The nature of stochastic transport of non interacting atoms in adiabatically driven optical lattices

Das Verhalten von stochastischem Transport von nicht wechselwirkenden Atomen in adiabatisch getriebenen optischen Gittern

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Abstract

This Bachelor thesis looks at cold gases in optical lattices and the transport behavior of particles in the adiabatic limit of an applied driving force.

The aim of this Bachelor thesis is to find out whether particles for a system of an optical lattice move rather diffuse or ballistic and to test numerically how the system behaves in the adiabatic limit. It is expected and later confirmed that in the adiabatic limit the diffusive properties decrease slightly while ballistic motion increases.

Contents

Chapter 1

Introduction

1.1 Cold gases as a perfect laboratory to study fundamental properties of solid-state systems

The study of fundamental phenomena of crystalline solids is quite hard. This is caused by lattice defects, thermal vibrations and inter-particle-interactions which induce scattering of electrons. Lattice defects interrupt the perfect structure of a solid because samples are rarely pure. So the properties of the solid are also impurified. They can be one-dimensional like missing or additional atom or two-dimensional like interrupted atom rows. Also the surfaces have different properties since the surface atoms are not totally surround by other atoms. Interactions between the particles produce its properties like mechanical (hardness, elasticity), electrical (resistivity), thermal properties and so on. All these phenomena make the system of a solid very complicated and since one cannot control nearly all of them, the fundamental properties of (crystalline) solids are hard to study. [1, 2]

This problem of low controlability in solids is the foundation of the interest on cold gases. When cooling down gases to ultra cold temperatures and applying an optical standing wave one can create a system similar to the one of a crystalline solid by having one particle in each well. This recreates a crystalline solid with one particle per lattice point. Systems of cold gases in optical lattices have the advantage of high controlability (examples later) and purity. [3, 4]
In the following section 1.2 the tools to realize such a lattice from a cold gas are described.

1.2 Tools for realization of a cold gases lattice

1.2.1 Cooling

At first the gas needs to be cooled down. The most popular way to do so is to use lasers: There are many ways of using lasers to cool gases down. The most common version of **laser cooling** is **Doppler cooling**. In Doppler cooling, a gas of particles is exposed to laser beams from all (six (three degrees of freedom times two sides)) sites. These laser-beams all have the same frequency. When a particle in the gas has a non-zero velocity it sees different frequencies in the lasers caused by the Doppler effect. The photons coming from the direction the particle is heading to, have a higher frequency in the particle's system compared to the photons following the particle. The photons with the higher frequency give a stronger impulse to the particle ($\Delta p = \hbar k$). This raises the particle into an higher state and acts in the opposing direction of the motion of the particle. The particle eventually falls down to its original state and gets an impulse due to the emitted photon. This is at first (in average) no problem since the direction of the emitted photon is evenly distributed and so averaged over many particles it goes to zero. But this emitted impulse limits the cooling to

$$k_b T_{min} = \frac{\hbar\Gamma}{2} \tag{1.1}$$

where Γ is the line width. This limits Doppler cooling for the example of Sodium (Na) to $T_{min} = 240\mu k$. [5]

For even lower temperatures there was a new method developed which was called **Sisyphus cooling**. In Sisyphus cooling there are two optical potentials. The one consists of a σ_- polarized laser on the one side and a σ_+ polarized laser couterpropagating on the other side. In between the lasers the potential is alternating (σ^+, linear polarized (0°), σ^-, linear polarized (90°),...) over the axis of the lasers. Depending on the position of the particle, a photon (σ^+, σ^- *or linear*) can act on the particle. If that happens it jumps into the exited state ($J_g = \frac{1}{2} \to J_e = \frac{3}{2}$ for the example in fig. 1.1 a) and its angular momentum M_J changes according to the polarization of the photon ($\Delta M_{J,\sigma^+} = +1$; $\Delta M_{J,lin} = 0$; $\Delta M_{J,\sigma^-} = -1$). The other optical potential consists of two counter-propagating, linear polarized, orthogonal lasers. This creates a cosine-like potential for the particles. Depending on the ground-state the particle is in ($M_J = +\frac{1}{2}$ *or* $M_J = -\frac{1}{2}$) it sees opposing potentials (See fig. 1.1 b). This is where the cooling happens. The particles are exited into the higher state when they are at the top of one potential and then they drop down to the other potential ($-M_{J\ initial}$) which is in a minimum at that point. Hence the name from the Greek mythology Sisyphus who has to push a rock up a mountain which then just rolls down again. This method has a limit of

$$k_B T \propto E_R = \frac{(\hbar k)^2}{2m} \tag{1.2}$$

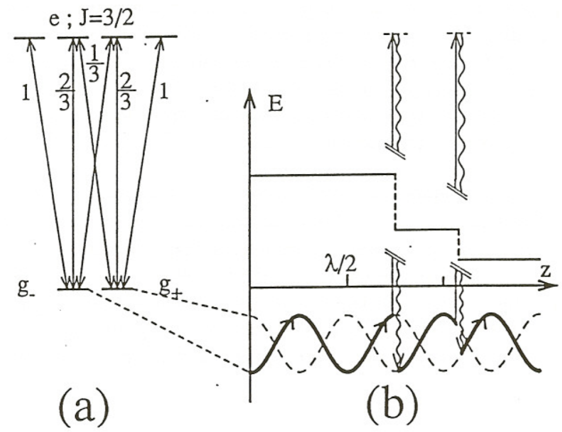


Figure 1.1: Sisyphus 1:
a) particle in the ground state is exited according to the absorbed photon
b) potential energy according to groundstate [23]

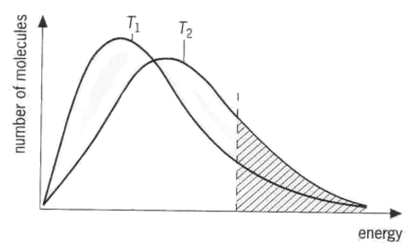


Figure 1.2: Evaporative cooling [24]

which is for Sodium (Na) $T_{min} \approx 400nk$ which is about 600 times smaller than the Doppler-cooling limit. [6]

In **Evaporative cooling** the most energetic molecules are taken away and so the average energy decreases. This has basically no limit in temperature. The only limiting part is the number of molecules, as it decreases. When repeatedly the hottest molecules are taken away in a too short time there are only a little number of molecules left.

