Ohmic and Step Noise from a Single Trapping Center Hybridized with a Fermi Sea

Rogerio de Sousa,[1] K. Birgitta Whaley,[1] Frank K. Wilhelm,[2] and Jan von Delft[2]
[1]*Department of Chemistry and Pitzer Center for Theoretical Chemistry, University of California, Berkeley, California 94720-1460, USA*
[2]*Department Physik, CeNS, and ASC, Ludwig-Maximilians-Universität, Theresienstrasse 37, D-80333 München, Germany*
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We show that single electron tunneling devices such as the Cooper-pair box or double quantum dot can be sensitive to the zero-point fluctuation of a single trapping center hybridized with a Fermi sea. If the trap energy level is close to the Fermi sea and has linewidth $\gamma > k_B T$, its noise spectrum has an Ohmic Johnson-Nyquist form, whereas for $\gamma < k_B T$ the noise has a Lorentzian form expected from the semiclassical limit. Trap levels above the Fermi level are shown to lead to steps in the noise spectrum that can be used to probe their energetics, allowing the identification of individual trapping centers coupled to the device.

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Currently vigorous efforts are underway to achieve robust coherent control over artificial two-level systems (TLS) based on single charge tunneling in superconducting [1] or quantized semiconductor [2] islands. While the main motivation is to build a prototype for future quantum information technologies, the associated need for unprecedented isolation from external noise also makes TLS structures sensitive probes of fundamental fluctuations in the solid state [3]. An interesting example is the recent charge echo experiment by Nakamura and collaborators [4], for which it has been claimed in [5] that a *single* background charge trapping center, henceforth called "local level," is responsible for most of the coherence decay, rather than $1/f$ noise originating from several charge traps [4,6]. Although $1/f$ noise is usually predominant in macroscopic samples, the signature of its individual fluctuators is often identified in sensitive mesoscopic devices [7,8]. The standard phenomenological description of random telegraph noise (RTN) of a fluctuating variable ξ arising from a single local level assumes a noise spectrum given by [5,9,10]

$$\tilde{S}_{\xi}(\omega) = \frac{1}{2\pi}\int e^{i\omega t}\langle \xi(t)\xi(0)\rangle dt = \frac{\tilde{S}(0)}{(\omega\tau_c)^2 + 1}, \quad (1)$$

where the trap fluctuation time scale τ_c can in principle be computed microscopically. The phenomenological Eq. (1) should be contrasted with the universal Johnson-Nyquist (JN) voltage noise of a circuit with impedance Z arising from particle-hole excitations in a conductor, $\tilde{S}_V(\omega) = \mathrm{Re}(Z)\hbar\omega \coth(\hbar\omega/2k_BT)$ [11,12]. At high frequencies ($\hbar\omega \gg k_BT$) the linear dispersion (Ohmic, or more generally f-noise) regime resulting from JN noise was measured long ago with the help of the Josephson effect [13]. Here we show how a microscopic model that in the semiclassical limit, whose regime of applicability will be clarified later, leads to RTN [Eq. (1)], crosses over to f noise at low temperatures. Monitoring the excited state population of a single charge tunneling device allows the detection of the crossover from semiclassical to quantum fluctuations.

We focus on the noise generated by a local trapping center (TC), e.g., a dangling bond, located in the dielectric barrier close to one of the electrodes [6,14]. When the trap energy level $\tilde{\epsilon}$ is close to the Fermi level and the temperature is lower than its linewidth $\gamma \sim \hbar/\tau_c$, we find that the zero-point fluctuation of the trap generated by coupling to the Fermi sea becomes evident through the appearance of *universal* linear dispersion in the noise spectrum, which reflects the Ohmic spectrum of electron-hole excitations in the underlying Fermi sea. At high temperatures we recover Eq. (1). Furthermore, we show that trap levels above the Fermi level ($\tilde{\epsilon} > \epsilon_{\mathrm{F}}$) reveal themselves as steps in the noise spectrum, so that multiple levels may give rise to a staircase spectrum. We discuss the relevance of these results for recent measurements of noise by fast single-shot detection of the excited state population of a Cooper-pair box [15].

The model.—The Hamiltonian describing electrostatic coupling between a single electron tunneling device in the TLS regime and a TC is given by

$$\mathcal{H} = \frac{1}{2}\delta E_C\sigma_z + \frac{1}{2}E_J\sigma_x + (\epsilon_d + \lambda\sigma_z)d^{\dagger}d + \sum_k \epsilon_k c_k^{\dagger}c_k + \sum_k (V_{dk} + V'_{dk}\sigma_z)(d^{\dagger}c_k + c_k^{\dagger}d), \quad (2)$$

where σ_i are Pauli operators acting on the states $|\downarrow\rangle$ and $|\uparrow\rangle$, corresponding, respectively, to zero and one excess Cooper pair in the superconducting island [1,4]. The model also applies to an electron localized in the left or right dot of a double quantum dot structure [2]. The last two terms of Eq. (2) describe the tunneling of an electron in one of the metallic gates controlling the TLS to a TC located in the dielectric interface [6,14]. $d^{\dagger}$ is a creation operator for an electron in the TC, whose energy can assume the values $\epsilon_d \pm \lambda$ depending on the state of the TLS. The matrix element $V_{dk} \pm V'_{dk}$ models the tunneling amplitude between TC and an electron in the metallic gate ($c_k^{\dagger}$) with energy ϵ_k ($c_k^{\dagger}$ anticommutes with $d^{\dagger}$). The coupling between TLS and TC arises due to the modulation of TC parameters modeled by the electrostatic coupling constants λ and V'_{dk}. Previous work [6] did not contain the V'_{dk} term in Eq. (2), which arises due to the sensitivity of the TC wave function to the TLS state (see estimates below). For simplicity we study a model of spinless electrons, as is appropriate if TC double occupation is impossible due to Coulomb repulsion in Eq. (2). The last three terms of Eq. (2) compose the spinless Fano-Anderson Hamiltonian [16], which can be diagonalized exactly in the case $\lambda = V'_{dk} = 0$. This is achieved by the transformation

