Exploiting environmental resonances to enhance qubit quality factors

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We discuss dephasing times for a two-level system (including bias) coupled to a damped harmonic oscillator. This system is realized in measurements on solid-state Josephson qubits. It can be mapped to a spin-boson model with a spectral function with an approximately Lorentzian resonance. We diagonalize the model by means of infinitesimal unitary transformations (flow equations) and calculate correlation functions, dephasing rates, and qubit quality factors. We find that these depend strongly on the environmental resonance frequency Ω; in particular, quality factors can be enhanced significantly by tuning Ω to lie *below* the qubit frequency Δ.

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I. INTRODUCTION

A key feature in qubit design is to gain good control of dephasing induced by the environment. A much-studied model that has yielded considerable insight into the dephasing of qubits (more generally, two-state systems) is the spin-boson model.[1] Most studies of this model assume a spectral function $J(\omega)$ that has a power-law form. However, several qubit systems of current interest are coupled to an environment that features rather strong resonances, which would correspond to a spectral function $J(\omega)$ with well-defined peaks at characteristic frequencies. A prominent example is the case of flux-qubits,[2] which are read out using a superconducting quantum interference device (SQUID), with a characteristic resonance frequency Ω (of order 3 GHz) that is in order of magnitude comparable to the characteristic qubit energy scale (10 GHz).[3,4]

The presence of environmental resonances raises several interesting questions with both fundamental and practical implications: How is the qubit dynamics influenced by the presence of environmental resonances? Can the latter be used to indirectly tune qubit properties, such as the tunneling rate or q-factor? Is it more advantageous to have the resonance frequency higher or lower than the characteristic qubit energies?

Here, we explore these questions in the framework of a model that has been used with great success to describe and optimize recent generations of flux qubits:[3] it involves a spin degree of freedom (qubit) coupled to an harmonic oscillator with frequency Ω (modeling the environmental resonance), which in turn is coupled to a bath of harmonic oscillators (to provide damping).[5] It can be mapped[6] onto a regular spin-boson model with a spectral function $J(\omega)$ featuring an almost Lorentzian resonance peak near Ω. We are interested not only in the regime where the qubit tunneling rate Δ is much smaller than Ω (which would correspond to the standard spin-boson model, with Ω playing the role of the bath cutoff frequency), but also in the hitherto unexplored regime $\Delta > \Omega$. Here a standard weak-coupling, poor-man scaling approach that predicts a downward renormalization of Δ is insufficient; instead, we need a method sufficiently powerful to deal with all ratios of Δ/Ω. To this end, we use the flow-equation renormalization (FER) group approach of Wegner[7] and of Glazek and Wilson.[8] Interestingly, we find that Δ is renormalized *upwards* if the initial Δ is greater than Ω, and that correspondingly, the dephasing times and q-factors are strongly *increased*. These results have the very important implication that *by appropriately tuning the environmental resonance frequency* Ω, *significant additional control of the qubit dynamics can indeed by obtained.*

II. SPIN-BOSON-MODEL

We consider the Hamiltonian

$$\tilde{\mathcal{H}} = -\frac{\Delta}{2}\sigma_x + \frac{\varepsilon}{2}\sigma_z + (B^\dagger + B)\left[g\sigma_z + \sum_k \kappa_k(\tilde{b}_k^\dagger + \tilde{b}_k)\right] + \Omega B^\dagger B + \sum_k \tilde{\omega}_k \tilde{b}_k^\dagger \tilde{b}_k + (B^\dagger + B)^2 \sum_k \frac{\kappa_k^2}{\tilde{\omega}_k}, \quad (1)$$