By such low temperatures quantum phenomena get important and phenomena like Bose-Einstein-Condensation (BEC) which also entail phenomena like superconductors, superfluids, or fractional quantum Hall liquids appear.

1.2.2 Controlling interactions

A further way to control an attribute of the system is the Feshbach resonance. The Feshbach resonance allows one to change the scattering length over the average interparticle spacing in ultra-cold, dilute gases. By that the strong-interacting regime is reached. In a two-particle collision, for example, Feshbach resonance occurs when one particle is in a bound state in a closed channel and it is resonantly coupled with the scattering continuum of an open channel particle. These particles are described by an effective pseudo-potential. After the collision the particles are temporarily held in a quasi-bound state. The scattering length is easily tunable by the magnetic field as shown in the following equation:

$$a(B) = a_{bg} \left(1 - \frac{\Delta B}{B - B_0} \right) \tag{1.3}$$

where $a(B)$ is the scattering length according to the magnetic field B, a_{bg} is the (off-resonant) background scattering length, ΔB is the width of the resonance and B_0 is the position of the resonance. With the Feshbach resonance the interaction between particles is well controllable. [4]

1.2.3 Examples of quantum phenomena

As the gas is cooled down to very cold temperature quantum-phenomena are important. **BCS-BEC Crossover**: In the BCS (Bardeen, Cooper, Schrieffer) theory two electrons (or other fermions) are weakly attracted and form so called Cooper-pairs. These Cooper-pairs then gain some bosonic properties and can make a superconductor. These fermions in the BCS-theory pair in the momentum-space while the particles in the BEC pair tightly in (real) position space. This interaction distance can also be varied by Feshbach resonance. However, BCS-superfluidity is not BEC of Cooper-pairs as the pairs do not obey Bose-Einstein-statistics. Even so in the high density limit the BCS-state becomes a condensate of pairs which may be even smaller than the lattice-distance and should be described by BE-statistics. The BCS-formalism and its ansatz are, besides the condensate of Cooper-pairs, useful for a BE-condensate of dilute gases of tightly bound pairs. [4, 7]
Superconductivity: The resistivity of normal metals decreases with decreasing temperature and saturates at a value $\rho > 0$ for $T \to 0$ as the electrons still scatter. Superconductors abruptly fall to the state with $\rho = 0$ below a critical temperature $T_c > 0$. [8] The critical temperature T_c of elementary metals and their compounds is normally not bigger than a few kelvin. This transformation is a phase transition. In superconductors there is a current which can flow without dissipation. But they also expulse a (constant) magnetic field which is negative proportional to the rotation of the current density (j). $\vec{\nabla} \times \vec{\jmath}_s \propto -\vec{B}$. This is called the Meissner effect.
Something similar is seen with superfluidity. Instead of a vanishing resistivity their viscosity vanishes at very low temperatures if the velocity is smaller than a critical value. The most astonishing thing about superfluids might be that they can overcome potential barriers to end up in a lower potential height. The best known example of this is the cup with a superfluid where the superfluid runs up the walls to exit the cup.

1.2.4 Optical lattices

When the gas is cooled down it can be formed by an optical lattice. This gives the gas a crystal-like form. The optical lattice creates a dipole force which acts on the (lattice) particles in the following manner:

$$\vec{F} = \frac{1}{2}\alpha(\omega_L)\vec{\nabla}[|\vec{E}(\vec{r})|^2] \tag{1.4}$$

where α is the polarizability and since the motion of the atom is slower then the light-frequency ω_L, $\vec{E}$ is averaged and squared so that only the intensity $I \propto |\vec{E}(\vec{r})|^2$ has an impact. In general the optical lattice is formed by a standing wave. This atom lattice is very controllable as you can change the intermolecular distance by varying the lights wavelength and if you are close to the atoms resonance you can control whether the atoms are attracted to the nodes or antinodes. For more information about this see [4].

1.2.5 Electron motion in a lattice

In this artificial lattice electrons (or other particles) can move around. This motion of electrons is described by the Bloch-wave and other theories.

Bloch waves When solving the single-electron Hamiltonian ($H = -\frac{\hbar^2\nabla^2}{2m} + U(\vec{r})$) one finds the wave-function for the Bloch wave:

$$\Psi_{n\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \cdot u_{n\vec{k}}(\vec{r}) \tag{1.5}$$

This consists of a plane wave function ($e^{i\vec{k}\cdot\vec{r}}$) with the wave-vector $\vec{k}$ and a function ($u_{n\vec{k}}(\vec{r})$) with the same periodicity of the Bravais lattice ($\vec{R}$). In a Bloch-wave there are no collisions and thus no scattering when the lattice is perfectly periodic ($U(\vec{r}+\vec{R}) = U(\vec{r})$).
While the Bloch wave is a quantum-mechanical theory one can use the simpler and more transparent semiclassical approach instead. The semiclassical model also looks only at the movement of electrons in a general periodic potential between collisions. The Bloch theory the semiclassical theory has no interband transitions and the evolution of the position and wave vector in a band are determined by specific equations of motion (see [2]).

