Strong coupling of a qubit to shot noise

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(Received 12 January 2007; published 10 April 2007)

We perform a nonperturbative analysis of a charge qubit in a double-quantum-dot structure coupled to its detector. We show that a strong detector-dot interaction tends to slow down and halt coherent oscillations. The transitions to a classical and a low-temperature quantum overdamping (Zeno) regime are studied. In the latter, the physics of the dissipative phase transition competes with the effective shot noise.

DOI: 10.1103/PhysRevB.75.165308 PACS number(s): 73.21.La, 72.70.+m, 03.67.Lx, 05.40.Ca

The study of fluctuations and noise provides deep insights into quantum processes in systems with many degrees of freedom. If coupled to a few-level system such as a qubit, fluctuations usually lead to destabilization of general qubit states and induce decoherence and energy relaxation. One important manifestation is the back-action of detection on qubits.[1] This topic has been extensively studied in the regime of weak coupling between qubit and noise source.[2] It has been shown that the qubit dephases into a mixture of qubit eigenstates (dephasing), whose classical probabilities thermalize to the noise temperature at a longer time scale. Another recent work[3] looked into a strong-coupling situation in a single degree of freedom between a driven qubit and its detector, which in turn has been weakly coupled to a heat bath. Unlike this setup, we will examine in the following a qubit that is uniformly strongly coupled to a nonequilibrium heat bath. Mesoscopic charge detectors such as quantum point contacts[4] (QPC's) and radio-frequency single electron transistors[5] (rf-SET's), whose low-temperature noise is shot noise,[6,7] are particular powerful detectors as they provide high resolution[8] and potentially reach the quantum limit. A particular attractive regime for qubit applications is the quantum nondemolition (QND) regime, realized if the qubit Hamiltonian and the qubit-detector coupling commute.[1,9]

We study a quantum point contact potentially strongly coupled to the coordinate (left or right) of a double-quantum-dot charge qubit[10,11] by a nonperturbative approach involving Gaussian and noninteracting blip approximations. We analyze the qubit at the charge degeneracy point, where the two lowest-energy eigenstates are delocalized between the qubits. In the weak-coupling regime, low-temperature relaxation would thus always delocalize charge. We show that, in strong coupling, the qubit state gets localized in one of the dots. Localization is manifest by a suppression of both the coherent oscillations and the incoherent tunneling rate. This "freezing" of the state also occurs at high bias and can, e.g., lock an excited state. Thus, in the strong-coupling regime, the environment naturally pushes the effective *dynamics* naturally to the QND limit even though the bare Hamiltonian is *not* QND, because the qubit Hamiltonian and the qubit-detector coupling *do not* commute in the model under consideration. With QND dynamics, we refer to the case of no error-inducing transitions between the qubit eigenstates and no oscillations of the measured observable even if the measurement takes significant time—both satisfy the requirement that the measurement can be repeated over and over without deteriorating the result. We point out the analogy of this physics to the case of the dissipative phase transition in oscillator bath models,[12] which in the QPC competes with the nonequilibrium induced by the voltage driving the shot noise.

We consider the case of a degenerate two-state system (TSS), realized by the charge states in a double quantum-dot-structure (see Fig. 1). For this approximation to hold, all energies quoted henceforth should be lower than the double-dot charging energy. These charge states can be read out by the current through a nearby quantum point contact. The Hamiltonian for the TSS with time-dependent fluctuation $\tilde{\varepsilon}(t)$ reads

$$H_{\rm sys} = \frac{\hbar}{2}\begin{pmatrix} \tilde{\varepsilon}(t) & \Delta \\ \Delta & -\tilde{\varepsilon}(t) \end{pmatrix} \rightarrow \tilde{H}_{\rm sys} = \frac{\hbar\Delta}{2}\begin{pmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{pmatrix}. \quad (1)$$

In the last step of Eq. (1), we applied a Polaron transformation[13] introducing the fluctuating phase $\phi = \int^t dt' \tilde{\varepsilon}(t')$, with $\tilde{\varepsilon}(t) = \varepsilon + \delta\varepsilon(t)$, for the tunneling matrix elements in the qubit. The microscopic foundation of the noise term $\delta\varepsilon(t)$ for a QPC is given in Refs. 6 and 14 and for an SET in Refs. 15–18.