which describes a two-state system with asymmetry energy ε and tunneling matrix element Δ, coupled linearly with strength g to a harmonic oscillator with frequency Ω, which is itself linearly coupled with strengths κ_k to a bath of harmonic oscillators. The coupling to the environment is completely defined by the spectral function $\tilde{J}(\omega) \equiv \Sigma_k \kappa_k^2 \delta(\omega - \tilde{\omega}_k) = \Gamma\omega\Theta(\omega_c - \omega)$, which is as usual taken to be of ohmic form to model the dissipative environment. This system can be mapped to a spin-boson model[6]

$$\mathcal{H} = -\frac{\Delta}{2}\sigma_x + \frac{\varepsilon}{2}\sigma_z + \frac{1}{2}\sigma_z \sum_k \lambda_k(b_k^\dagger + b_k) + \sum_k \omega_k b_k^\dagger b_k, \quad (2)$$

where spin dynamics depends only on the structured spectral function $J(\omega) \equiv \Sigma_k \lambda_k^2 \delta(\omega - \omega_k)$ given by[3]

$$J(\omega) = \frac{2\alpha\omega\Omega^4\Theta(\omega_c - \omega)}{(\Omega^2 - \omega^2)^2 + (2\pi\Gamma\omega\Omega)^2}, \quad \text{with } \alpha = \frac{8\Gamma g^2}{\Omega^2}. \quad (3)$$

III. FLOW EQUATION RENORMALIZATION

FER is based on infinitesimal unitary transformations of the Hamiltonian.[7] We follow the approach of Ref. 9 and mention only the main steps here:

(a) In order to decouple the two-level system from its environment we apply a sequence of *unitary transformations* $U(l)$ to Eq. (2): $\mathcal{H}(l)=U(l)\mathcal{H}U^{\dagger}(l)$. Here $\mathcal{H}(l=0)=\mathcal{H}$ is the initial Hamiltonian; $\mathcal{H}(l=\infty)$ is the final, diagonal Hamiltonian; and l denotes the flow parameter, which characterizes the square of the inverse energy scale being decoupled. In differential formulation this transformation reads

$$\frac{d\mathcal{H}(l)}{dl}=[\eta(l),\mathcal{H}(l)] \quad \text{with } \eta(l)=\frac{dU(l)}{dl}U^{-1}(l). \tag{4}$$

(b) The canonical choice for the *generator* η suggested by Wegner is $\eta_c=[\mathcal{H}_0,\mathcal{H}]$ with $H_0=-\Delta/2\sigma_{\rm x}+\varepsilon/2\sigma_{\rm z}+\sum_{\rm k}\omega_{\rm k}b_{\rm k}^{\dagger}b_{\rm k}$.[7] However, because η_c generates coupling terms originally not present in the Hamiltonian, it is advisable to modify our generator. For $\varepsilon\neq 0$ we choose[10]

$$\eta=i\sigma_{\rm y}\sum_{\rm k}\eta_{\rm k}^{\rm y}(b_{\rm k}+b_{\rm k}^{\dagger})+\sigma_{\rm z}\sum_{\rm k}\eta_{\rm k}^{\rm z}(b_{\rm k}-b_{\rm k}^{\dagger})+\sigma_{\rm x}\sum_{\rm k}\eta_{\rm k}^{\rm x}(b_{\rm k}-b_{\rm k}^{\dagger}) + \sum_{{\rm k}q}\eta_{{\rm k}q}(b_{\rm k}+b_{\rm k}^{\dagger})(b_q-b_q^{\dagger}), \tag{5}$$

