PHYSICAL REVIEW B **86**, 241102(R) (2012)

Fermionic and bosonic ac conductivities at strong disorder

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(Received 15 October 2012; published 6 December 2012)

We study the ac conduction in a system of fermions or bosons strongly localized in a disordered array of sites with short-range interactions at frequencies larger than the intersite tunneling but smaller than the characteristic fluctuation of the on-site energy. While the main contribution $\sigma_0(\omega)$ to the conductivity comes from local dipole-type excitations on close pairs of sites, coherent processes on three or more sites lead to an interference correction $\sigma_1(\omega)$, which depends on the statistics of the charge carriers and can be suppressed by a magnetic field. For bosons the correction is always positive, while for fermions it can be positive or negative depending on whether the conduction is dominated by effective single-particle or single-hole processes. We calculate the conductivity explicitly assuming a constant density of states of single-site excitations. Independently of the statistics, $\sigma_0(\omega) =$ const. For bosons, $\sigma_1(\omega) \propto \log(C/\omega)$. For fermions, $\sigma_1(\omega) \propto \log[\max(A,\omega)/\omega] - \log[\max(B,\omega)/\omega]$, where the first and the second term are, respectively, the particle and hole contributions, A and B being the particle and hole energy cutoffs. The ac magnetoresistance has the same sign as $\sigma_1(\omega)$.

DOI: 10.1103/PhysRevB.86.241102 PACS number(s): 71.10.−w, 77.22.Gm, 74.25.N−, 67.85.−d

Interestingly, interference and many-body phenomena in strongly disordered insulators may be comparable to or even greater than those in clean materials. Long ago fermionic many-body cotunneling at strong disorder had been addressed in the context of variable-range hopping in semiconductors.[1] The advent of modern cold-atom techniques[2] and recent experiments on disordered superconductive films[3–5] offered ample new opportunities to observe interference and many-body effects at strong disorder, yet to access them in both fermionic and bosonic systems. Recently it was shown in Ref. 6 and later discussed in Ref. 7 that interference effects in bosonic systems are particularly strong due to the constructive interference between all low-energy single- and many-boson cotunneling processes, which leads, e.g., to a huge positive magnetoresistance[8,9] and broadens the superfluid phase of the bosons.

The dc transport at strong disorder requires inelastic hopping of charge carriers through the whole sample, assisted by absorption and emission of neutral excitations, e.g., phonons.[10,11] The ac conductivity may come from local excitations and does not vanish even in the limit of infinitesimal dissipation.[12] The simplest local excitation is a particle-antiparticle dipole created by an external field on a pair of neighboring sites[13]—impurities or quasilocalized states. To study ac magnetoresistance and the Hall effect in hopping insulators[14–17] one has to consider processes on three or more sites.[13] So far studies of magnetotransport and interference phenomena have concerned fermionic charge carriers (electrons) and have not addressed the influence of quantum statistics.

In this Rapid Communication we calculate the ac conductivity of strongly disordered fermionic and bosonic insulators at zero temperature. We demonstrate that the coherent processes on three or more sites cause the interference correction to the conductivity, which depends on the statistics of the charge carriers. For bosons the correction is always positive, while for fermions it may be positive or negative if the conductivity is dominated by particles or holes, respectively. The latter holds even in the case of noninteracting fermions. We calculate the conductivity explicitly for sufficiently large frequencies, assuming that the density of the charge carrier states is constant at respective energies.

Our results apply equally to charged particles in disordered media and to neutral cold atoms in optical lattices. In the latter case the conductivity should be understood as a response to a tilt of external potential.

Model. A strongly disordered insulator may be modeled as an array of random-energy sites, e.g., electron puddles, grains, impurities, quasilocalized states, with weak intersite tunneling. We assume that the tunneling conserves particle spin and that the characteristic on-site interactions significantly exceed the characteristic energies of conducting excitations. Then one may consider spinless particles, multiplying the conductivity by the spin degeneracy in the end. The generic Hamiltonian of the charge carriers reads

$$\hat{\mathcal{H}} = \sum_{\mathbf{r}} U_{\mathbf{r}}(n_{\mathbf{r}}) - \sum_{\mathbf{r}\neq\mathbf{r}'} t_{\mathbf{r}\mathbf{r}'} \hat{b}^\dagger_{\mathbf{r}} \hat{b}_{\mathbf{r}'}, \tag{1}$$