Without loss of generality, we assume $\langle\hat{\sigma}_z(0)\rangle = 1$. We can now formally solve the Liouville equation. The expectation value of $\hat{\sigma}_z$, the difference of occupation probabilities of the dots, satisfies a closed equation

$$\begin{aligned}\langle\dot{\hat{\sigma}}_z(t)\rangle &= -\Delta^2\int_0^t dt' \cos[\varepsilon(t-t')]\mathrm{Re}[\langle e^{i\delta\phi(t)}e^{-i\delta\phi(t')}\rangle\langle\hat{\sigma}_z(t')\rangle] \\ &= -\Delta^2\int_0^t dt' \cos[\varepsilon(t-t')]e^{J(t-t')}\langle\hat{\sigma}_z(t')\rangle, \quad (2)\end{aligned}$$

where the first step of Eq. (2) is formally exact and the second step is equivalent to the noninteracting blip approxima-

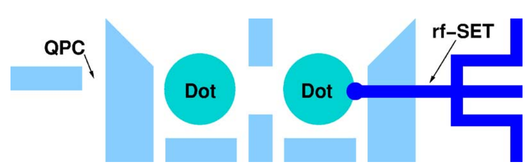

FIG. 1. (Color online) Schematic view of the double-dot system analyzed; see, e.g., Refs. 4 and 5. The QPC and rf-SET detectors can be used *alternatively*; both options are discussed in the paper.

tion (NIBA) usually obtained by path integrals.[19,20] This automatically includes a Gaussian approximation to the shot noise.[6] This approach is nonperturbative in ϕ and a good approximation in the two cases $\varepsilon=0$ and $|\varepsilon|\gg\Delta|$.

We start with the charge-degeneracy case $\varepsilon=0$. Here, we can solve Eq. (2) in Laplace space and find

$$\mathcal{L}[\langle\hat{\sigma}_z(t)\rangle]=\frac{1}{s+\Xi(s)}, \tag{3}$$

with the Laplace-transformed self-energy $\Xi(s)=\Delta^2\int_0^\infty dte^{-st}e^{J(t)}$. The phase correlation function $J(t)$ as seen by the dots reads[6]

$$J(t)=\frac{2\pi}{\hbar R_K}\int_{-\infty}^{\infty}d\omega\frac{|Z(\omega)|^2}{\omega^2}S_I(\omega)(e^{i\omega t}-1), \tag{4}$$

where $S_I(\omega)$ is the full current noise in the QPC that for sufficient environmental impedance greater than R_K is given[6] by

$$S_I(\omega)=\frac{4}{R_K}\sum_m^N D_m(1-D_m)\left\{\frac{\hbar\omega+eV}{1-e^{-\beta(\hbar\omega+eV)}}+\frac{\hbar\omega-eV}{1-e^{-\beta(\hbar\omega-eV)}}\right\}+\frac{4}{R_K}\sum_m^N D_m^2\frac{2\hbar\omega}{1-e^{-\beta\hbar\omega}} \tag{5}$$

and the transimpedance $Z(\omega)$ between qubit and point contact. In Eq. (5), V is the bias voltage of the QPC, R_K is the quantum resistance, and D_m is the transmission eigenvalue of the mth conductance channel. We observe that the finite voltage enhances the noise at low frequencies $|\omega|<V$ for both signs of ω, which can be interpreted as the occurence of hot electrons.

