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Fermionic Mach-Zehnder interferometer subject to a quantum bath

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Abstract. – We study fermions in a Mach-Zehnder interferometer, subject to a quantum-mechanical environment leading to inelastic scattering, decoherence, renormalization effects, and time-dependent conductance fluctuations. We present a method to derive both the loss of interference contrast as well as the shot noise, using equations of motion and leading-order perturbation theory. The dependence of the shot noise on the Aharonov-Bohm phase acquires an unexpected average phase shift, due to correlations between the fluctuating renormalized phase shift and the output current. We discuss the limiting behaviours at low and high voltages, compare with simpler models of dephasing, and present implications for experiments.

Introduction. – Quantum interference effects and their destruction by scattering play a prominent role in mesoscopic physics. In contrast to the usual Aharonov-Bohm ring setups, the recently introduced Mach-Zehnder interferometer for electrons [1] offers an exciting possibility to study an ideal two-way interference geometry, with chiral single-channel transport and in the absence of backscattering. The loss of visibility with increasing bias voltage or temperature has been observed, and the idea of using shot noise measurements to learn more about potential dephasing mechanisms has been introduced.

On the theoretical side, the loss of interference contrast in the current had been studied for the Mach-Zehnder setup [2] prior to this experiment. More recently, the influence of dephasing on shot noise has been analyzed [3], revealing important differences between phenomenological and microscopic approaches. However, both of these works treat a *classical* noise field acting on the electrons, and thus they are dealing essentially with a single-particle problem. Therefore, experimentally observed features such as the increase of the dephasing rate with rising bias voltage could not be studied, as this effect is due to lifting the restrictions of Pauli blocking on the scattering of particles, representing a many-body effect absent for classical noise.

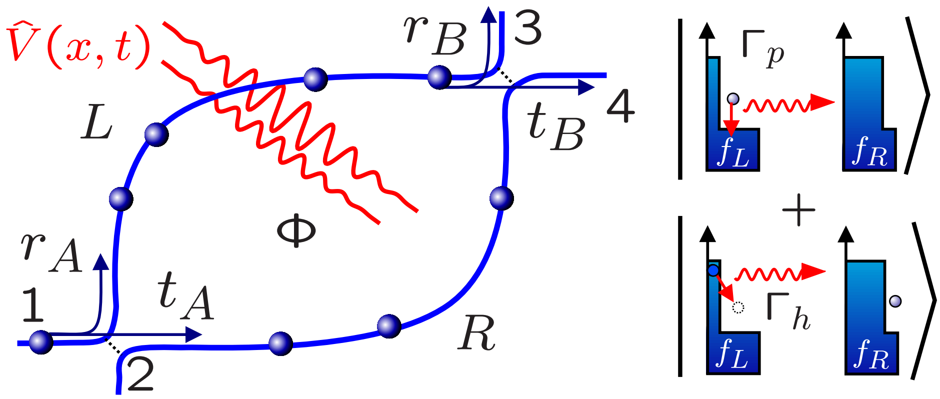

Fig. 1 – (Color online) Left: schematic of the Mach-Zehnder setup. Right: particle- and hole-scattering processes contributing to the dephasing rate, see discussion after eq. (14).

In this work, we study the influence of any true quantum bath (phonons, Nyquist noise, etc.) on a fermionic Mach-Zehnder interferometer (fig. 1). Besides its experimental relevance [1], this setup represents an ideal model problem in which many of the features of a quantum bath acting on a fermion system can be analyzed more easily and/or thoroughly than in more complicated situations such as weak localization [4, 5]. We fully account for Pauli blocking in a nonequilibrium transport situation (*i.e.* arbitrary bias) and derive both the dephasing rate, as well as the effects on the current noise. We introduce a physically transparent equations-of-motion approach that is analogous to the case of classical noise, but keeps the Pauli principle via the back-action of the bath onto the system. The evaluation will be performed perturbatively, to leading order in the system-bath interaction.