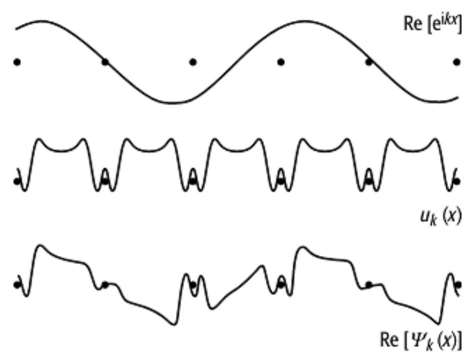


Figure 1.3: Bloch wave [25]

1.3 stochastical transport

Classical transport When one goes from the single-particle perspective to the more general stochastic perspective, different variables are needed to describe the system. For the case of gases the stochastic motion is described by the Fokker-Planck-Kolmogorov equations (FPK). This can later be generalized for a periodic potential.
The **Fokker-Planck-Kolmogorov equations (FPK)** uses two terms: $\langle \Delta x \rangle$ and $\langle (\Delta x)^2 \rangle$. With the Kolmogorov conditions it can be said that $\langle (\Delta x)^m \rangle = 0$ for $m > 2$. Where $\langle ... \rangle$ means averaging over the initial conditions. In a gas $\langle \Delta x \rangle$ can be interpreted as the movement of the center of mass. While $\langle (\Delta x)^2 \rangle$ is a measure for the width of the Gaussian distribution of the gas. In normal diffusion one can find

$$\langle (\Delta x)^2 \rangle = D \cdot t^\mu \tag{1.6}$$

with $\mu = 1$ *and* $D = const.$ while it can also happen that $\mu \neq 1$ but $0 < \mu < 2$. $\mu < 1$ is then called sup-diffusion and $\mu > 1$ is called super-diffusion or anomalous transport. For derivation of this see [9, 10].
For the system in this thesis the standard deviation $\langle (\Delta x)^2 \rangle$ will be fitted in the manner of $f(t) = D \cdot t + v^2 \cdot t^2$. This polynom is an overlap of $D \cdot t$ which can be interpreted as normal diffusion and $v^2 \cdot t^2$ which can be interpreted as ballistic motion.

Symmetrical potentials/ Example for FPK in electron dynamics The FPK theory can be nicely applied to the systems of Renzoni et al. [11, 12, 13, 14]. In [11] periodic ($U(x) = U(x + \lambda)$) symmetric ($U(-x) = U(x)$) potential with a symmetric ($F(t + \frac{T}{2} = -F(t)$) periodic ($F(t) = F(t+T)$) driving forces are introduced. If the potential and the driving force have the mentioned properties there is no directed diffusion (DD) ($\neg DD \rightarrow \langle \Delta x \rangle = 0$). According to [11] for DD to happen a symmetry has to be broken.[9, 10, 15]
In experiments this can be achieved by applying two non-monochromatic light beams on a gas of atoms. In [11], for example, two orthogonal, counter-propagating lasers with the frequencies ω and 2ω are applied on a gas of atoms. This breaks the symmetry of the driving force. By then varying the phase difference ϕ Renzoni et al. get DD for $\phi \neq n \cdot \pi$; $n \in N$. The interesting fact in this system is that even so the forces are zero-mean ac forces the system has DD caused by the broken symmetry.
In [12] a similar system is used to realize a kind of Brownian motor. The time-symmetry is broken as well. It is shown that noise affects the DD in a resonance like fashion.
In both examples FPK can be applied to describe the dynamics of the system.

1.4 Outline of the thesis and statement of the problem

In this thesis stochastic transport of electrons in a lattice of a cold gas is discussed. This is simulated by a (mathematical) model of a driven mathematical pendulum. The specific question is, how does the nature of the stochastic transport change in the adiabatic limit of the frequency of the driving force $\omega_f \rightarrow 0$. Similar systems are already discussed in [15, 16, 17] and it has been suggested in [16] that as $\omega_f \rightarrow 0$ the diffusion const D decreases as ballistic motion (v^2) increases. To prove this numerically is the main objective of this thesis.
The rest of the thesis is organized as follows: a theoretical chapter describes dynamical chaos. It will give a short summary of the origin of chaos and its most important properties. This is important to understand the behavior of a single particle in the system. The chapter about chaos also shows the effect of the unlimited growth of the upper energy of the chaotic layer as the frequency of the driving force $\omega_f \rightarrow 0$. This is unique for the physical significant systems. The transport coefficients are shifted towards ballistic motion as an AC driving force is present. Especially for low frequencies ω_f.
The higher energy boarder of the chaotic layer can be used to increase the rate of threshold devices. A threshold device resets to an initial state when it overcomes a specific threshold. If there is a driving force it is easier for the electron to overcome the threshold.
After the part about chaos and the behavior of a single particle in the system follows the presentation of the results and the conclusion.

Chapter 2

Theoretical background

2.1 Chaos

In a chaotic system the trajectories mix in the phase space. That means their trajectories are not laminar as in an ordered system. Chaotic systems are also sensitive to small changes in initial conditions. This means that the difference between two trajectories grows exponentially over time.
In general exists chaos only in a subspace of the phase space. Regions of chaos and order are separated. These are then called stochastic sea and non-chaotic islands (more in sec. 2.1.1).
Chaos generally develops around separatices. The separatrix is the border between the two regions in a phase space map (of a differential equation) in which the particle has different behaviors of motion (example fig 2.1). As for example oscillating and ballistic motion in a cosine-potential.