for which Eq. (4) closes for terms linear in bosonic operators. We neglect small higher-order terms in $[\eta,\mathcal{H}]$ that contain a coupling of the system to two bosonic modes. The parameters $\eta_{\rm k}$ and $\eta_{\rm kq}$ in Eq. (5) are given by $\eta_{\rm k}^{\rm x}=-(\lambda_{\rm k}/2)\times(\varepsilon\Delta/\omega_{\rm k})f(\omega_{\rm k},l)$, $\eta_{\rm k}^{\rm y}=-(\lambda_{\rm k}/2)\Delta f(\omega_{\rm k},l)$, $\eta_{\rm k}^{\rm z}=-(\lambda_{\rm k}/2)[(\omega_{\rm k}^2-\varepsilon^2)/\omega_{\rm k}]f(\omega_{\rm k},l)$, and $\eta_{\rm kq}=\Delta^2/(2\Delta_\varepsilon)\tanh(\beta\Delta_\varepsilon/2)\lambda_{\rm k}\lambda_{\rm q}\omega_{\rm q}/(\omega_{\rm k}^2-\omega_{\rm q}^2)[f(\omega_{\rm k},l)+f(\omega_{\rm q},l)]$ with $\Delta_\varepsilon=\sqrt{\Delta^2+\varepsilon^2}$. We choose $f(\omega_{\rm k},l)=[\omega_{\rm k}^2(\omega_{\rm k}-\Delta_\varepsilon)]/[\Delta_\varepsilon^2(\omega_{\rm k}+\Delta_\varepsilon)]$. By comparing numerical results for the $\varepsilon=0$ (see Ref. 10) and the $\varepsilon\neq 0$ Ansatz we see that for $\varepsilon\neq 0$, due to our particular choice of $f(\omega_{\rm k},l)$, we are restricted to couplings $\alpha\lesssim 0.02$, which is a reasonable bound for experimental realizations. For an alternative Ansatz (for $\varepsilon\neq 0$) see also Ref. 11.

(c) Equations (4) and (5) give us a set of differential equations (flow equations) for the parameters in the Hamiltonian, namely $\varepsilon(l)$, $\Delta(l)$, and $\lambda_{\rm k}(l)$ [respectively $J(\omega,l)$]:

$$-\partial_l\Delta/\Delta=\int d\omega\coth\left[\frac{\beta\omega}{2}\right]J(\omega,l)f(\omega,l),\ \partial_l\varepsilon=0, \tag{6}$$

$$\partial_l J(\omega,l)=-2f(\omega,l)(\omega^2-\Delta_\varepsilon^2)J(\omega,l)+\tanh\frac{\beta\Delta_\varepsilon}{2} \times\frac{2\Delta^2}{\Delta_\varepsilon}J(\omega,l)\int d\omega'\frac{\omega' J(\omega',l)}{\omega^2-\omega'^2}[f(\omega,l)+f(\omega',l)]. \tag{7}$$

Note that according to (6) the bias ε is not renormalized. The renormalization of the bath frequencies $\omega_{\rm k}$ vanishes in the thermodynamic limit of infinitely many modes.

(d) *Observables*, such as $\sigma_{\rm z}$, have to be subject to the same sequence of infinitesimal transformations as the Hamiltonian: $d\sigma_{\rm z}(l)/dl=[\eta(l),\sigma_{\rm z}(l)]$. For the flow of $\sigma_{\rm z}$ we make the Ansatz[10]

$$\sigma_{\rm z}(l)=h(l)\sigma_{\rm z}+s(l)\sigma_{\rm x}+r(l)+i\sigma_{\rm y}\sum_{\rm k}\mu_{\rm k}^{\rm y}(l)(b_{\rm k}-b_{\rm k}^{\dagger}) +\sum_{\rm k}[\sigma_{\rm x}\chi_{\rm k}^{\rm x}(l)+\sigma_{\rm z}\chi_{\rm k}^{\rm z}(l)](b_{\rm k}+b_{\rm k}^{\dagger}). \tag{8}$$

We neglect (small) terms in $[\eta,\sigma_{\rm z}]$ that contain a coupling to two bosonic modes. The calculation of the flow equations for the six parameters h, s, r, $\chi_{\rm k}^{\rm x}$, $\chi_{\rm k}^{\rm z}$, and $\mu_{\rm k}^{\rm y}$ in Eq. (8) is straightforward.[12]