The model. – We consider a model of spinpolarized fermions, moving chirally and without backscattering through an interferometer at constant speed v_F. The two beamsplitters A and B connect the fermion fields $\hat{\psi}_\alpha$ of the input ($\alpha = 1, 2$) and output ($\alpha = 3, 4$) channels to those of the left and right arm ($\alpha = L, R$), which we take to be of equal length l:

$$\begin{aligned}
\hat{\psi}_L(0,t) &= r_A\hat{\psi}_1(0,t) + t_A\hat{\psi}_2(0,t), && (1)\\
\hat{\psi}_R(0,t) &= t_A\hat{\psi}_1(0,t) + r_A\hat{\psi}_2(0,t), && (2)\\
\hat{\psi}_3(l,t) &= r_B e^{i\phi}\hat{\psi}_L(l,t) + t_B\hat{\psi}_R(l,t), && (3)\\
\hat{\psi}_4(l,t) &= t_B e^{i\phi}\hat{\psi}_L(l,t) + r_B\hat{\psi}_R(l,t). && (4)
\end{aligned}$$

The transmission (reflection) amplitudes $t_{A/B}$ ($r_{A/B}$) fulfill $t_j^* r_j = -t_j r_j^*$ due to unitarity, and we have included the Aharonov-Bohm phase difference ϕ. The input fields $\alpha = 1, 2$ obey $\langle \psi_\alpha^\dagger(0,0)\psi_\alpha(0,t)\rangle = \int_{-k_c}^{k_c} (\mathrm{d}k)\, f_{\alpha k} e^{-iv_F kt}$ (note $\hbar = 1$), with a band-cutoff k_c. We use the notation $(\mathrm{d}k) \equiv \mathrm{d}k/(2\pi)$.

The particles are assumed to have no intrinsic interaction, but are subject to an external free bosonic quantum field $\hat{V}$ (linear bath) during their passage through the arms L, R: $\hat{H}_{\mathrm{int}} = \sum_{\lambda=L,R} \int \mathrm{d}x\, \hat{V}_\lambda(x)\hat{n}_\lambda(x)$ with $\hat{n}_\lambda(x) = \hat{\psi}_\lambda^\dagger(x)\hat{\psi}_\lambda(x)$.

General expressions for current and shot noise. – We focus on the current going into output port 3, which is related to the density: $\hat{I}(t) = ev_F\hat{n}_3(t)$ with $\hat{n}_3(t) = \hat{\psi}_{3t}^\dagger\hat{\psi}_{3t}$, where we take fields $\hat{\psi}_{\alpha t} = \hat{\psi}_\alpha(l,t)$ at the position of the final beamsplitter B (except where noted otherwise). In the following we set $e = v_F = 1$, except where needed for clarity. We have

$$\left\langle \hat{I} \right\rangle = R_B \left\langle \hat{\psi}_L^\dagger \hat{\psi}_L \right\rangle + T_B \left\langle \hat{\psi}_R^\dagger \hat{\psi}_R \right\rangle + e^{i\phi} t_B^* r_B \left\langle \hat{\psi}_R^\dagger \hat{\psi}_L \right\rangle + \text{c.c.} \tag{5}$$

We have set $T_B = |t_B|^2$ and $R_B = 1 - T_B$. Without bath, the interference term is given by $\left\langle \hat{\psi}_R^\dagger \hat{\psi}_L \right\rangle_{(0)} = r_A t_A^* \int (\mathrm{d}k) \delta f_k = r_A t_A^* (eV/2\pi)$, where we define $\delta f_k \equiv f_{1k} - f_{2k}$ and $\bar{f}_k \equiv (f_{1k} + f_{2k})/2$ for later use, and $eV = \mu_1 - \mu_2$.

The zero-frequency current noise power is

$$S \equiv \int_{-\infty}^{+\infty} \mathrm{d}t \left\langle \left\langle \hat{I}(t) \hat{I}(0) \right\rangle \right\rangle , \tag{6}$$

where the double bracket denotes the irreducible part. The dependence on ϕ and T_B, R_B is explicit,

$$\begin{aligned} S &= R_B T_B C_0 + R_B^2 C_{0R} + T_B^2 C_{0T} + \\ &+ 2\mathrm{Re}\left[e^{i\phi}(t_B^* r_B)(R_B C_{1R} + T_B C_{1T}) - e^{2i\phi} T_B R_B C_2\right] \end{aligned} \tag{7}$$

with the coefficients following directly from inserting eq. (3) into (6), for example $C_2 = \int \mathrm{d}t \left\langle \left\langle \hat{\psi}_{Rt}^\dagger \hat{\psi}_{Lt} \hat{\psi}_{R0}^\dagger \hat{\psi}_{L0} \right\rangle \right\rangle$. $C_{0(R/T)}$ are real-valued, the other coefficients may become complex. The free values correspond to the result given by the well-known scattering theory of shot noise of non-interacting fermions [6, 7]:

$$S_{(0)} = \int (\mathrm{d}k)(f_{2k} + \delta f_k \mathcal{T})(1 - (f_{2k} + \delta f_k \mathcal{T})) , \tag{8}$$

where $\mathcal{T}(\phi) = T_A T_B + R_A R_B + 2 t_A^* r_A t_B^* r_B \cos(\phi)$ is the transmission probability from 1 to 3.