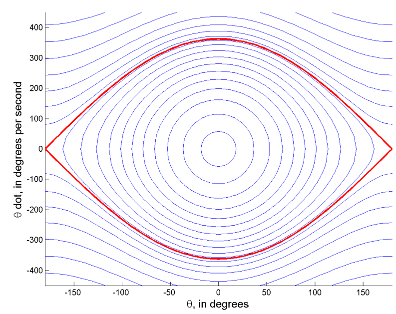


Figure 2.1: Separatrix of harmonic oscillator [26]

This chaotic layer first grows until its at its borders and then fill the space between the borders up.
For chaos to appear there has to be a non-linearity and more than one degree of freedom. Non-linearity is needed as 1^{st} order ordinary differential equation have a unique solution and thus no chaos. More degrees of freedom are needed so there are too many variables of motion for the integrals of motion to get a unique solution (more in section 2.2).
The Hamiltonian $H(I,\theta,t)$ of a system can be splitted into a linear part $H_0(I,\theta)$ and a non-linear part $H_1(I,\theta,t)$. There are some standard models for chaotic dynamics. Two examples will be quickly introduced now. The so called Web-map or kicked Oscillator is a discrete model. Its Hamiltonian $H = \underbrace{\frac{1}{2}(p^2+\omega_0^2x^2)}_{H_0(p,x)} - \underbrace{\frac{\omega_0 k}{T}cos(x)\cdot\sum_{n=-\infty}^{\infty}\delta\left(\frac{t}{T}-n\right)}_{H_1(p,x,t)}$ shows $1^1/_2$ degrees of freedom as a dependency on time can be counted as an additional half degree of freedom. The model also shows non-linearity as $cos(x)$ is in the same term as the time. So the requirements for chaos are given. The δ-function gives regular kicks to the system. So the variable p jumps every $\Delta t = T$.
The other model comes quite close to the system used later in this thesis. It is the pertubed Pendulum. Its Hamiltonian $H = \underbrace{\frac{1}{2}(p^2-\omega_0^2cos(x))}_{H_0(p,x)} - \underbrace{\varepsilon\omega_0^2cos(kx-\nu t)}_{H_1(p,x,t)}$ also shows $1^1/_2$ degrees of freedom as it is

time dependent and the non-linearity is in the last term where x and t are used with a cos-function. This time the model is continues. In this model the stochastic motion develops in the chaotic layer around the separatrix of H_0 for $E = \frac{1}{2}\omega_0^2$.
There are lots of different methods trying to describe chaos as for example the Poincaré map which shows stochastic seas and non-chaotic islands (for more see next subsection 2.1.1). Even so there are lots of different analytic and graphic methods they are only good for two or less degrees of freedom. [10, 18]

2.1.1 Poincaré map: stochastic seas and non chaotic islands

A Poincaré section is a phase space diagram where points $(q(t_n), p(t_n))$ are set at specific times t_n. In these maps one can spot regions of chaos called statistical seas and regions of non-chaotic islands where non chaotic trajectories lie. On a closer look one can also see lakes of chaos inside the islands. All these regions are separated and don't mix. In fig. 2.2 one can very well see the islands in the chaotic sea and their separation. [10, 19]
The equations of motion of a chaotic system are also derived from a Hamiltonian. The following section will show why more than one degree of freedom is needed for chaos.

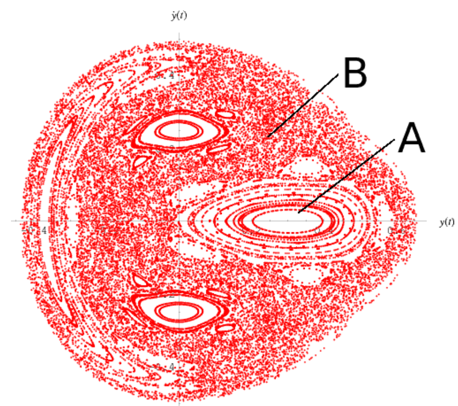


Figure 2.2: Poincaré map - Hénon and Heiles Problem
A non-chaotic island; B statistical sea [27]

2.2 Hamiltonian systems with 1½ degrees of freedom

As it is well known, Hamiltonian systems (here only one degree of freedom) can be described like this:

$$H(q, p, t) = T(p) - U(q, t) \tag{2.1}$$

$$\dot{p} = -\frac{\partial H}{\partial q}, \qquad \dot{q} = \frac{\partial H}{\partial p}, \tag{2.2}$$

In this case there is only one degree of freedom which means that there is a unique solution since there are enough integrals of motion, as the Energy $E = H(q, p)$ or conservation of angular momentum etc., since (example: energy):

$$dH = -\dot{p}dq + \dot{q}dp + \underbrace{\left(\frac{\partial H}{\partial t}dt\right)}_{\substack{=0\ if\ no\ explicit \\ timedependence}} \Rightarrow \frac{dH}{dt} = 0 \Rightarrow H(q, p) = E = const. \tag{2.3}$$

