 0$ and the electron interaction is SU(2) symmetric, the exponent $\tilde{K}$ in Eq. (11) reduces to $(K_c + 1)/2 > 1/2$. In this case, the integral in Eq. (13) converges at small times, the theory is local in time and the effective Hamiltonian approach is valid. Therefore, the above described crossover in the behavior of $E_{\rm RKKY}$ is absent and a new phase does not appear in the nonhelical spinful Luttinger liquid. Note that 1D interacting systems driven far from the SU(2) symmetry may possess an emergent helicity with physics being similar to that of our helical model [50].

When the RKKY approach is valid, it is easy to calculate different spin correlation functions, e.g., $G_{zz} = -\hat{T}_\tau\langle\hat{S}_1^z(\tau')\hat{S}_2^z(\tau)\rangle$, by using the effective Hamiltonian $\hat{H}_{\rm RKKY} = -E_{\rm RKKY}(\hat{S}_1^+\hat{S}_2^- + \text{H.c.})$, which corresponds to the local action Eq. (12). Calculations at $T \to 0$ and the analytical continuation to the upper half-plane yield the retarded Green's function

$$G_{zz}^R(\omega) = -\frac{\pi}{2}\frac{|E_{\rm RKKY}|}{\omega_+^2 - (2E_{\rm RKKY})^2}; \qquad \omega_+ \equiv \omega + i0. \tag{15}$$

RKKY-Kondo transition.—To understand the transition to the new phase, let us switch from the perturbation theory to the one-loop RG. We still work with the theory of Eq. (8), where the dimension of the backscattering vertex equals $\tilde{K}$. Thus, the leading in $J_\perp$ RG equation for this coupling constant reads as

$$\partial_l J_\perp = (1 - \tilde{K})J_\perp. \tag{16}$$

Here l is the logarithm of the energy Ω. The difference between RG for one [24] and two impurities is not visible at this level. Moreover, Eq. (16) looks precisely like RG for the backscattering amplitude of the static impurities [51,52], though with renormalized K. These two analogies are not accurate because the renormalization of $J_\perp$ stops quickly.

Let us find the RG cutoff by adapting the scattering approach of Refs. [53,54] to the problem we study. The main idea of that approach is to consider the weak electron interaction and to find logarithmic corrections to Green's function of the backscattered electron, $G_{\rm bs} = -\hat{T}_\tau\langle\hat{\psi}_L(\tau_f, x_f)\hat{\psi}_R^\dagger(\tau_i, x_i)\rangle$. Here $x_{i,f} \to -\infty$, $\hat{\psi}_R^\dagger$ ($\hat{\psi}_L$) is the creation operator for the right- (the annihilation operator for left-) moving fermion. The leading correction to backscattering caused by the static impurity appears in the first order in the interaction, $\delta G_{\rm bs} \sim (1-K)\log(D/\Omega)$, $\Omega > T$.

Now we recall that backscattering in HLL is caused by KI and requires spin flip. Hence, Green's function describing backscattering must account for changing the spin state. Formally, one has to add the spin operator in the definition of Green's function,

$$G_{\rm bs}^{(\rm KI)} = -\hat{T}_\tau \langle \hat{S}^-(\tau_f)\hat{\psi}_L(\tau_f, x_f)\hat{\psi}_R^\dagger(\tau_i, x_i)\rangle; \qquad (17)$$

the impurity number is omitted here. The leading in $(1-K)$ and $J_{\perp,z}$ correction to $G_{\rm bs}^{(\rm KI)}$, $\delta G_{\rm bs}^{(\rm KI)}$, is given by the diagram shown in Fig. 2. The difference between $\delta G_{\rm bs}$ and $\delta G_{\rm bs}^{(\rm KI)}$ is due to the spin propagator $G_{-+} = -\hat{T}_\tau\langle \hat{S}^-(\tau_f)\hat{S}^+(\tau_1)\rangle$. Using the parametrization Eq. (7), we obtain $G_{-+} = -2\langle c(\tau_f)\bar{c}(\tau_1)\rangle\langle d(\tau_f)\bar{d}(\tau_1)\rangle$; d fields have the bare Lagrangian $\mathcal{L}_f[\bar{d}, d]$. Because of the interimpurity correlations, the spin flip of one KI costs the energy of the gap $E_{\rm RKKY}$, which can be qualitatively described by adding the mass term to the Lagrangian of c fields: $\mathcal{L}_f[\bar{c}, c] \to \mathcal{L}_f[\bar{c}, c] + E_{\rm RKKY}\bar{c}c$. This yields

$$G_{-+} = 2\theta[(\tau_f - \tau')E_{\rm RKKY}]e^{-(\tau_f-\tau')E_{\rm RKKY}}, \qquad (18)$$

with the step function $\theta(x \geq 0) = 1$. G_{-+} changes the cutoff of the logarithm from Ω to $\max[\Omega, E_{\rm RKKY}]$.