Symmetries of shot noise. – For our model, the full shot noise power S may be shown to be invariant under each of the following transformations, if the bath couples equally to both arms of the interferometer: i) $t_A \leftrightarrow r_A$, $\phi \mapsto -\phi$; ii) $V \mapsto -V$, $\phi \mapsto -\phi$; iii) $t_B \leftrightarrow r_B$. As a consequence, $C_{1T} = -C_{1R}$. Note that the free result (8) is invariant under $\phi \mapsto -\phi$ and $V \mapsto -V$ separately, but these symmetries may be broken by a bath-induced phase-shift.

Equations of motion. – Here we introduce an equations-of-motion approach that is set up in analogy to the simpler case of classical noise [3] but keeps many-body effects such as Pauli blocking. We start from Heisenberg's equations of motion for the fermions and the bath. The fermion field in each arm obeys (omitting the index L/R for now):

$$i(\partial_t - v_F \partial_x)\hat{\psi}(x,t) = \int \mathrm{d}x' K(x - x') \hat{V}(x', t) \hat{\psi}(x', t) , \tag{9}$$

where $\hat{V}$ evolves in the presence of the interaction, see below. The kernel $K(x - x') = \{\hat{\psi}(x), \hat{\psi}^\dagger(x')\} \neq \delta(x - x')$ appears because we have to consider states within a finite band. Nevertheless, for the purpose of our subsequent leading-order approximation, it turns out we can replace the right-hand side by $\hat{V}(x,t)\hat{\psi}(x,t)$ (neglecting, *e.g.*, velocity-renormalization in higher orders). The corresponding formal solution describes the accumulation of a random "quantum" phase:

$$\begin{aligned} \hat{\psi}(x,t) &= \hat{T} \exp\left[-i \int_{t_0}^{t} \mathrm{d}t_1 \, \hat{V}(x - v_F(t - t_1), t_1)\right] \times \\ & \quad \times \hat{\psi}(x - v_F(t - t_0), t_0) . \end{aligned} \tag{10}$$

In contrast to the case of classical noise [3], the field $\hat{V}$ contains the response to the fermion density, in addition to the homogeneous solution $\hat{V}_{(0)}$ of the equations of motion (*i.e.* the free fluctuations):

$$\hat{V}(x,t) = \hat{V}_{(0)}(x,t) + \int_{-\infty}^{t} \mathrm{d}t'\, D^R(x,t,x',t')\hat{n}(x',t') \,. \tag{11}$$

Here D^R is the unperturbed retarded bath Green's function, $D^R(1,2) \equiv -i\theta(t_1 - t_2)\big\langle[\hat{V}(1), \hat{V}(2)]\big\rangle$, where $\hat{V}$-correlators refer to the free field. This (exact) step is analogous to the derivation of an operator quantum Langevin equation [8]. Together with (10), it correctly reproduces results from lowest-order diagrammatic perturbation theory.

Accounting for cross-correlations between the fluctuations in both arms is straightforward for a geometry with symmetric coupling to parallel arms at a distance d (assuming $d \ll l$). Then, in the following results (*e.g.*, phase shift and total dephasing rate Γ_φ), we have to set $\langle \hat{V}\hat{V}\rangle = \langle \hat{V}_L\hat{V}_L\rangle - \langle \hat{V}_L\hat{V}_R\rangle$ and $D^R = D^R_{LL} - D^R_{LR}$. These correlators derive from the three-dimensional version, *e.g.* $\left\langle \hat{V}_L(x,t)\hat{V}_R(x',t)\right\rangle = \left\langle \hat{V}(x,y+d,z,t)\hat{V}(x',y,z,t')\right\rangle$.