So for the easy example of a one dimensional system without time-dependence, there is only energy as integral of motion. So one can express the momentum in terms of the energy (constant) and the position:

$$H(q, p) = E = const. \ \rightarrow \ p = p(E, q) \tag{2.4}$$

With given initial values this gives an unique trajectory.
But when you have explicit time-dependence (or another degree of freedom and non-linearity) in your Hamiltonian H(q,p,t), the energy is not conserved anymore so you loose it as an integral of motion

and you gain the time t as another variable:

$$\frac{dH}{dt} = \underbrace{\frac{\partial H}{\partial q}}_{-\dot{p}} \frac{\partial q}{\partial t} + \underbrace{\frac{\partial H}{\partial p}}_{\dot{q}} \frac{\partial p}{\partial t} + \frac{\partial H}{\partial t} = \frac{\partial H}{\partial t} \rightarrow H \neq const. \rightarrow E\ is\ no\ integral\ of\ motion \tag{2.5}$$

Then there is a trick to extract an integral of motion: let the time and the original hamiltonian be a new ($1/2$) degree of freedom and create a new Hamiltonian $\tilde{H}$, like:

$$\tilde{q} := t; \quad \tilde{p} := -H; \quad \tilde{H} := H(q, p, \tilde{q}) + \tilde{p} \tag{2.6}$$

with

$$\dot{t} = \dot{\tilde{q}} = \frac{\partial \tilde{H}}{\partial \tilde{p}} = 0 + \frac{dt}{dt} = 1 \checkmark \ and\ \dot{\tilde{p}} = -\frac{\partial \tilde{H}}{\partial \tilde{q}} = \frac{\partial H}{\partial t} \checkmark \tag{2.7}$$

with this we get the new Energy $\tilde{E}$ as an integral of motion:

$$\frac{d\tilde{H}}{dt} = \underbrace{\frac{\partial \tilde{H}}{\partial q}\frac{\partial q}{\partial t} + \frac{\partial \tilde{H}}{\partial p}\frac{\partial p}{\partial t}}_{0} + \underbrace{\frac{\partial \tilde{H}}{\partial \tilde{q}}\frac{\partial \tilde{q}}{\partial t} + \frac{\partial \tilde{H}}{\partial \tilde{p}}\frac{\partial \tilde{p}}{\partial t}}_{0} + \underbrace{\frac{\partial \tilde{H}}{\partial t}}_{0} = 0 \rightarrow \tilde{H}(t) = const.\ over\ time \tag{2.8}$$

So for one and a half degrees of freedom and three variables (q,p,t) there is only one integral of motion. This gives the problem that there is no unique solution for the equations of motion. This is the origin of the chaos in such systems. [10, 19, 21]

2.3 Specific system

The systems Hamiltonian used in this bachelor thesis looks as follows:

$$H(q, p, t) = \frac{p^2}{2m} - \omega_0^2 \cdot cos(q) + qh\omega_f^2 \cdot sin(\omega_f t) \tag{2.9}$$

Where H is the hamiltonian, q the position, p the momentum, t the time, m the mass, ω_0 the period of the potential, h a parameter and ω_f is the frequency of the light or driving force. For simplicity it has only one degree of freedom and the mass $m = 1$ which later implies that the linear momentum is equal to the velocity $p = \dot{q}(= v)$

Elements of hamiltonian This hamiltonian has three terms. The kinetic energy ($T = \frac{p^2}{2m}$), the oscillator potential energy ($U_0 = \omega_0^2 \cdot cos(q_0)$) and the timedependent potential energy ($U_f = qh\omega_0^2 \cdot sin(\omega_f t)$). The kinetic energy T is quite standard linear type. The oscillator potential energy U_0 is also quite trivial. This gives a local periodic potential $U_0(q) = U_0(q + \lambda)$ of a mathematical pendulum and is symmetric. Last but not least there is the timedependent potential energy U_f. This part gives the additional half degree of freedom and the non-linearity to originate chaos.

Both the static potential and the timedependent driving force are symmetric and periodic. So according to [11] there should be no directed diffusion (DD) (see 1.3 for summary).

By using the equations for the hamilton you get the following equations of motion:

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{m} \xrightarrow{m=1} \dot{q} = p \tag{2.10}$$

$$\dot{p} = -\frac{\partial H}{\partial q} = -\omega_0^2 \cdot sin(q) - h\omega_0^2 \cdot sin(\omega_f t) \tag{2.11}$$

Putting these two equations together you get to

$$\ddot{q} = -\omega_0^2 \cdot sin(q) - h\omega_0^2 \cdot sin(\omega_f t) \tag{2.12}$$

This equation is solved numerically (with symplectic integrator) and the results are presented later on.

Discussion of single trajectory

short range Generally the particles move close to the separatrix ($\rightarrow q(t_0) \approx \pi,\ p(t_0) = 0$). The parameter h is set $h \ll 1$ so the timedependent potential is weak compared to the oscillator. In general the timedependent potential U_f is slower than a particle oscillating in a well of the static potential U_0. So the potential can be seen as a wave potential with a slight overall slope of $h\omega_0^2 \cdot sin(\omega_f t)$. This is well visualized in fig. 2.3. There the particle starts with two oscillations. This can be explained by the proximity to the border of the chaotic layer. The speed is changing periodically due to the periodic potential. After these two oscillations ($t \approx 400$) the particle overcomes the barrier and starts ballistic motion. The direction can be explained by the driving force (U_f) which acts to the positive direction. The change in direction soon after ($t \approx 700$) can also be explained by the driving force (U_f) which has now changed its direction. This change in driving force direction happens every $\Delta t = 100 \cdot \pi$. Later on ($t \approx 1200, t \approx 1800$) the particle remains in a well for one oscillation. Weather or not the particle does this or probably even changes its direction is chaotic. There sometimes are ballistic segments which have neither of the above two phenomena for a longer period of time. Also good to see is the proximity to the separatrix since the particle nearly reaches 0 speed each time it is at top of a potential hill during its ballistic motion into one direction. For a theoretical discussion about the movement of the particle in this system see [15]

medium range While in the detailed view of the trajectory there is some typical way of moving in the medium-range movement it is quite chaotic. As one can see in fig 2.4 in medium-range-movement one might only guess two kinds of movement: the ballistic like movement, this is when most of the displacement is made, and the diffusional movement.