Semiclassical limit: We now discuss the resulting dynamics in a number of limiting cases. We start by first taking the limit $\omega\to 0$. This corresponds to $\hbar\Delta,\hbar\varepsilon\ll eV,k_BT$; i.e., the qubit probes the shot noise at energy scales much lower than its internal ones. Here, the noise expression [Eq. (5)] becomes frequency independent.[7] We can then compute the semiclassical spectral function $J_c(t)=-\gamma_c t$. Here, we have assumed a frequency-independent transimpedance controlled by a dimensionless parameter κ, $|Z(\omega)|^2\approx\kappa^2R_K^2$ and $\gamma_c=2\pi^2\kappa^2R_KS_I(0)/\hbar$ with $S_I(0)=\frac{4}{R_K}\sum_m^N D_m(1-D_m)eV\coth\left(\frac{\beta eV}{2}\right)+\frac{4}{R_K}\sum_m^N D_m^2\frac{2}{\beta}\cdot S_I(0)$ represents the shot noise of the QPC in the low-frequency regime.[21] The self-energy is then readily calculated and analytical, so we can go back from Laplace to real time and obtain

$$\langle\hat{\sigma}_z(t)\rangle=\left[\cos(\omega_{\mathrm{eff,c}}t)+\frac{\gamma_c}{2\omega_{\mathrm{eff,c}}}\sin(\omega_{\mathrm{eff,c}}t)\right]e^{-(\gamma_c/2)t}, \tag{6}$$

where $\omega_{\mathrm{eff,c}}=\sqrt{\Delta^2-\frac{\gamma_c^2}{4}}$. We observe that the coherent oscillations of the qubit decay on a time scale γ_c^{-1} and get slowed down. At $\gamma_c=2\Delta$, the damping becomes critical and the oscillations disappear, ending up with a purely exponential overdamped regime at $\gamma_c>2\Delta$. This crossover corresponds to the classical overdamping of a harmonic oscillator. Even in the overdamped regime, the qubit decays exponentially to $\langle\hat{\sigma}_z(t)\rangle\to 0$ at long times; e.g., it gets completely mixed by the shot noise, whose noise temperature is high, $k_BT_{\mathrm{noise}}\simeq\max\{eV,k_BT\}\gg\hbar\Delta$. Note that it is possible to discuss the overdamped regime, where γ_c is *not* a small parameter and our theory is also non-Markovian [see Eq. (2)], capturing the necessary time correlations arising in strong coupling. Additionally, changes in the QPC transmissions (and therefore the Fano factor) do not play any role other than entering the total noise level.

Quantum limit: Now, we let $T\to 0$ and leave ω arbitrary. $S_I(\omega)$ reads in this limit

$$S_I(\omega)=\frac{4}{R_K}\left[\sum_m^N D_m(1-D_m)\{(\hbar\omega+eV)\theta(\hbar\omega+eV)+(\hbar\omega-eV)\theta(\hbar\omega-eV)\}+\sum_m^N D_m^2 2\hbar\omega\theta(\hbar\omega)\right]. \tag{7}$$

This shape is dominated by two terms, which resemble the Ohmic spectrum at low T, $S_\Omega\propto\omega\theta(\omega)$ with shifted origins of energy. For computing the quantum correlation function $J_q(t)$, an ultraviolet cutoff ω_c has to be introduced, which physically originates either from the finite bandwidth of the electronic bands in the microscopic Hamiltonian or from the high-frequency limitations of the transimpedance $Z(\omega)$. We end up with the long-time limit for $J_q(t)$ applicable at $\hbar\Delta\ll eV$:

$$J_q(t)=-\alpha_1+\alpha_2\ln\left[\left(\frac{eV}{\hbar}\right)^F\frac{1}{\omega_c}t^{F-1}\right]-\gamma_q t+i\alpha_3. \tag{8}$$