where $\hat{b}^\dagger_{\mathbf{r}}$ and $\hat{b}_{\mathbf{r}}$ are the (bosonic or fermionic) particle creation and annihilation operators on site $\mathbf{r}$, $t_{\mathbf{r}\mathbf{r}'}$ is the intersite tunneling element, and $U_{\mathbf{r}}(n_{\mathbf{r}})$ is the energy of $n_{\mathbf{r}} = \hat{b}^\dagger_{\mathbf{r}}\hat{b}_{\mathbf{r}}$ particles on site $\mathbf{r}$, a large random function of $n_{\mathbf{r}}$, which accounts for the interaction of particles and a random potential. In particular, these energies prevent superfluidity in the case of bosonic particles. The disorder manifests itself in the fluctuations of the on-site energies $U_{\mathbf{r}}(n_{\mathbf{r}})$, intersite couplings $t_{\mathbf{r}\mathbf{r}'}$, and random locations of the sites.

The intersite interactions are assumed to be negligible, which corresponds to a system of neutral cold atoms,[2] a Josephson network with large self-capacitances,[18] and can be also justified in systems with weak Coulomb interactions, provided all relevant energies sufficiently exceed the Coulomb gap.[11] We assume that each occupation number $n_{\mathbf{r}}$ uniquely determines the many-particle state on site $\mathbf{r}$ because of sufficiently strong on-site interactions, leading to large energy gaps between different orbital states of the $n_{\mathbf{r}}$ particles. Equation (1) is a generic Hamiltonian that describes, for instance, the Anderson model or the disordered Bose-Hubbard model in the case of fermions or bosons, respectively.

The strongly insulating regime requires the tunneling $t_{\mathbf{rr}'}$ to be small compared to the fluctuations of the on-site energies and to sufficiently quickly decay with distance. Thus, the ground state of the Hamiltonian (1) is close to the ground state of the on-site energies, which corresponds to the configuration of integers $n_{\mathbf{r}}^0$ minimizing the first term on the right-hand side of Eq. (1).[6] At low temperatures and frequencies only the lowest-energy excitations are important on each site,[6] corresponding to the occupation numbers $n_{\mathbf{r}}^0+1$ and $n_{\mathbf{r}}^0-1$, which will be referred to as particle and hole excitations on site $\mathbf{r}$, respectively, both in the case of bosons and fermions.

In the case of fermions the important lowest-energy excitations correspond to only one orbital state on each site, the interaction with the other electrons being equivalent to a static potential, and, without loss of generality, we may assume that the occupation numbers take only values $n_{\mathbf{r}}=0$ and $n_{\mathbf{r}}=1$. In the case of bosons we assume $n_{\mathbf{r}}^0 \gg 1$ for simplicity; however, the conclusions of this Rapid Communication hold at arbitrary average characteristic occupation numbers.

We calculate the conductivity assuming that on each site the density of states ν is constant up to the characteristic cutoffs E^p and E^h of the particle and hole excitation energies, and that the frequency exceeds the characteristic matrix elements of the intersite couplings but is smaller than the characteristic excitation energies,

$$J_{\mathbf{rr}'} \ll \omega \ll \nu^{-1}, \quad \max(E^p, E^h), \tag{2}$$

where $J_{\mathbf{rr}'} = t_{\mathbf{rr}'}$ for fermionic charge carriers, while for bosons $J_{\mathbf{rr}'} = t_{\mathbf{rr}'}(n_{\mathbf{r}}^0 n_{\mathbf{r}'}^0)^{1/2}$.

A uniform external field $\mathbf{E}(\omega)$ induces a current $I_{12}(\omega)$ on each pair of sites 1 and 2, which at low temperatures can be expressed through the retarded correlator of I_{12} and currents $I_{\mathbf{rr}'}$ on all the other pairs of sites using the Kubo formula:

$$I_{12}(\omega) = i(2\omega)^{-1} \sum_{\mathbf{r},\mathbf{r}'} \Pi_{12,\mathbf{rr}'}(\omega) U_{\mathbf{rr}'}, \tag{3}$$

$$\Pi_{12,\mathbf{rr}'}(\omega) = -i \int_0^\infty \langle [\hat{I}_{12}(t), \hat{I}_{\mathbf{rr}'}(0)] \rangle e^{i\omega t} dt. \tag{4}$$