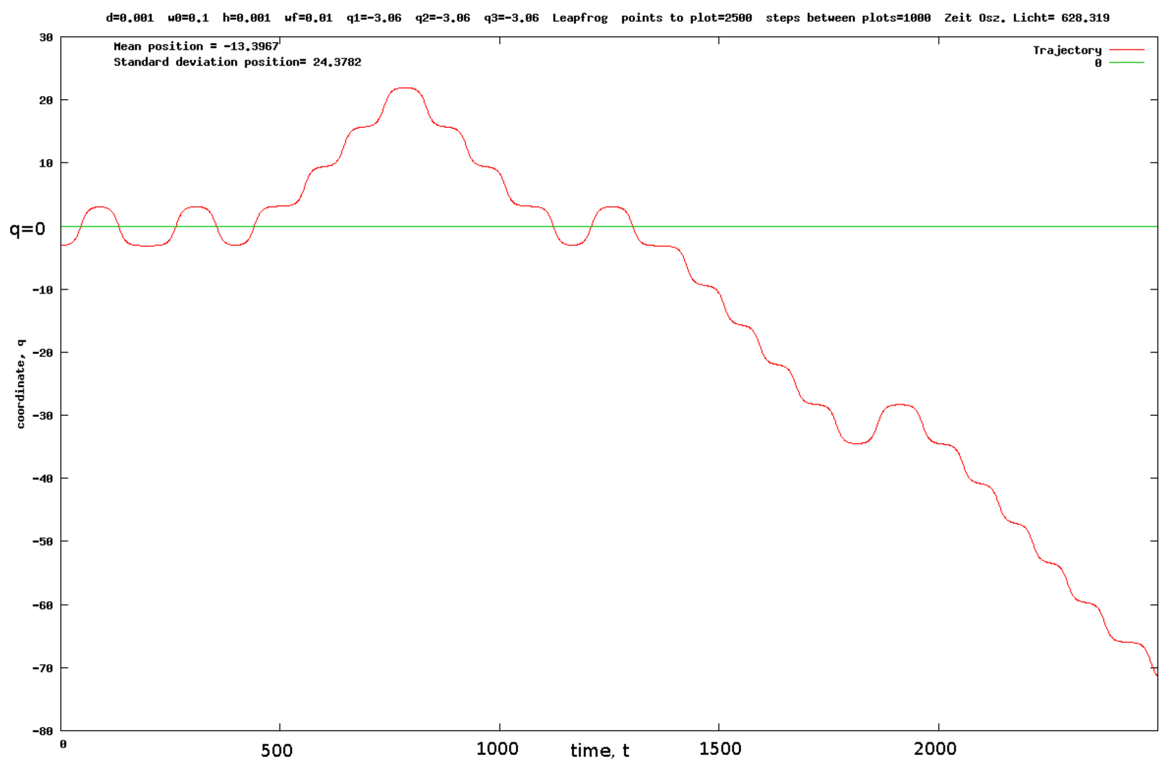
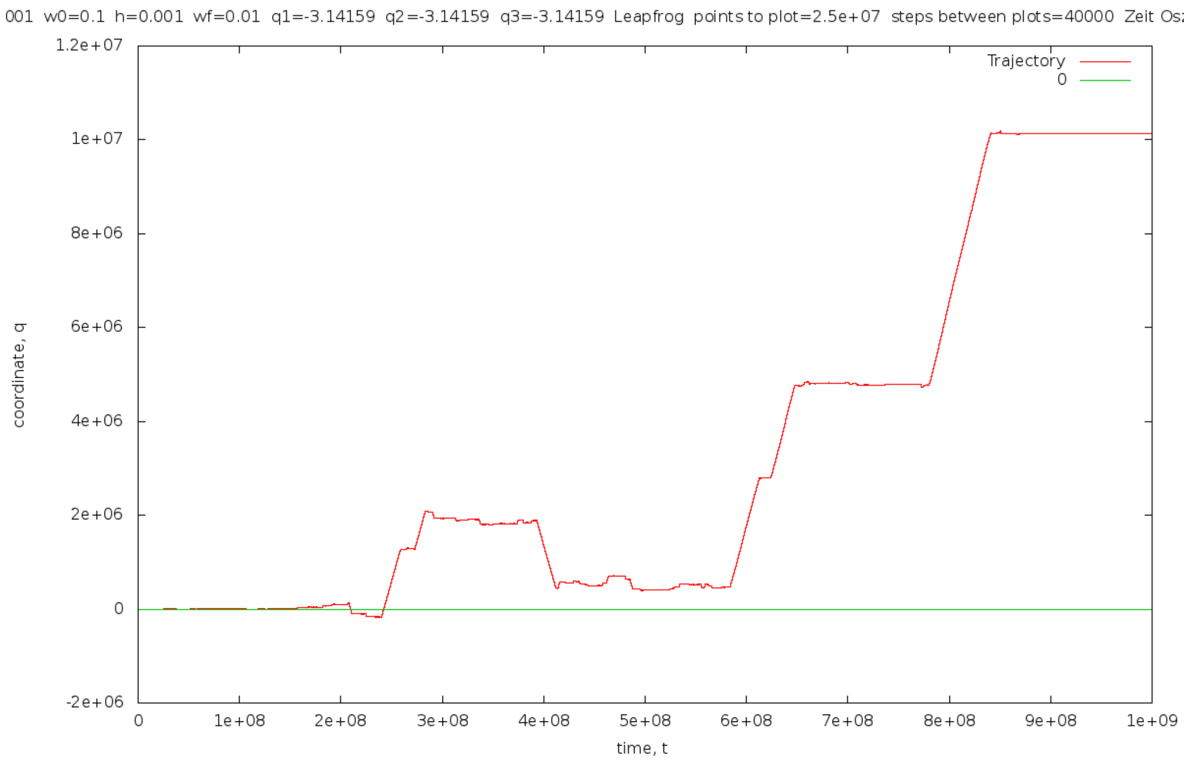