(e) To calculate *correlation functions* of the form $C(t)=1/2\langle\sigma_{\rm z}(t)\sigma_{\rm z}(0)+\sigma_{\rm z}(0)\sigma_{\rm z}(t)\rangle$ we use the decoupled Hamiltonian $\mathcal{H}(\infty)=-[\Delta(\infty)/2]\sigma_{\rm x}+[\varepsilon/2]\sigma_{\rm z}+\sum_{\rm k}\omega_{\rm k}b_{\rm k}^{\dagger}b_{\rm k}$, and Eq. (8) for $l=\infty$. For $T=0$ the Fourier transform $C(\omega)$ of $C(t)$ then takes the form (here all parameters are taken at $l=\infty$):[10]

$$C(\omega)=\left[\frac{\varepsilon s}{\Delta_\varepsilon}+\frac{\Delta h}{\Delta_\varepsilon}\right]^2\delta(\omega-\Delta_\varepsilon)+\left[\frac{\varepsilon h}{\Delta_\varepsilon}-\frac{\Delta s}{\Delta_\varepsilon}-r\right]^2\delta(\omega) +\sum_{\rm k}\left[\frac{\varepsilon}{\Delta_\varepsilon}\chi_{\rm k}^{\rm x}-\mu_{\rm k}^{\rm y}+\frac{\Delta}{\Delta_\varepsilon}\chi_{\rm k}^{\rm z}\right]^2\delta(\omega-[\omega_{\rm k}+\Delta_\varepsilon]) +\sum_{\rm k}\left[\frac{\varepsilon}{\Delta_\varepsilon}\chi_{\rm k}^{\rm z}-\frac{\Delta}{\Delta_\varepsilon}\chi_{\rm k}^{\rm x}\right]^2\delta(\omega-\omega_{\rm k}). \tag{9}$$

Numerically, one finds that $h(\infty)=s(\infty)=0$, and (i) $r(\infty)=0$ for $\varepsilon=0$ or (ii) $r(\infty)\neq 0$ for $\varepsilon\neq 0$. Therefore, of the terms in the first line of (9) only $\delta(\omega)$ remains and describes the nonzero expectation value of σ_z for systems with asymmetry.

In order to obtain quantitative results for the correlation function $C(\omega)$, we numerically integrate the flow equations up to some value l_0, which is taken sufficiently large that the final results do not depend on it. From the numerical results for $C(\omega)$, which reflects the dynamics of the two-level system, we extract "dephasing times," defined as the widths at half maximum of the resonances occurring in $C(\omega)$ (as depicted in the inset of Fig. 1). For zero bias ($\varepsilon=0$) a sum rule of the form $\int_0^\infty d\omega C(\omega)=1$ should hold.[9]

Ohmic bath

We start by comparing our results for the dephasing time for a spin-boson model with an ohmic bath, $J(\omega)=2\alpha\omega\Theta(\omega_c-\omega)$, with results from real-time renormalization group (RTRG)[13] and weak coupling calculations (WCC).[14] Figure 1 shows the dephasing time τ as a function of the

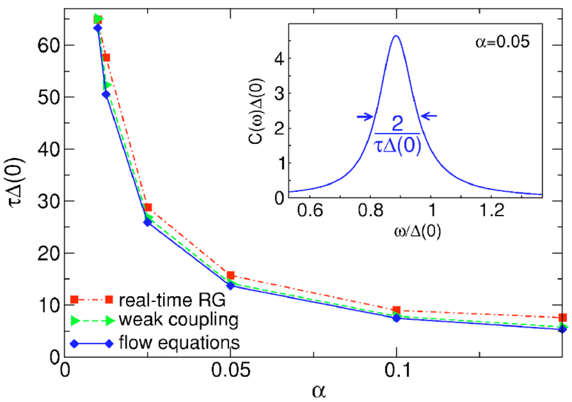

FIG. 1. (Color online) Dephasing times for an ohmic bath with spectral function $J(\omega)=2\alpha\omega\Theta(\omega_c-\omega)$ as a function of α. The FER result $[\varepsilon=0$ and $\omega_c=10\Delta(0)]$ is compared with results from RTRG calculations[13] and WCC.[14] The inset shows a typical FER spin-spin correlation function.

coupling strength α. For weak coupling the dephasing time (at $T=0$) is given by[14]

$$\tau_w = 4/J[\Delta_\varepsilon(\infty)]. \tag{10}$$

We find very good agreement with RTRG and WCC.