Here $\hat{I}_{\mathbf{rr}'} = iq(t_{\mathbf{r}'\mathbf{r}} \hat{b}^\dagger_{\mathbf{r}'} \hat{b}_{\mathbf{r}} - t_{\mathbf{rr}'} \hat{b}^\dagger_{\mathbf{r}} \hat{b}_{\mathbf{r}'})$ and $U_{\mathbf{rr}'} = (\mathbf{r}' - \mathbf{r})\mathbf{E}$ are the current operator and the effective voltage drop between sites $\mathbf{r}$ and $\mathbf{r}'$, q is the particle charge, and $\mathbf{E}$ is the uniform amplitude of the electric field. In the absence of an external magnetic field the average current density evaluates

$$j = \frac{1}{2} n^2 \int \langle I_{\mathbf{rr}'} U_{\mathbf{rr}'} \rangle_{\text{dis}} |\mathbf{E}|^{-1} d^d(\mathbf{r}' - \mathbf{r}), \tag{5}$$

with n being the concentration of the sites, and $\langle \cdots \rangle_{\text{dis}}$ is our convention for the disorder averaging.

Two-site conduction. At infinitesimal dissipation the conduction comes from the resonant absorption of the electromagnetic field quanta ω by local particle-antiparticle excitations on sparse pairs of sites.[13] Creating more complicated excitations is suppressed by the smallness of the tunneling,

$$\alpha \equiv \left\langle \nu \sum_{\mathbf{r}'} J_{\mathbf{rr}'} \right\rangle_{\text{dis}} = n\nu \int \langle J_{\mathbf{rr}'} \rangle_{\text{dis}} d\mathbf{r}' \ll 1. \tag{6}$$

Clearly, the particle hopping between the two sites is not affected by quantum statistics, except maybe for the value of the coefficient before the hopping rate. We evaluate first the contribution of these trivial two-site processes to the conductivity before analyzing the interference and many-body corrections to it. We consider the limit of low temperatures $T \ll \omega$.

Two-site processes correspond to the terms with $\mathbf{r} = 1(2)$, $\mathbf{r}' = 2(1)$ in Eq. (3). Evaluating the correlator of the current I_{12} with itself we obtain the real part of the two-site conductance:

$$G_{12} = \pi q^2 \omega^{-1} |J_{12}|^2 \delta(E_2 - E_1 - \omega), \tag{7}$$

where E_1 and E_2 are the energies of the particle on the respective sites, the ground state being close to the particle residing on site 1. The delta function in Eq. (7) reflects the resonant absorption. Actually, in the presence of a finite weak dissipation it should be ascribed a certain width γ, the concentration of resonant pairs of sites $\sim n\gamma\nu$ being very small. Unless quenched disorder is long correlated, the intersite couplings $J_{12} = J_{|\mathbf{r}_2 - \mathbf{r}_1|}$ and the on-site energies $E_{1,2}$ fluctuate independently.

The conductances (7) between pairs of sites lead to the conductivity

$$\sigma_0 = \pi n^2 q^2 (2\omega d)^{-1} \int \left\langle J_\xi^2 \right\rangle_{\text{dis}} \xi^2 \nu_{\text{dip}}(\omega, \xi) d^d \boldsymbol{\xi}, \tag{8}$$

with $\nu_{\text{dip}}(\omega,\xi) = 2\langle \delta(E_2 - E_1 - \omega) \rangle_{\text{dis}}$ being the density of states of dipole excitations, particle-hole pairs of size ξ, factor 2 accounting for the two possible polarizations of a dipole at the same E_1 and E_2.

For a constant density of states ν of the single-site particle and hole excitations considered in this Rapid Communication, we find $\nu_{\text{dip}} = 2\nu^2\omega$, which yields a frequency-independent conductivity

$$\sigma_0 = \pi n^2 \nu^2 q^2 d^{-1} \int \left\langle J_\xi^2 \right\rangle_{\text{dis}} \xi^2 d^d \boldsymbol{\xi}. \tag{9}$$

Let us emphasize that the conductivity $\sigma_0(\omega)$ is constant only in the frequency range under consideration. At higher frequencies, violating the second of inequalities (2), the conductivity decays due to the lack of sufficiently high-energy states. At smaller frequencies, when the first of inequalities (2) no longer holds, the conductivity is described by the famous Mott's formula[12,19] and decreases $\propto \omega^2$.