Interference term, renormalized phase shift and dephasing rate. – The loss of interference contrast, as observed in ref. [1], is a way to quantify "dephasing" also in a nonequilibrium situation ($|V| > 0$). In order to obtain the interference term in the current, we expand the exponential (10) to second order, insert the formal solution (11), and perform Wick's averaging over fermion fields, while implementing a "Golden Rule approximation", *i.e.* keeping only terms linear in the time-of-flight τ. Then we obtain the following leading correction to the interference term:

$$\delta\left\langle \hat{\psi}^\dagger_R\hat{\psi}_L\right\rangle = r_A t^*_A \int (\mathrm{d}k)\delta f_k [i\delta\bar{\varphi}(k) - \Gamma_\varphi(k)\tau]. \tag{12}$$

Here the effective average k-dependent "renormalized" phase shift induced by coupling to the bath is

$$\delta\bar{\varphi}(k) = \tau(R_A - T_A)\int (\mathrm{d}q)(\mathrm{Re}D^R_{q,q} - D^R_{0,0})\delta f_{k-q} \,, \tag{13}$$

which vanishes for $T_A = 1/2$, since then there is complete symmetry between both arms. The interference term is suppressed according to the total dephasing rate $\Gamma_\varphi(k) = \Gamma^L_\varphi(k) + \Gamma^R_\varphi(k)$, with

$$\begin{aligned}\Gamma^L_\varphi(k) &= \int (\mathrm{d}q)\left[\frac{1}{2}\langle \hat{V}\hat{V}\rangle_{q,q} + \mathrm{Im}D^R_{q,q} f_{Lk-q}\right] \\ &= -\frac{1}{2}\int (\mathrm{d}q)\,\mathrm{Im}D^R_{q,q}\left[\coth\frac{\beta q}{2} - (f_{Lk-q} - f_{Lk+q})\right] .\end{aligned} \tag{14}$$

The "back-action" $\propto D^R$ is crucial, since it introduces the nonequilibrium Fermi functions ($f_L = R_A f_1 + T_A f_2$, $f_R = T_A f_1 + R_A f_2$) which capture the physics of Pauli blocking: Large energy transfers $v_F|q| \gg eV, T$ are forbidden for states k within the transport region. As a result, the interference contrast becomes perfect for $V, T \to 0$. On the other hand, increasing the bias voltage diminishes the contrast (as observed in the experiment [1]), since the phase space for scattering is enhanced.

The dephasing rate is the sum of particle- and hole-scattering rates, $\Gamma^L_\varphi = (\Gamma^L_p + \Gamma^L_h)/2$, with $\Gamma^L_p(k) = \int (\mathrm{d}q)\left\langle \hat{V}\hat{V}\right\rangle_{q,q}(1 - f_{Lk-q})$ and $\Gamma^L_h(k) = \int (\mathrm{d}q)\left\langle \hat{V}\hat{V}\right\rangle_{q,q} f_{Lk+q}$. This is because both kinds of scattering processes destroy the superposition of many-particle states (kets in fig. 1, right) that is created when a particle passes through the first beam splitter, entering the left or the right arm (see [9] for the same kind of physics in weak localization).

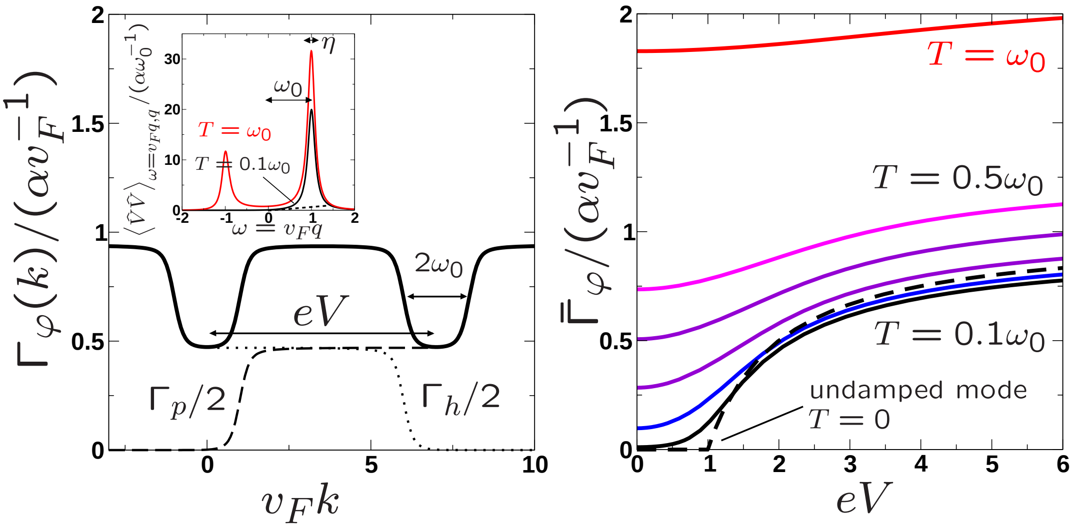

Fig. 2 – (Color online) Left: energy-resolved dephasing rate for a sample bath spectrum (inset). Right: energy-averaged dephasing rate $\bar{\Gamma}_\varphi(V,T)$. Energies in units of ω_0. The interference visibility $(I_{\max}-I_{\min})/(I_{\max}+I_{\min})$ is given by $1-\bar{\Gamma}_\varphi\tau$, in the leading order considered here.