Structured bath/weak coupling

We now turn to the structured spectral density given by Eq. (3). The main features of the corresponding system [Eq. (1)] can already be understood by analyzing only the coupled two-level-harmonic oscillator system (without damping, i.e., $\Gamma=0$). For $\varepsilon=0$ this system exhibits two characteristic frequencies, close to Ω and Δ, associated with the transitions 1 and 2 in Fig. 2(c). These should also show up in the correlation function $C(\omega)$; and indeed Fig. 2(a) displays a *double-peak* structure with the peak separation somewhat larger than $(\Delta-\Omega)$, due to level repulsion. The coupling to the bath will in general lead to a broadening of the resonances and an enhancement of the repulsion of the two energies. Due to the very small coupling ($\alpha=0.0006$), peak positions of $C(\omega)$ in Fig. 2 can with very good accuracy be derived from a second-order perturbation calculation for the coupled two-level-harmonic oscillator system, yielding the following transition frequencies [depicted in Fig. 2(c)]: $\omega_{1,+}-\omega_{0,+}=\Omega-g^2 2\Delta(0)/[\Delta^2(0)-\Omega^2]\approx 0.987\Omega$ and $\omega_{0,-}-\omega_{0,+}=\Delta(0)+g^2 2\Delta(0)/[\Delta^2(0)-\Omega^2]\approx 1.346\Omega$. With the two peaks we associate two different dephasing times, τ_Ω and τ_Δ, as shown in Figs. 2(a) and 2(b). In Fig. 3(a) both these dephasing times are shown as functions of $\Delta(0)/\Omega$ for $\alpha=0.0006$. Moreover, τ_Δ is compared to the WCC result τ_w of Eq. (10). This comparison is expected to work well for $J(\Omega)/\Omega \ll 1$, and indeed it does [here $J(\Omega)/\Omega\approx 0.08$]. Due to the small coupling between the two-level-system and harmonic oscillator, $g\approx 0.06\Omega$, the dependence of τ_Ω on $\Delta(0)$ is very weak.

For $\Delta(0)$ close to Ω, the two resonances in $C(\omega)$ merge to a symmetric double-peak structure as shown in the inset of Fig. 3(a). Here a characterization by two different time scales becomes difficult. Therefore the corresponding data points in Fig. 3(a) have not been included.

Stronger coupling to bath

Figure 3(b) shows τ_Δ, τ_Ω, and τ_w for a larger coupling strength of $\alpha=0.01$. Figure 4(a) shows one of the calculated correlation functions. Note that the stronger coupling α leads to a larger separation, or "level repulsion," between the Δ- and Ω- peaks than in Fig. 2. The inset of Fig. 3(b) shows the renormalized tunneling matrix element $\Delta(\infty)$ as a function the initial matrix element $\Delta(0)$. Very importantly, for $\Delta(0)\gtrsim\Omega$, Δ increases during the flow, whereas for $\Delta(0)\gtrsim\Omega$, it decreases.[15] This behavior can be understood from the fact that $f(\omega,l)$ in Eq. (6) changes sign at $\omega=\Delta$. Note also, that the upward renormalization toward larger $\Delta(\infty)$ in the inset of Fig. 3(b) is stronger than the downward one toward smaller values, i.e., the renormalization is *not symmetric* with respect to $\Delta(0)=\Omega$. The reason for this *asymmetry* lies in the fact that $f(\omega,l)$ has a larger weight for $\omega<\Delta$ than for $\omega>\Delta$. Also τ_Δ and even $\tau_w=1/J[\Delta(\infty)]$ in Fig. 3(b) show an asymmetric behavior with a steep but continuous increase at

