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The spin–split incompressible edge states within empirical Hartree approximation at intermediately large Hall samples

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Abstract

A self-consistent Thomas–Fermi–Poisson based calculation scheme is used to achieve spin resolved incompressible strips (ISs). The effect of exchange and correlation is incorporated by an empirically induced g factor. A local version of Ohm's law describes the imposed fixed current, where the discrepancies of this model are resolved by a relevant spatial averaging process. The longitudinal resistance is obtained as a function of the perpendicular (strong) magnetic field at filling factor one and two plateaus. Interrelation between the ISs and the longitudinal zeros is explicitly shown.

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Devising spin degree of freedom as a coherent state to carry information became an essential phenomena in today's quantum information processing research, also in the integer quantized Hall effect (IQHE) [1] regime. However, a consistent picture describing the current distribution within these samples is still under debate. Recent attempts to calculate these quantities can be categorized under two essential titles: (i) a semi-classical solution of the Poisson and Schrödinger equations to obtain electrostatic quantities [2,3], i.e. a Hartree type (self-consistent) Thomas–Fermi(–Poisson) approximation (SCTFPA), in which the imposed current is treated within a local version of Ohm's law [4,5] (LOL) and (ii) a full quantum mechanical solution of the Hartree–Fock Hamiltonian, either using direct diagonalization [6] or local spin density approximation (LSDA) plus density functional theory (DFT) [7]. The latter two approximations, although much powerful to describe the electrostatics, lack presenting the global longitudinal or Hall resistances, since the direct diagonalization model is essentially focused on describing the local compressibility or conductivity and the second one uses the commonly used Landauer–Büettiker type conductance phenomena. On the other hand, the simpler approach, i.e. SCTFPA + LOL, was able to provide information of the local current distribution and the global resistances as a function of magnetic field considering spinless electrons [8]. Here, we extend our previous works [5,8] to include the effect of spin, i.e. Zeeman splitting, by considering an experimentally induced g factor [9]. The simplest, but not the most trivial, treatment of Zeeman splitting, conductivity model and exchange–correlation effects is used to describe the mentioned quantities. We essentially incorporate Zeeman effect by introducing a gap in the energy dispersion, then calculate the electron and the current densities together with the electrostatic and electrochemical potentials as a function of the lateral coordinate. Moreover, the longitudinal resistance (R_{xx}) is calculated for varying magnetic field. In our calculations we assume a translational invariant sample in y direction, where an electron channel is formed in the interval $-b<x<b$, with b being the depletion length. The geometry under consideration is the so-called floating gate structure [4]. Given the boundary conditions and external potentials one can solve the Poisson equation, starting from the zero temperature, zero

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magnetic field solution. The basic ingredients of the SCTFPA are the iterative solution of the electron density,

$$n_{\text{el}}(x) = \int \mathrm{d}E D(E) f(E - V(x)), \tag{1}$$

where $f(\alpha)$ is the Fermi function, $D(E)$ a relevant density of states, and the total potential energy,

$$V(x) = V_{\text{ext}}(x) + (2e^2/\kappa) \int_{-d}^{d} n_{\text{el}}(x') K(x, x')\, \mathrm{d}x' + g^* \sigma \mu_{\text{B}} B. \tag{2}$$

Here, the first term stands for the external potential composed of gates and donors. In the second (Hartree) term κ is the dielectric constant of the material and $K(x, x')$ is the solution of the Poisson equation preserving the boundary conditions $V(-d) = V(d) = 0$, where $2d$ is the sample width. The third term is the Zeeman energy with effective g factor ($g^* = 5.2$ from experimental and theoretical findings [9] and the references therein), $\sigma = \pm\frac{1}{2}$ spin polarization and μ_{B} the effective Bohr magnetron considering GaAs/AlGaAs heterostructures. The well-established wisdom is that the exchange–correlation effects enhance the Zeeman splitting; however, direct numerical calculations of these potentials without taking into account screening lead to discrepancies in explaining the experimental data. In this work we take the experimental value of the g factor without dealing with the difficulties that arise from explicit calculation of the effects of exchange and correlations [6,7].