This holds for any number of channels. for simplicity we concentrate henceforth on the single-channel case. The Fano factor then is given by $F=1-D$. Here, we can introduce $\alpha_2=g=16\pi\kappa^2D$, the dimensionless conductance as seen by the qubit, $\alpha_1=g\gamma D$, $\alpha_3=\pi g/2$ and $\gamma_q=\pi g(1-D)eV/2\hbar$. The resulting self-energy is now nonanalytical:

$$\Xi(s)=\Delta_{\mathrm{eff}}^2\frac{(s+\gamma_q)^{gD-1}}{\left(\frac{eV}{\hbar}\right)^{gD}}e^{i\pi g/2}, \tag{9}$$

where we have introduced the effective tunnel splitting $\Delta_{\mathrm{eff}}^2=\Delta^2e^{-\gamma gD}\left(\frac{eV}{\hbar\omega_c}\right)^g\Gamma(-gD+1)$. In our regime $\omega_c\gg eV/\hbar\gg 1/t\simeq\Delta$, this expression resembles the renormalized Δ of the spin-boson model[19] and we have $\Delta_{\mathrm{eff}}\ll\Delta$. This is a sign of massive entanglement between system and detector.[22,23] Note that similar to the adiabatic scaling treatment in Ref. 12, the NIBA is compatible with forming entangled states between system and bath. This has been numerically confirmed, for the spin-boson model, in Ref. 22. An elegant approach to this system reflecting entanglement and use of the measurement result in the perturbative regime has been given in Ref. 24. The main difference in our shot noise case is that the infrared cutoff entering the renormalization and controlling the final expressions is V instead of Δ. In particular, Δ_{eff} grows with eV, which indicates that the hot electrons in nonequilibrium shot noise compete with the spin-boson-like suppression.

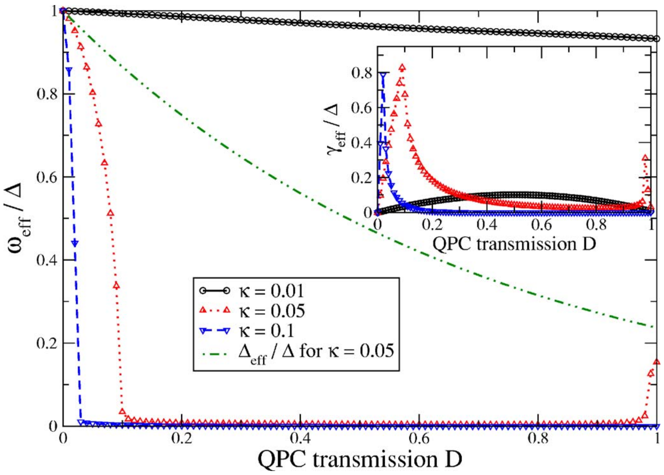

FIG. 2. (Color online) Quantum limit: imaginary parts of the numerically determined poles as a function of the QPC transmission D. The other parameters are $eV=10^2\hbar\Delta$ and $\omega_c=10^{12}\Delta$. Inset: real parts of the poles as a function of the QPC transmission D.

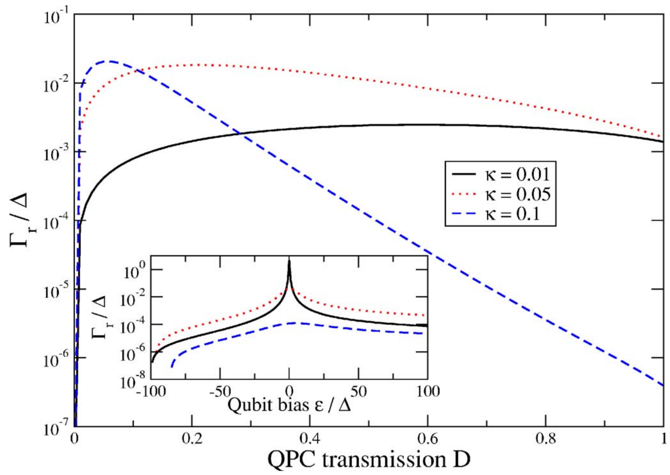

FIG. 3. (Color online) Quantum limit: relaxation rate Γ_r as a function of the QPC transmission D. The other parameters are $\varepsilon=10\Delta$, $\Delta=1.524\times10^9$ 1/s, $eV=10^2\hbar\Delta$, and $\omega_c=10^{12}\Delta$. Inset: relaxation rate Γ_r as a function of the qubit bias ε. Other parameters as above, but with $D=0.5$.