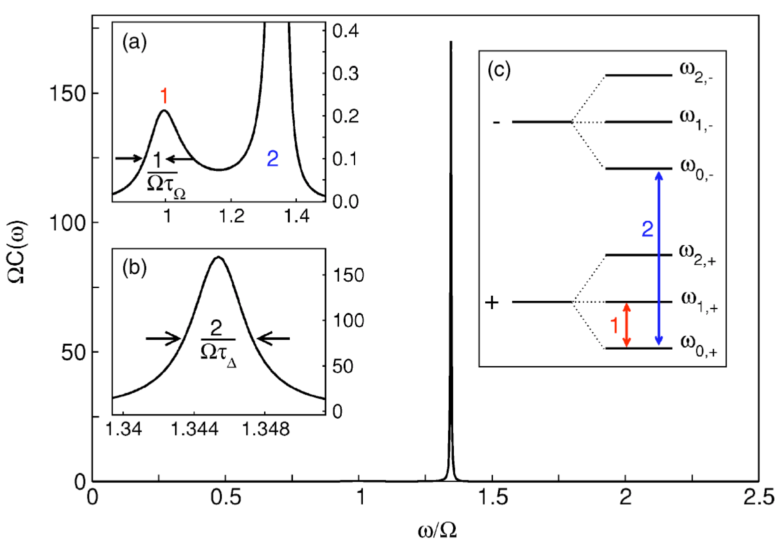

FIG. 2. (Color online) Spin-spin correlation function as a function of frequency for experimentally relevant parameters discussed in Ref. 3: $\alpha=0.0006$, $\Delta(0)=4$ GHz, $\varepsilon=0$ (this is the so-called idle state), $\Omega=3$ GHz, $\Gamma=0.02$, and $\omega_c=8$ GHz. The sum rule is fulfilled with an error of less than 1%. (a) Blowup of the peak region reveals a double peak, (b) blowup of the larger peak, (c) term scheme of a two-level system coupled to an harmonic oscillator, drawn for $\Delta(0)\gg\Omega$ ($\alpha=0.0006$ corresponds to $g/\Omega\approx 0.06$).