Frequency-independent conductivity, Eq. (9), can be understood from the Mott's formula as follows. A small voltage V, applied to a resonant pair of sites, $E_1 - E_2 = \omega$, with a small intersite coupling $J \ll \omega$, makes a perturbation with the off-diagonal entry $\sim qVJ/(E_1 - E_2) \propto \omega^{-1}$. The Mott's formula, on the opposite, applies when the typical coupling exceeds the frequency, $\omega \lesssim \langle J \rangle_{\text{dis}}$. Then the matrix element of the intersite transitions is nonperturbative in J/ω and does not depend on frequency (except, maybe, for a logarithmic factor). Because the conductivity is quadratic in the matrix element, $\sigma(\omega) \propto \sigma_{\text{Mott}}(\omega)\omega^{-2} = \text{const}$ at high frequencies under consideration, $\omega \gg \langle J \rangle_{\text{dis}}$.

Three close sites is the smallest cluster of sites that accounts for the interference and nontrivial quantum statistics effects. The contribution of larger clusters to the conductivity is suppressed due to the smallness of the intersite tunneling, Eq. (6).

There are two possibilities for the lowest-energy single-site excitations in a cluster of three sites: (1) A hole can occur on one site and an extra particle on each of the other two, or, vice versa, (2) a particle excitation can occur on one site and a hole on each of the other two.

In the first case the dynamics of the three sites is equivalent to the hopping of a single particle between these sites, described, both in the case of bosons and fermions, by the effective Hamiltonian

$$H_3^{\text{particle}} = \begin{pmatrix} E_1 & -J_{12} & -J_{13} \\ -J_{21} & E_2 & -J_{23} \\ -J_{31} & -J_{32} & E_3 \end{pmatrix}, \tag{10}$$

where $E_{\mathbf{r}}$ is the energy of the particle on site $\mathbf{r}$, $E_1 < E_2, E_3$. The three-site Hamiltonian in the form (10) has been used in a number of works (cf., e.g., Refs. 14, 15, and 20) to study the Hall effect in hopping insulators.

Single-hole hopping. In the second case the dynamics of the three sites is equivalent to the hopping of a lack of a particle between these sites. However, the effective Hamiltonian will depend dramatically on the statistics of the charge carriers.

Indeed, the state of several fermions on several sites is antisymmetric with respect to the permutations of the fermions, which makes the sign of the tunneling element between two sites depend on the occupation numbers of the other sites, $\langle 1_i 0_k | \hat{b}_i^\dagger \hat{b}_k | 0_i 1_k \rangle = (-1)^{\sum_{j=i+1}^{k-1} n_j}$ for $i \leqslant k-2$.[21] For bosons all the signs are the same. From Eq. (1) we find the effective Hamiltonian which describes the hopping of a hole on three sites:

$$H_3^{\text{hole}} = \begin{pmatrix} E_1 & -J_{21} & \mp J_{31} \\ -J_{12} & E_2 & -J_{32} \\ \mp J_{13} & -J_{23} & E_3 \end{pmatrix}, \tag{11}$$

where the upper and lower signs apply to bosons and fermions, respectively, and $E_{\mathbf{r}}$ is the energy of a state with a hole on site $\mathbf{r}$, $E_1 < E_2, E_3$.

The hopping of a single particle or hole is not a many-body problem effectively. Nevertheless, Eqs. (10) and (11) show that the parameters of the respective single-body Hamiltonian *qualitatively depend on the statistics*: for bosons (particles and holes) and fermionic particles the signs of all tunneling elements are the same, while for a fermionic hole the sign is alternating. Below we demonstrate that this difference manifests itself, under certain conditions, in the sign of the interference correction to the ac conductivity.

The currents between three sites may be found, using Eqs. (3) and (4), straightforwardly from the Hamiltonians (10) and (11). Because the interference correction to the conductivity has a relative smallness α, Eq. (6), compared to the two-site contribution (9), it is sufficient to find the currents up to the third order in the small couplings J, corresponding to the first-order-in-α correction to the two-site current $I_{12} = G_{12}U_{12}$, Eq. (7).

Again, conduction requires some resonant excitation be present on the three sites. Below we assume that sites 1 and 2 in the cluster under consideration are resonant, i.e., $E_2 - E_1 \approx \omega$. The probability of finding one more resonant pair in the same cluster $\sim\gamma\nu$ is negligible.