Figure 2.3: detailed view of a trajectory
$\omega_0 = 0.1;\ \omega_f = 0.01;\ q_0 = -3.06 = -\pi + 0.081;\ \Delta t = 0.001$
time-coordinate-diagram

Figure 2.4: some Trajectory



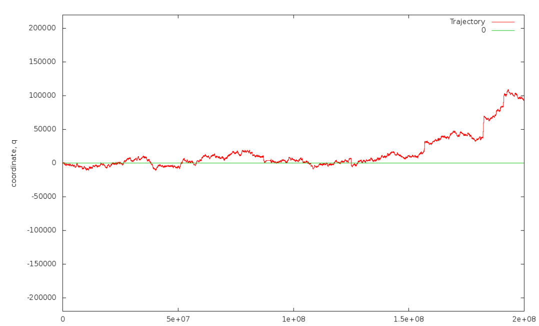


Figure 2.5: View of start diffusional motion
abscissa $(0 < t < 2 \cdot 10^8)$
ordinate $(-2.2 \cdot 10^5 < q < 2.2 \cdot 10^5)$

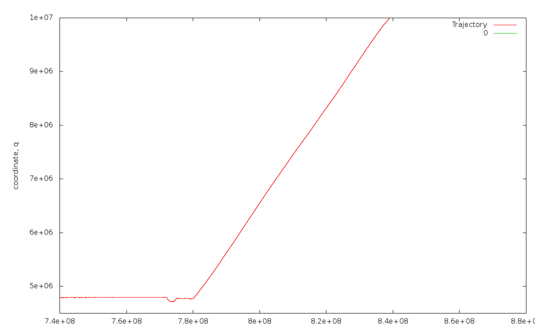


Figure 2.6: View of ballistic motion
abscissa $(7.4 \cdot 10^8 < t < 8.8 \cdot 10^8)$
ordinate $(4.5 \cdot 10^6 < q < 10^7)$

Chapter 3

Simulation/ (Presentation of) results

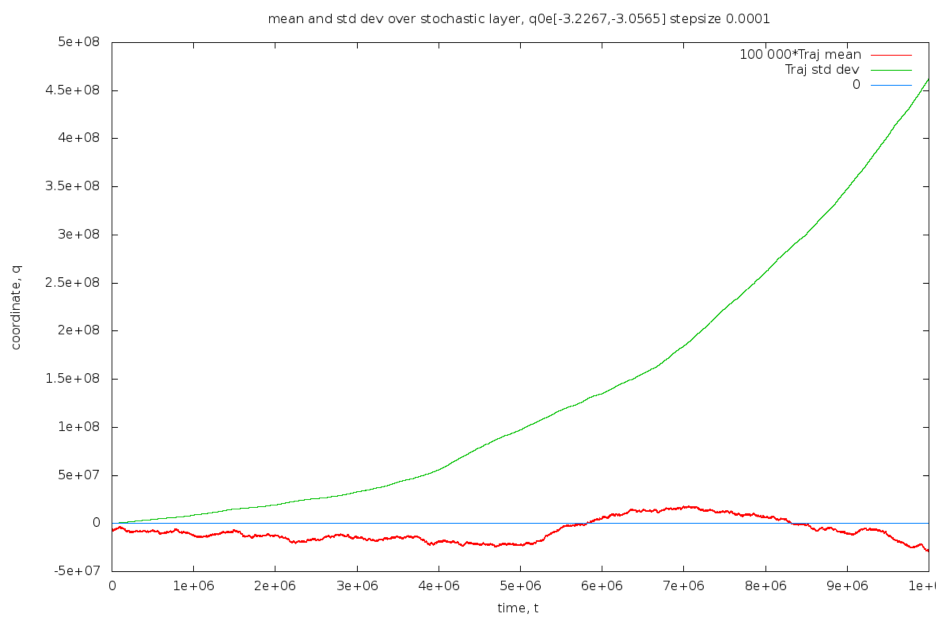


Figure 3.1: (100'000*)mean and std deviation of 1703 trajectories; $q_0 \in [-3.2267, -3.0565]$ Steppsize=0.0001

In fig. 3.1 multiple trajectories with different starting-positions were averaged and the standard deviation was taken. The constants of the Hamiltonian were: $\omega_0 = 0.1$; $\omega_f = 0.01$; $h = 0.001$. This lead to chaotic motion in the limits of $q_0 \in [-3.2267, -3.0565]$ and a time $t_{ch} \approx 10^5$ to once spread over the whole chaotic layer. The total time T was chosen much bigger than t_{ch} to get values which are representatively spread over the chaotic layer ($10^7 = T \gg t_{ch} = 10^5$). On fig. 3.1 all 1703 trajectories were taken. These trajectories were evenly spread over the chaotic layer. The linear mean is very small (already multiplied by 10^5) and can be said to go to 0 for more particles. The standard deviation $\langle(\Delta x)^2\rangle$ follows a mixture of a diffusive and ballistic behavior. When fitting fig. 3.1 as shown in fig. 3.2 one get a the curve $f(t) = 0.999 \cdot t + 4.07 \cdot 10^{-6} \cdot t^2$.
This was done for $\frac{\omega_f}{\omega_0} = [0.6; 0.5; 0.1]$.