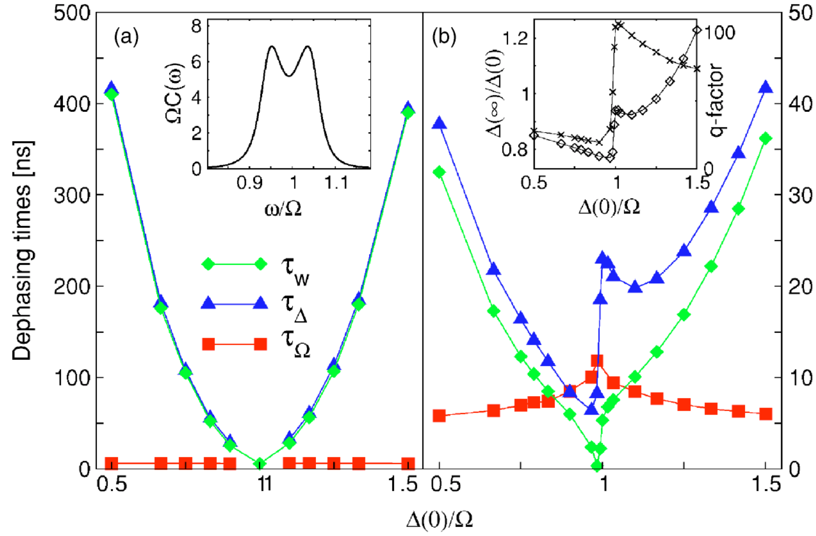

FIG. 3. (Color online) FER results for dephasing times (τ_Δ and τ_Ω) for the structured bath [Eq. (3)] compared to results from WCC (τ_w) given by Eq. (10), with $\Delta(\infty)$ occurring therein calculated using FER. Parameters are $\varepsilon=0$, $\Omega=3$ GHz, $\Gamma=0.02$. (a) weak coupling: $\alpha=0.0006$ and $\omega_c=8$ GHz. Sum rules are fulfilled with errors of less than 1%. Inset: Spin-spin correlation function for $\Delta(0)=0.987\Omega$. (b) stronger coupling: $\alpha=0.01$ and $\omega_c=9$ GHz. Sum rules are fulfilled with errors of less than 3%. Inset: $\times$—Renormalized tunneling matrix element; $\diamond$—quality factor $q=\tau_\Delta\Delta(\infty)/2$.

$\Delta(0)\approx\Omega$: dephasing times for $\Delta(0)>\Omega$ are larger than for $\Delta(0)<\Omega$. That this happens, although $J(\omega)$ is more or less symmetric around its maximum, is a direct consequence of the stronger renormalization of Δ for the latter case. Also the quality factor [q-factor, see inset in Fig. 3(b)] defined as $q=\tau_\Delta\Delta(\infty)/2$ shows this asymmetric behavior [being larger for $\Delta(0)>\Omega$ than for $\Delta(0)<\Omega$] with a steep increase at $\Delta(0)=\Omega$ from 8 to 43, i.e., by a remarkably large factor of ≈ 5. We consider the asymmetry of the renormalized tunneling matrix element, the dephasing time and the quality factor as the central results of this paper: By tuning Ω such that $\Delta(0)>\Omega$, dephasing times can be significantly enhanced (as compared to $\Delta(0)<\Omega$).[16] Note also that τ_Ω in Fig. 3(b) shows a much stronger dependence on $\Delta(0)$ than in Fig. 3(a). This is due to the stronger coupling ($g\approx 0.3\Omega$) of the two-level system to the bath in (b).

Nonzero bias

We now turn to the case of nonzero bias, $\varepsilon\neq 0$. A second-order perturbation calculation, analogous to the zero-bias case, shows that a third resonance in $C(\omega)$ is expected to show up at an energy scale $\Delta_\varepsilon+\Omega$. Indeed it does, as exemplified in Fig. 4(b), which shows a typical result for $C(\omega)$ for nonzero bias.[18] With every resonance we associate a dephasing time (analogous to the zero bias case). Figure 5(a) shows all three dephasing times (τ_Δ, τ_Ω, and $\tau_{\Delta+\Omega}$) as a function of ε. τ_Δ is compared to the weak coupling result τ_w. As expected, τ_Δ shows a minimum at $\varepsilon_{\min}\approx\sqrt{\Omega^2-\Delta^2(0)}$, which corresponds to the maximum of $J(\omega,l=0)$. Beyond $\varepsilon_{\min}$, τ_Δ increases whereas τ_Ω and $\tau_{\Delta+\Omega}$ decrease. This is the expected behavior: In the limit $\varepsilon\to\infty(\Delta\to 0)$, $C(t)$, respectively, $C(\omega)$ should become independent of all bath characteristics, i.e., $C(t)\to 1$ and $C(\omega)\to\delta(\omega)$. In this limit the dephasing times should show the following behavior: $\tau_\Delta\to\infty$, $\tau_\Omega\to 0$, and $\tau_{\Delta+\Omega}\to 0$. In Figs. 5(b) and 5(c) and the renormalized tunneling matrix element and the quality factor are shown as function of the bias. Note that since ε is not renormalized [see Eq. (6), $\Delta_\varepsilon(\infty)$ as a function of ε/Ω does not show a strong asymmetry, in contrast to the case $\varepsilon=0$. As a direct consequence, dephasing times and quality factors do not change much at $\varepsilon_{\min}$. Finally, τ and q as a function of $\Delta(0)$ for fixed ε can be shown[12] to show a qualitatively similar behavior to Fig. 3.