In the absence of an external magnetic field all the couplings may be chosen to be real and positive, $J_{\mathbf{rr}'} = |J_{\mathbf{rr}'}|$. Diagonalizing the Hamiltonians (10) and (11), and evaluating the correlators of the currents, from Eqs. (3) and (4) we find the currents due to the hopping of particles and holes on three sites:

$$I_{12} = \pi q^2 \omega^{-1} \delta(E_2 - E_1 - \omega) \bigg\{ |J_{12}|^2 U_{12} \pm J_{12} J_{23} J_{13} \left[\frac{U_{13}}{E_3 - E_2} + \frac{U_{32}}{E_3 - E_1} \right] \bigg\}, \tag{12}$$

$$I_{13} = \pm \pi q^2 \omega^{-1} \delta(E_2 - E_1 - \omega) J_{12} J_{23} J_{13} \frac{U_{12}}{E_3 - E_2}, \tag{13}$$

$$I_{23} = \pm \pi q^2 \omega^{-1} \delta(E_2 - E_1 - \omega) J_{12} J_{23} J_{13} \frac{U_{12}}{E_3 - E_1}, \tag{14}$$

where the upper signs apply to fermionic particles and to bosonic particles and holes, and the lower signs to fermionic holes only.

The first line of Eq. (12) is the current due to the processes on the two resonant sites, while the second line of Eq. (12) and Eqs. (13) and (14) are, respectively, the interference correction to the two-site current and the extra currents due to the presence of the third site.

Bosonic interference correction. In most three-site clusters under consideration, the excitation energy E_3 on the third site greatly exceeds the energies $E_1, E_2 \sim \omega$ on the two resonant sites due to the condition (2).

In the case of bosons the direct current between sites 1 and 2 is increased by the presence of the third site at $E_3 \gg E_1, E_2$, yet the extra current $I_{13} \approx I_{32}$, flowing through the third site, additionally enhances transport between sites 1 and 2. This reflects the general principle that all low-energy cotunneling processes in a bosonic system interfere constructively, effectively enhancing low-energy transport.[6,22]

To find the conductivity it is convenient to average first the currents in a three-site cluster with respect to the energy of the excitation on site 3. The positions of sites fluctuate independently of the couplings and can be averaged separately. Using Eqs. (5) and (12)–(14) we find the interference correction to the conductivity σ_0, Eq. (9), for a bosonic system with a constant density of states of single-site excitations at $\omega \ll E^{p,h}$:

$$\sigma_1(\omega) = A \log(E^p/\omega) + A \log(E^h/\omega), \tag{15}$$

$$A = 2\pi d^{-1} n^3 \nu^3 \int \langle J_\zeta J_\xi J_{|\boldsymbol{\xi}-\boldsymbol{\zeta}|} \rangle_{\text{dis}} \xi^2 d^d \boldsymbol{\xi} d^d \boldsymbol{\zeta}. \tag{16}$$

The logarithms in Eq. (15) come from integrating Eqs. (12)–(14) with respect to the third-site energies at $\omega \ll E_3 \ll E^{p,h}$ and must be smaller than α^{-1} to ensure the validity of the perturbation theory. The frequency ω serves as an effective low-energy cutoff due to the saturation of the average current corrections at small energies on the third site, $0 < E_3 - E_1 \lesssim E_2 - E_1 = \omega$. If the frequency ω exceeds the energy cutoffs E^p or E^h of the particle or hole excitations, the respective contribution in Eq. (15) vanishes.

Equations (15) and (16) show that in a bosonic system both particle and hole contributions to the interference correction to the conductivity are positive.

Fermionic interference correction. The single-particle and single-hole processes contribute to the interference correction with different signs in the case of fermions:

$$\sigma_1 = A\log[\max(E^p,\omega)/\omega] - A\log[\max(E^h,\omega)/\omega], \quad (17)$$

where constant A is defined by Eq. (16). The cutoffs E^p and E^h depend on the level of doping of the insulator. If the band of the localized states is nearly empty, then $E^p \gg E^h$, and the interference correction to the conductivity is positive. An almost filled band corresponds to $E^p \ll E^h$, leading to a negative correction (17).

In the whole frequency range, Eq. (2), the correction is positive or negative if the conduction is dominated, respectively, by particles or holes. At sufficiently small frequencies, $\omega \ll E^{p,h}$, the correction is frequency independent, $\sigma_1 = A\log(E^p/E^h)$. Despite this, it can be separated from the trivial two-site contribution σ_0 using the suppression of interference by the magnetic field, as we discuss below. If the frequency lies between the two cutoffs, $E^{p,h} \ll \omega \ll E^{h,p}$, the conduction is dominated by the excitations with the higher cutoff, $\sigma_1 = \pm A\log(E^{p,h}/\omega)$. Let us notice that in the latter regime of an effectively one type of fermionic charge carriers, the frequency dependency of σ_1 is the same as for bosons, Eq. (15), but has a different sign if the charge carriers are holes.