The self-energy is analytical only at $F=1$, which corresponds to the no-noise case $D=0$. Due to the generally nonanalytic self-energy, it is difficult to compute the full real-time dynamics by back-transformation to the time domain. The structure of the result will be $\langle\hat{\sigma}_z(t)\rangle=P_{\text{cut}}(t)+P_{\text{coh}}(t)+P_{\text{incoh}}(t)$.[19] For our case of $\varepsilon=0$, there is no incoherent exponential decay P_{incoh}. P_{cut} is a nonexponential branch cut contribution. In the following, we concentrate on the coherent part $P_{\text{coh}}(t)$, given through the poles $s_i=-\gamma_{\text{eff}}\pm i\omega_{\text{eff}}$ of Ξ with finite imaginary part, and hence this leads to damped harmonic oscillations with frequency ω_{eff} and decay rate γ_{eff}.

Close to $D=0$, we can characterize these poles perturbatively. We find a renormalized oscillation frequency ω_{eff}: namely, $\omega_{\text{eff}}=\text{Re}\Big[\sqrt{\Delta_p^2\big(1+\frac{i\pi}{2}g\big)-\frac{\gamma_q^2}{4}}\Big]$ whereas $\gamma_{\text{eff}}=\frac{\gamma_q}{2}\mp\text{Im}\Big[\sqrt{\Delta_p^2\big(1+\frac{i\pi}{2}g\big)-\frac{\gamma_q^2}{4}}\Big]$. Here, Δ_p^2 is defined as $\Delta_p^2=\Delta^2\big[1+g\ln\big(\frac{eV}{\hbar\omega_c}\big)\big]$.

For arbitrary F or D, we can solve the pole equation numerically: see Fig. 2. With the numerical results from Fig. 2, one can again calculate the Laplace back-transformation, where the two residues of the kind $a_{-1}=\frac{e^{s_i t}(s_i+\gamma_q)}{s_i(2-gD)+\gamma_q}$ have to be summed up. This leads finally again to decaying oscillations as already mentioned above. Note that the strong coupling does not only influence the dynamics in the form of a large decay rate but also through a strong renormalization of the effective tunnel coupling Δ_{eff}. In fact, we will discuss later in the quantum regime how the decay rate becomes small and can even vanish even though the qubit-detector coupling is strong.

We see that at sufficiently strong coupling to the detector, a finite Fano factor can lead to a complete suppression of the coherent oscillations, whereas the decay rate increases. Both these tendencies *together* show that a finite Fano factor brings the system closer to charge localization.

In fact, for sufficient damping, we can tune the tunneling frequency all the way to zero by increasing D. On the other hand, also γ_{eff} can become very small—in these points the detector completely localizes the particle up to nonexponential contributions. At other values of D, unlike the dissipative phase transition in the spin-boson model, the hot electrons driving the shot noise again drive the relaxation rate close to its bare value, and thus this resembles the classical overdamping case.

This scenario is not limited to $\varepsilon=0$. The NIBA permits one to reliably study the opposite regime $\varepsilon\gg\Delta$ as well. As already shown in Refs. 12 and 25, the resulting dynamics is dominated by incoherent exponential relaxation dominating over P_{coh} and P_{cut}. The relaxation rate is

$$\Gamma_r=2\,\text{Re}[\Xi(i\varepsilon+0)]=2\Delta_{\text{eff}}^2\,\text{Re}\left[\frac{(i\varepsilon+\gamma_q)^{gD-1}}{(eV)^{gD}}e^{i\pi g/2}\right]. \tag{10}$$

This again demonstrates the slowdown (through Δ_{eff}) of the decay to the other dot due to the interaction with the detector. Notably, this rate does not display standard detailed balance at $T=0$; rather, around $\varepsilon=0$, the rate is smeared out on a scale of γ_q, reflecting the role of the nonequilibrium shot noise temperature. We have plotted this result in Fig. 3.