The imposed current densities are described by an LOL, by making use of the translation invariance and equation of continuity, namely

$$E_y(x) \equiv E_y^0 \quad \text{and} \quad j_x(x) = 0 \tag{3}$$

and

$$j_y(x) = E_y^0/\rho_{xx}(x), \quad E_x(x) = \frac{\rho_{xy}(x)}{\rho_{xx}(x)} E_y^0, \tag{4}$$

where $\rho_{xx}(x)$ and $\rho_{xy}(x)$ are the diagonal (longitudinal) and off-diagonal (Hall) components of (local) resistivity tensor, $\hat{\rho(x)} = [\hat{\sigma(x)}]^{-1}$, respectively. Hence, given the local conductivity tensor elements one can obtain the current density in x direction and thereby the global resistances. We choose a simple (generalized) description of the conductivity model, namely the Hall component is $\sigma_{xy} = (e^2/h)\nu(x)$, where $\nu(x)$ is the local filling factor and the longitudinal component is $\sigma_{xx} =$

$$\begin{aligned} &\sigma_{xy}[\varepsilon + (1 - \nu(x))^2/4], \quad 0 < \nu(x) < 1.5, \\ &\sigma_{xy}[\varepsilon + (2 - \nu(x))^2/4], \quad 1.5 \leqslant \nu(x) < 2.5, \end{aligned} \tag{5}$$

where ε is a cut-off parameter defining the accuracy of the numerics ($\sim 2 \times 10^{-6}$). The above scheme enclosures the simplest set of assumptions to provide a qualitative understanding of the current distribution and the calculation of the global resistances and can be improved in many aspects, however, as a first attempt grasps most of the essential physics.

Fig. 1 depicts the calculated local filling factors versus the lateral coordinate, when the $\nu(0) < 2$ (black solid line) and $2 < \nu(0) < 3$ (red solid line). The ellipses encircle the