The one-loop RG comes from resummation of the leading logarithms. Therefore, we conclude that RKKY correlations change the scale at which the RG flow stops, from a self-consistently obtained scale $E_{\rm sc}$, which marks the strong coupling limit of the RG flow, to $\max[E_{\rm sc}, E_{\rm RKKY}]$. According to Eq. (16), $E_{\rm sc}$ coincides with T_K in the second line of Eq. (2) with $\tilde{K}$ being substituted for K. The crossover occurs at $T_K(\tilde{K}) \sim E_{\rm RKKY}$, which obviously means the transition between RKKY and Kondo physics at

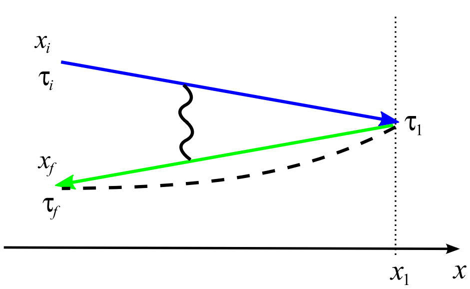

FIG. 2. The diagram that yields the leading in interaction (wavy line) and in the coupling constants $J_{\perp,z}$ correction to Green's function describing backscattering of a helical electron by a Kondo impurity. KI is located at the position x_1. Solid lines show the electron propagators before (blue) and after (green) backscattering. The dashed line denotes the spin propagator G_{-+}, which stops logarithmic divergences of the theory of Refs. [53,54] at $E_{\rm RKKY}$.

$$\tilde{K} = 1/2. \qquad (19)$$

This explains failure of the paradigmatic theory for RKKY when $\tilde{K} < 1/2$. The RKKY-Kondo transition is illustrated by the phase diagram in Fig. 1. We have restricted axes to the relevant range of K and $|\rho_0 J_z| < 1$ and have excluded the extremely strong coupling and the second critical line from this figure. The phase diagram of Fig. 1 is different from that for the single KI [24]: the border between two phases is defined by Eq. (19) for two KIs and by $\tilde{K} = 1$ for the single KI.

Kondo phase.—Two impurities coupled to HLL is not the exactly solvable model; therefore, one cannot say much about the Kondo phase without numerics. One possibility for analytics is provided by the vicinity of the so-called decoupling limit [24], which can be conveniently analyzed by using Eq. (9) with $|a_{\rm fs}| \ll 1$ [55]. In this case, the spin Green's function G_{zz} can be calculated perturbatively in $(\rho_0 J_z a_{\rm fs})$ and exactly in $J_\perp$. Similar to Eq. (15), we do the analytic continuation to the upper half-plane at $T \to 0$ and find G_{zz}^R near the decoupling limit

$$G_{zz}^R(\omega) \simeq i\left(\frac{\pi}{2}\right)^3(\rho_0 J_z a_{\rm fs})^2\left(\frac{\Omega_\perp}{\omega_+^2 - \Omega_\perp^2}\right)^2 \frac{\omega}{K} e^{i\frac{R\omega_+}{u}}, \qquad (20)$$

with $\Omega_\perp \equiv J_\perp/2\pi\xi$.

The difference between two phases becomes obvious after comparing the frequency dependence of G_{zz}^R in Eqs. (15) and (20). In the Hamiltonian description of the RKKY phase, there is no retardation and G_{zz}^R becomes constant at $|\omega| \ll E_{\rm RKKY}$. This reflects the RKKY-induced interimpurity correlation. The retardation is present in Eq. (20) (note the oscillating exponential) and, much more importantly, G_{zz}^R decays as $\omega/\Omega_\perp$ at $|\omega| \ll \Omega_\perp$. This decay shows the absence of the noticeable interimpurity correlation near the decoupling limit. When $\omega \to 0$, i.e., the observation time goes to infinity, (almost) uncorrelated dynamics of two KIs leads to the suppression of G_{zz}^R. If the interimpurity correlation is weak, we can make use of the RG for the single KI, which shows the flow toward the decoupling limit where KIs are not correlated and only the Kondo-like backscattering remains relevant [24].

All these observations confirm that the Kondo physics fully dominates at $\tilde{K} < 1/2$.

Summary.—We have shown that the paradigmatic RKKY theory is not applicable if the indirect exchange interaction of two spin-1/2 Kondo impurities is mediated by strongly correlated helical electrons with the effective Luttinger parameter $\tilde{K} < 1/2$ [Eq. (3)]. The physical reason for this counterintuitive finding is the competition between RKKY-induced spin correlations and Kondo screening of localized spins. This competition is crucially intensified by helicity. Phenomenological arguments combined with the perturbation theory and with a scaling analysis of the one-loop renormalization group have allowed us to identify a border between phases where either the RKKY or the Kondo physics dominates (Fig. 1). These phases emerge when the (effective) electron interaction is weak or strong, respectively.

We have encountered an instructive example of the interacting system where the usual description of a subsystem in terms of an effective Hamiltonian is impossible due to helicity and strong interaction. Physical situations where the effective Hamiltonian of a subsystem cannot be constructed put forward a conceptual problem of treating such strongly correlated systems.

Our results give new insight into the fundamental phenomenon of the RKKY-Kondo competition. In particular, they indicate that the Doniach phase diagram [56] can be very nontrivial in systems with spin-orbit interaction. This famous diagram describes a crossover of a Kondo lattice between magnetically ordered phases and phases of heavy fermion Fermi liquid, which are dominated by correlations between local magnetic moments and by the Kondo screening, respectively.

Our predictions may serve as a basis for describing an influence of a rare Kondo array on transport in helical systems. Measurements of Ref. [57] suggest that HLL on the edges of 2D topological insulators made of InAs/GaSb can have a really small Luttinger parameter, $K \sim 0.2 < 1/2$. We thus expect that our predictions are relevant for the experimental studies of the topological insulators. Another possible platform, where the unusual RKKY-Kondo competition can be detected, is provided by recently fabricated 1D wires with interaction-induced helicity [58–61]. Further development of the theory may include a detailed study of a vicinity of the transition and an extension to the case of larger spins.

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