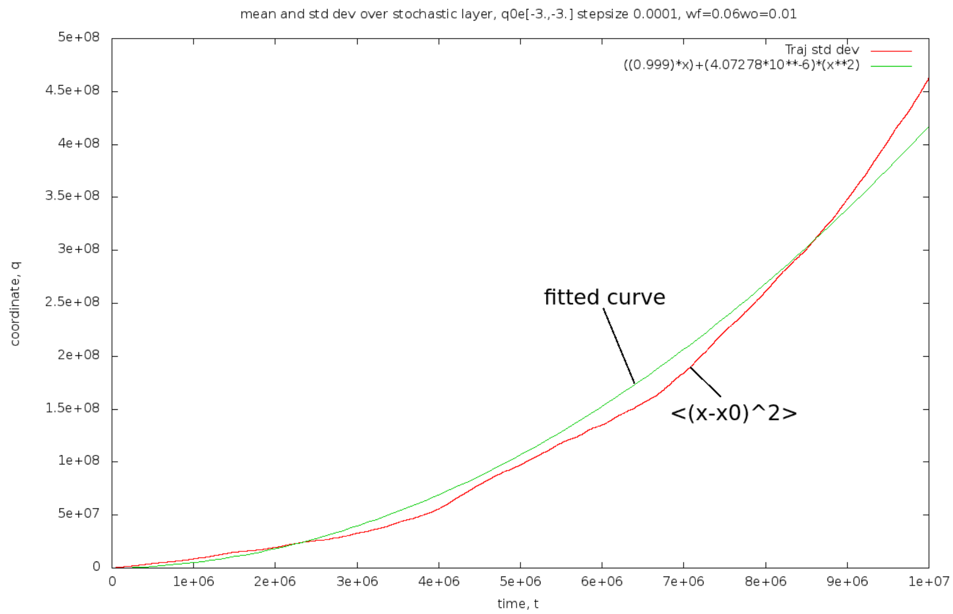


Figure 3.2: fit for $\omega_f = 0.1 \cdot \omega_0 = 0.01$: $fit\ of\ \langle (\Delta x)^2 \rangle = 0.999 \cdot t + 4.07 \cdot 10^{-6} \cdot t^2$

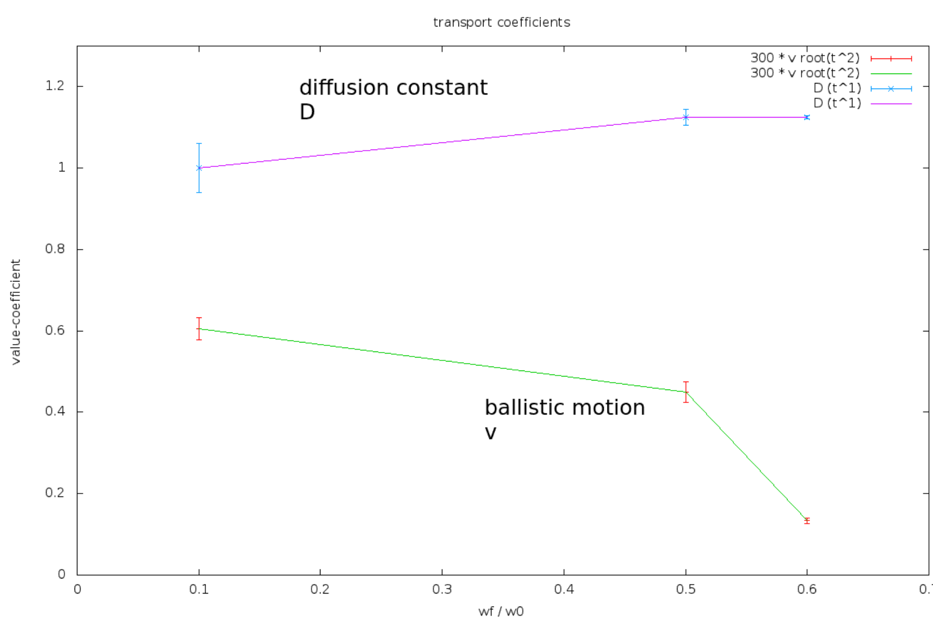


Figure 3.3: transport coefficients D and $300 \cdot v$ plotted over $\frac{\omega_f}{\omega_0}$ for $\langle (\Delta x)^2 \rangle = D \cdot t + v^2 \cdot t^2$

Chapter 4

Conclusion

It has been shown that the nature of stochastic properties of particles in a periodic, symmetric potential with a symmetric, periodic driving force is changed as the frequency of the driving for vanishes $\omega_f \to 0$. In the adiabatic limit ($\omega_f \to 0$) the diffusion constant D decreases slightly while v increases (see fig. 3.3). This means a noticeable ballistic contribution to the diffusive transport. This can be explained as the ballistic flights become longer in the adiabatic limit.
So with an AC driving force the stochastic transport in an optical lattice can be made faster.

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Erklärung

Hiermit erkläre ich, dass ich meine Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

München, 5.7.2013
Ort, Datum

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