Another view of this is that the effective size of the noncommuting term between qubit and detector, given by Δ_{eff}, is reduced; hence, the strong interaction brings the effective Hamiltonian closer to a QND situation.

On the other hand, such dynamics is known as the quantum Zeno effect. Note that unlike standard derivations,[1,9,26] we have derived our result in a nonperturbative way which is consistent with the necessary strong coupling and which retains the non-Markovian structure. The main difference between our work and the already mentioned references[1,9,26] is that in our self-energy $\Xi(s)$ also higher orders in the coupling (between qubit and detector) are included.

Summarizing the QPC results, we can observe that, on the one hand, the system shows traces of the physics of environment-induced localization, which competes with classical overdamping by effectively "hot" electrons at finite voltage and somewhat reinforced at finite Fano factor. This can be understood as follows: the dissipative phase transition occurs when the environmental noise is highly asymmetric in frequency and when the full bandwidth plays a role. At high voltage, the asymmetry of the shot noise spectrum is reduced.[6] In fact, the $\gamma_q t$ contribution in the correlation function $J_q(t)$ resembles the finite temperature term in the correlation function of the Ohmic spin-boson model—both terms originate from the zero-frequency part of the noise.

A similar analysis on back-action by strong coupling of a QPC to a quantum device—there an Aharonov-Bohm experiment[27]—has been done in Ref. 28. That work concentrates on a stationary situation and weak hopping into the dot, whereas in our case the dots are not connected to leads. The interdot interaction, however, is strong, and we concentrate on the real-time dynamics.

These results can be extended to shot noise sources other than QPC's. In fact, it may today be quite challenging to reach κ values high enough, such that slowdown and localization can be observed, when the noise source has only a few open channels. An attractive alternative is given by readout using *metallic* SET's fabricated on another sample layer;[5] see Fig. 1. In these devices, there are a number of rather opaque conductance channels.

In that case, we use the expression[15–18] of the voltage noise of the SET (only valid for small frequencies):

$$S_V(\omega,\omega_I) = 4\frac{E_{\mathrm{SET}}^2}{e^2}\frac{4\omega_I}{\omega^2+16\omega_I^2}, \tag{11}$$

where $E_{\mathrm{SET}}=\frac{e^2}{2C_{\mathrm{SET}}}$ is the charging energy of the SET and $\omega_I=I/e$ is the tunneling rate through the SET. Then the final result for $\langle\hat{\sigma}_z(t)\rangle$ is again the same as in Eq. (6). The difference, of course, is that γ_c is now defined as $\gamma_c=\frac{2\pi^2\kappa^2E_{\mathrm{SET}}^2}{\hbar R_K e^2\omega_I}$. The full quantum mechanical analysis in the low-temperature regime works along the same lines as the QPC case but goes beyond the scope of this article.

We performed a nonperturbative analysis of the quantum dynamics of a double-quantum-dot coupled to shot noise. We analyzed the crossover from underdamped to overdamped oscillations in the classical case. In the quantum case, we demonstrated that at strong coupling the oscillations show the same behavior, competing with a critical slowdown similar to the dissipative phase transition. This can be interpreted as the onset of a Zeno effect.

We thank M. J. Storcz, M. Sindel, L. Borda, A. Käck, J. von Delft, L. P. Kouwenhoven, and U. Weiss for clarifying discussions. This work was supported by DFG through SFB 631 and in part by the National Security Agency (NSA) and Advanced Research and Development Activity (ARDA) under Army Research Office (ARO) Contract No. P-43385-PH-QC.

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