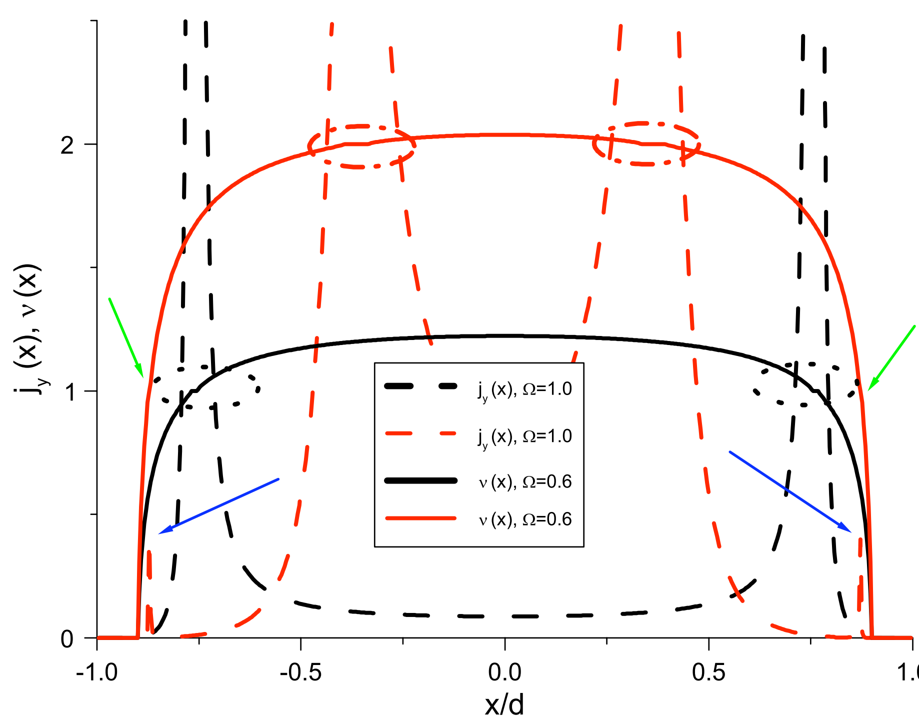

Fig. 1. The local filling factors (solid lines) calculated at two representative magnetic field values, $\Omega = \hbar\omega_c/E_0$, where $\hbar\omega_c = \hbar eB/m$ is the cyclotron energy and E_0 is the pinch-off energy given by $2\pi e^2 n_0/\kappa$, with the fixed donor number density ($n_0 = 4 \times 10^{11}\,\text{cm}^{-2}$). Corresponding to current densities (broken lines) a small current ($I = U_H/\Omega = 0.01$, see Ref. [4] for the definition) is driven in y direction, which is still in the linear response regime. The calculations are done at the default temperature $k_{\text{B}}T/E_0 = 0.001$ for a sample width $2d = 6.3\,\mu\text{m}$.

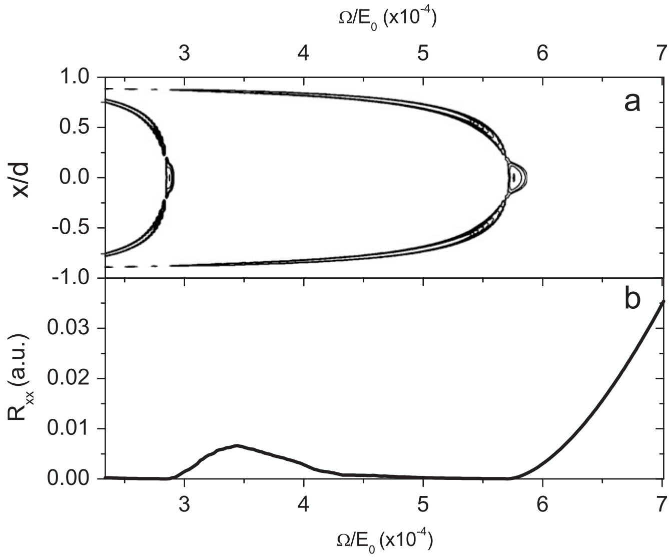

Fig. 2. (a) The numerically calculated current distribution as a function of external magnetic field, lines present the equi-current contours. (b) R_{xx} against magnetic field, obtained at default temperature.

region where the local filling factor is one (dotted) or two (dash-dotted). It is seen apparently that the current (broken lines) is essentially confined into a region where an IS exists. In the case of $\nu(0)<2$, most of the current is flowing either inside the IS or in the very close vicinity, whereas for $2<\nu(0)<3$ beside the main two peaks near filling factor 2, we observe that there exists two other peaks on both sides coinciding with the positions of filling factor 1 regions (highlighted by arrows). This essentially indicates that although most of the current is carried by the innermost IS, if an external current is imposed there is a possibility for electrons to flow from these very narrow local regions, due to a local minimum of the longitudinal conductivity. This finding coincides with the very recent model by Neder and Marquardt [10], considering a non-Gaussian noise to explain the visibility oscillations performed at the Mach–Zehnder interference experiments. In Fig. 2 we show the local current intensity (a) as a function of magnetic field together with the longitudinal resistance (b). The evolution of the reminiscence IS with $\nu(x)=1$ can also be observed in the left side of the current distribution; however, the longitudinal resistance vanishes within the numerical accuracy in this interval. If the ISs become larger than the magnetic length or the Fermi wave length, the system exhibits finite R_{xx} and the current is distributed all over the sample, otherwise $R_{xx}=0$. The bubble-like features seen at the centers of the sample are simply due to the conductivity model we consider.

In summary, we have calculated the spatial distribution of the current carrying, spin–split ISs within a Hartree type approximation incorporating Zeeman splitting by an experimentally induced effective g factor. We have shown that the formation of (spin resolved) incompressible strips dominates the current distribution and thereby defines the magnetic field interval where one would observe the IQHE.

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