Suppression by gauge fields. In presence of a magnetic field the tunneling elements of the charged particles acquire phases

$$J_{\mathbf{r}\mathbf{r}'} = |J_{\mathbf{r}\mathbf{r}'}|\exp\left(iqc^{-1}\int_{\mathbf{r}}^{\mathbf{r}'}\mathbf{A}(\boldsymbol{\xi})d\boldsymbol{\xi}\right), \quad (18)$$

where $\mathbf{A}$ is the vector potential, related to the magnetic field, and we neglected the modification of the on-site wave functions. In the case of neutral atoms in optical lattices, the role of the magnetic field may be played by artificial gauge fields, induced by rotation or the Berry phases of atomic levels.[2]

The phase factors fluctuate randomly, due to the random relative positions of the sites. This does not affect the two-site contribution to the conductivity σ_0 but destroys the interference corrections to it if the magnetic flux through a characteristic three-site cluster exceeds one flux quantum,[6] $qB\lambda^2 \gg 1$, where λ is the characteristic radius of the coupling $J_{\mathbf{r}\mathbf{r}'}$.

Charge carriers with spin projections s_z acquire additional energies $-s_zB$ in the magnetic field, which has a negligible effect on the conductivity provided $|s_zB| \ll E^{p,h}$. Thus, the interference correction determines the sign of the magnetoresistance: positive for bosons, and positive or negative for fermions depending on the doping level.

Discussion. We studied ac transport in strongly disordered systems. The conductivity is dominated by statistics-independent processes on pairs of sites, while larger site clusters give rise to the interference correction, which is always positive for bosons and can have either sign for fermions.

Indeed, different commutation rules for bosonic and fermionic variables lead to a different character of the interference effects.[22] All low-energy bosonic processes interfere constructively, effectively enhancing transport[6] and leading to a positive bosonic interference correction.

Fermionic many-body processes can have different signs. We have shown that particle-dominated fermionic processes give a positive correction, which is hole dominated—negative, even in the case of noninteracting fermions. At first glance, the transport of noninteracting particles and holes is a single-body problem and should not depend on the statistics of the charge carriers. Nevertheless, the parameters of the effective single-body Hamiltonians depend on the statistics, which in the case of fermions manifests itself in the alternating signs of the effective intersite couplings, Eq. (11).

We have calculated the conductivity explicitly, Eqs. (9), (15), and (17), under the assumptions of a constant density of states, low temperature, and negligible intersite interactions at relevant energies. However, our qualitative conclusions for the sign of the ac magnetoresistance are valid for arbitrary interactions, densities of states, and temperatures smaller than the on-site energy fluctuations. Experimentally one can suppress the interference correction by the magnetic field or artificial gauge fields and thus verify its sign. In a strong fermionic insulator the interference contribution to the ac magnetoresistance changes sign when changing the doping level.

Our results can be tested straightforwardly in experiments on spin-polarized fermionic[23] and bosonic[24] cold atoms localized in incommensurate optical lattices[25] or in a random potential.[23,24] The interactions in these systems are short ranged, yet the disorder and the interaction strength are easily controlled.[2] These systems may be in the insulating state, which implies the smallness of the intersite coupling compared to the fluctuations of the on-site energies. The respective frequencies [cf. Eq. (2)] lie in an easily accessible kilohertz range and exceed the typical temperatures $T \leqslant 10$ nK. Thus, all the assumptions of this Rapid Communication are fulfilled in such experiments, which allows one to verify not only our qualitative conclusions for the signs of magnetoresistance but also the explicit dependencies of the conductivity on frequency.

The discussed effects can be observed also in disordered superconductive films in the insulating state. Again, strong insulation indicates the smallness of the tunneling. The characteristic frequencies used in the recent experiments, Refs. 26 and 27, $\omega \sim 10$–100 GHz, significantly exceed achievable temperatures $T \sim 0.1$ K ~ 1 GHz. The behavior of the ac conductivity in magnetic field would help one to identify which charge carriers, electrons or Cooper pairs, dominate transport in those materials.

We appreciate insightful discussions with B. L. Altshuler, D. M. Basko, and V. E. Kravtsov. We are particularly indebted to Yu. M. Galperin for valuable discussions and suggestions on improving the manuscript. This work has been financially supported by SFB Transregio 12.

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