For linear transport, $f_{Lk-q}-f_{Lk+q}\rightarrow -\tanh(\beta(k-q)/2)$ under the integral, leading to the result well known in the theory of weak localization [5], where ballistic motion in our case ($\omega = v_F q$) is replaced by diffusion.

Figure 2 displays both $\Gamma_\varphi(k)$ and the energy-averaged $\bar{\Gamma}_\varphi = (eV)^{-1}\int \mathrm{d}k\,\delta f_k \Gamma_\varphi(k)$ for the example of a damped optical phonon mode, $D^R_{q,\omega} = \alpha[(\omega-\omega_0+i\eta)^{-1}-(\omega+\omega_0+i\eta)^{-1}]$.

Discussion of shot noise correction. – In exactly the same manner, after a straightforward calculation, one can derive the leading-order corrections to the coefficients C_0, C_{1R}, C_2 in the noise power S (again keeping only terms $\propto \tau^1$). This is done by inserting the solutions of the equations of motion, eqs. (10) and (11), into the coefficients defined in (7) and proceeding as before. In the following, we provide a discussion of the results and illustrate them by plots. The rather lengthy full analytical expressions will be listed in a forthcoming extended article [10].

As expected, the ϕ-dependence of the shot noise (7) is suppressed, in a similar manner as the interference term in the current: $|C_2|$ and $|C_{1R}|$ decrease in magnitude.

There is no Nyquist noise correction, *i.e.* $\delta S(V=0)$ vanishes at arbitrary temperature T (fig. 3, left). This is plausible, as $S_{(0)}(V=0)$ does not depend on ϕ and thus is not affected by phase fluctuations. In the case of purely classical noise, we had found a finite Nyquist correction [3], but this is due to heating by a "bath" that is nominally at $T=\infty$.

At large voltage V (larger than the bath spectrum cutoff), there is a quadratic contribution $\propto V^2$ in δC_0 and $\mathrm{Re}\delta C_2$, due to time-dependent conductance fluctuations, corresponding precisely to the leading order of "S_{cl}" in ref. [3].

There are two peculiar features of those phase-shifts in the shot noise $S(\phi)$. First, the phase-shift in the $e^{2i\phi}$ term is twice as large as expected from the phase-shift in $I(\phi)$, shown in eq. (13). Second, even when there is no phase shift in the current pattern ($\delta\bar{\varphi}_k = 0$), the phase-shift of the $e^{i\phi}$ component in $S(\phi)$ does not necessarily vanish. There remains a $\phi \leftrightarrow -\phi$-asymmetry in δS even when both arms are completely symmetric and $T_A = 1/2$. Only the additional constraint $T_B = 1/2$ will guarantee the ϕ-symmetry.