IV. SUMMARY

We used FER to study a two-level-system coupled to a damped harmonic oscillator for arbitrary ratios of Δ/Ω. We

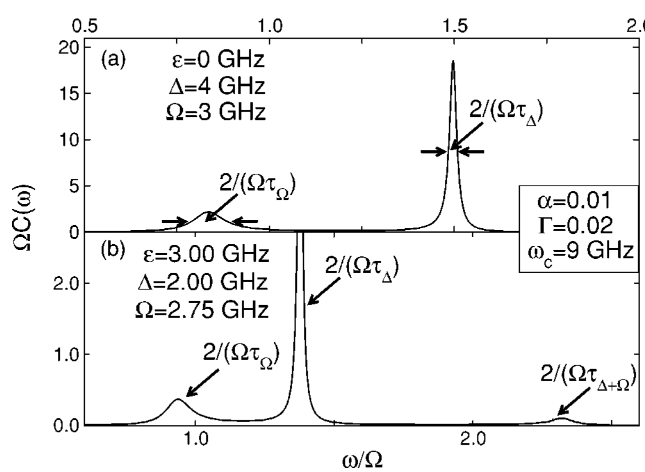

FIG. 4. Spin-spin correlation function for the structured bath [Eq. (3)] as a function of frequency.[18] The maximum height of the middle peak in (b) is ≈ 7.2.

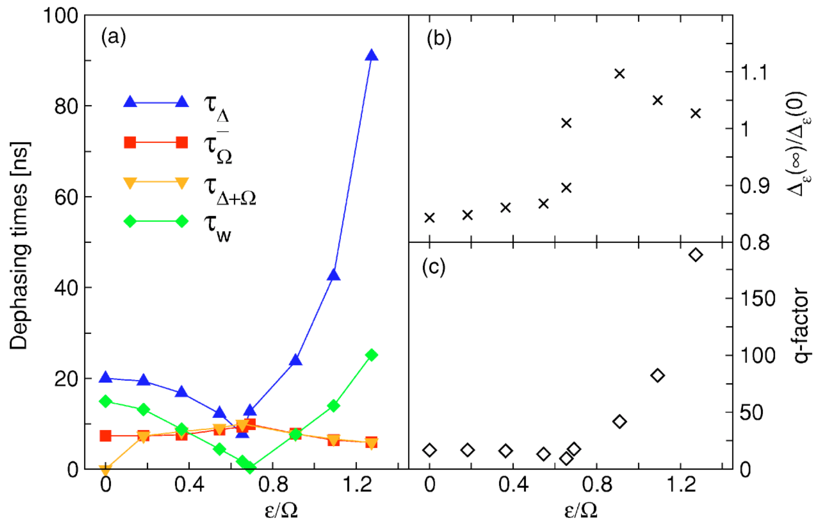

FIG. 5. (a) FER results for dephasing times (τ_Δ, τ_Ω, and $\tau_{\Delta+\Omega}$) for the structured bath [Eq. (3)], compared to results from WCC (τ_w) given by Eq. (10). (b) Renormalized tunneling matrix element. (c) q-factor $q=\tau_\Delta\Delta_\varepsilon(\infty)/2$. Parameters: $\alpha=0.01$, $\Delta(0)=2$ GHz, $\Omega=2.75$ GHz, $\Gamma=0.02$, and $\omega_c=9$ GHz.

find that by tuning the system into the regime $\Delta>\Omega$, which is studied here, dephasing times and q-factors can be significantly enhanced.

Note added in proof. Recent numerical simulations[19] using the QUAPI method for model (1) show similar behavior to our results. A quantitative comparision is, however, difficult since the QUAPI is restricted to finite temperature.

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