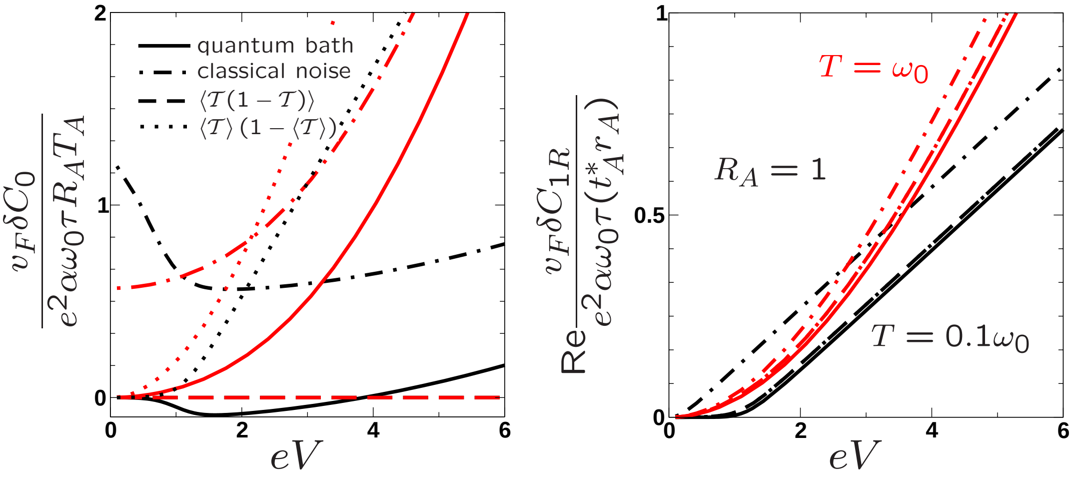

Fig. 3 – (Color online) Shot noise corrections $\delta C_0(V)$ and $\delta C_1(V)$ for $T/\omega_0 = 0.1, 1$ (black, red); spectrum as in fig. 2. Comparison with classical noise, $\langle \mathcal{T}(1-\mathcal{T})\rangle$ and $\langle \mathcal{T}\rangle(1-\langle \mathcal{T}\rangle)$, see text.

Both features arise because the phase shift fluctuates, due to the density fluctuations in both arms. Restricting attention to the k-independent part for ease of the discussion, we may interpret the phase-shift as an operator depending on the densities, schematically $\delta\hat{\varphi}[\hat{n}_{L/R}]$, whose expectation value determines the phase-shift of the current pattern. It is correlated with the output current, $\left\langle (\delta\hat{\varphi}(t) - \delta\bar{\varphi})(\hat{I}(0) - \bar{I}) \right\rangle \neq 0$, leading to an extra shot noise contribution and (together with the k-dependent part) accounting for the extra factor of two in the $e^{2i\phi}$ phase-shift, as well as the fact that $T_A = 1/2$ is not enough to obtain a ϕ-symmetric shot noise (since the correlator $\left\langle \delta\hat{\varphi}\hat{I} \right\rangle$ depends on T_B as well).

Comparison with simpler models. – The limit of classical noise (treated to all orders in ref. [3]) is recovered by setting $D^R = 0$ and using the symmetrized correlator $\langle V_{\rm cl} V_{\rm cl}\rangle = \left\langle \{\hat{V}, \hat{V}\} \right\rangle/2$ everywhere. However, it is impossible to mimick the features obtained for true quantum noise by any classical noise model, even with an arbitrary "effective" correlator (*e.g.* phase-shift terms are missing). In ref. [1], two formulas were introduced to describe the modification of the partition noise by dephasing or phase averaging: $\langle \mathcal{T}(\phi+\varphi)(1-\mathcal{T}(\phi+\varphi))\rangle_\varphi$ or $\langle \mathcal{T}(\phi+\varphi)\rangle_\varphi \langle 1-\mathcal{T}(\phi+\varphi)\rangle_\varphi$ (see also ref. [3]). To check such an ansatz, we introduce fluctuations $\phi \mapsto \phi + \delta\varphi_k$ into the scattering theory result, eq. (8), and average either in the form $\langle \mathcal{T}\mathcal{T}\rangle$ or $\langle \mathcal{T}\rangle\langle \mathcal{T}\rangle$, assuming Gaussian variables $\delta\varphi_k$, with $\delta\bar{\varphi}_k$ taken from eq. (13) and $\left\langle \delta\varphi_k^2 \right\rangle = 2\tau\Gamma_\varphi(k)$. This procedure is designed to reproduce the correct current. However, neither formula gives a good approximation to our result (fig. 3).

Conclusions and implications for experiments. – We have introduced an equations-of-motion method to describe a fermionic interferometer subject to quantum noise. The crucial Pauli blocking effects are described as a consequence of the back-action of the bath onto the system. The present approach lends itself naturally to a systematic extension to higher orders, calculations beyond the Golden Rule approximation, and the analysis of current cross-correlators. In addition, it would be interesting to evaluate the full counting statistics or to compare with a Luttinger liquid approach, where dephasing is due to intrinsic interactions (see the recent preprints refs. [11] and [12], respectively). Regarding future experiments following ref. [1], the following points should be checked first: i) the shot noise symmetries listed above; ii) the generic dependence of shot noise S on beam splitter transparency T_B and Aharonov-Bohm phase ϕ (in particular: only $e^{i\phi}$, $e^{2i\phi}$ contributions present) predicted in eq. (7); iii) observing different phase-shifts of the $e^{i\phi}$ and $e^{2i\phi}$-contributions to S would be particularly interesting, as this feature is not present in any simple phenomenological (or classical noise) model.

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