$$d^\dagger = \sum_k \nu_k \alpha_k^\dagger, \qquad c_k^\dagger = \sum_{k'} \eta_{k,k'} \alpha_{k'}^\dagger, \tag{3}$$

where α_k are dressed electron operators. In the thermodynamic limit $\sum_k \to \int d\epsilon g(\epsilon)$, with the bare electron density of states $g(\epsilon) = \sum_k \delta_{\epsilon,\epsilon_k}/\delta\epsilon_k = (3N/2\epsilon_{\rm F})\sqrt{\epsilon/\epsilon_{\rm F}}$. Here $\delta\epsilon_k$ is the mean energy spacing between nondegenerate levels of the electron gas and $\delta_{\epsilon,\epsilon_k}$ is the Kronecker delta. The continuous description of ν_k is then given by the spectral function $A(\epsilon) = \sum_k \frac{\nu_k^2}{\delta\epsilon_k}\delta_{\epsilon,\epsilon_k}$, which is interpreted as the mean density in energy of the admixture of the TC with the electron bath. The energy density for the electrons that do not participate in the admixture with the TC is given by $g'(\epsilon) = \sum_{k,k'} \frac{|\eta_{k',k}|^2}{\delta\epsilon_k}\delta_{\epsilon,\epsilon_k}$. Below we show that the response of the TLS to a TC depends only on the densities $A(\epsilon)$ and $g'(\epsilon)$. The exact solution for $A(\epsilon)$ in the thermodynamic limit has the Lorentzian form [16]

$$A(\epsilon) = \frac{\gamma/\pi}{(\epsilon - \tilde{\epsilon})^2 + \gamma^2}, \tag{4}$$

normalized to unity by virtue of the commutation relation $\{d, d^\dagger\} = 1$. The energy densities are related by

$$g'(\epsilon) = g(\epsilon) + \frac{d \ln g(\epsilon)}{d\epsilon}(\epsilon - \tilde{\epsilon})A(\epsilon). \tag{5}$$

Here the dressed trap energy $\tilde{\epsilon}$ differs from ϵ_d to second order in V_{dk}. The linewidth $\gamma = \pi V_{\tilde{\epsilon}}^2 g(\tilde{\epsilon})$ is essentially the Fermi golden rule for the decay of a localized state (TC) into the continuum (Fermi sea), with $V_{\tilde{\epsilon}}^2 \equiv \langle V_{dk}^2 \rangle_{\epsilon_k = \tilde{\epsilon}}$. Finally, we perform a rotation on the Pauli operators to get the transformed Eq. (2),

$$\mathcal{H} = \frac{1}{2}\hbar\omega\sigma_z + (\sin\theta\sigma_x + \cos\theta\sigma_z)\sum_{k,k'}\Big[\lambda\nu_k\nu_{k'}\alpha_k^\dagger\alpha_{k'} + V'_{dk'}\sum_{k''}(\nu_k\eta_{k',k''} + \nu_{k''}\eta_{k',k})\alpha_k^\dagger\alpha_{k''}\Big] + \sum_k \epsilon_k\alpha_k^\dagger\alpha_k, \tag{6}$$

with $\tan\theta = E_J/\delta E_C$ and $\hbar\omega = \sqrt{E_J^2 + \delta E_C^2}$. Equation (6) has the structure of a TLS coupling to electron-hole excitations in a new, effective Fermi sea, characterized by coupling constants $\lambda\nu_k\nu_{k'}$.

Ohmic JN noise.—The decay rate for a TLS initially prepared in the excited state $|\uparrow\rangle$ gives a measure of the noise spectral density at frequency ω through the golden-rule result [3]

$$\frac{1}{T_1} = \frac{\pi\sin^2\theta}{2\hbar^2}\Big[\tilde{S}_\lambda(\omega) + \tilde{S}_{V'}(\omega) + \tilde{S}_\lambda(-\omega) + \tilde{S}_{V'}(-\omega)\Big]. \tag{7}$$

Note that the weak coupling between the detector (TLS) and the noise source (TC) implies the TC remains in thermal equilibrium with the Fermi sea [12]. Here $\tilde{S}$ are Fourier transforms of the time-ordered correlation functions as in Eq. (1). $\tilde{S}_\lambda$ occurs due to the charging and discharging of the TC, where $\bar{n} = \langle d^\dagger d\rangle = \int A(\epsilon)f(\epsilon)d\epsilon$ is the average TC occupation and $f(\epsilon) = 1/[1 + e^{(\epsilon-\epsilon_{\rm F})/k_BT}]$ the Fermi function. The noise densities are calculated by Eqs. (3) and (6),

$$\tilde{S}_\lambda(\omega) = \frac{4\hbar\lambda^2}{2\pi}\int dt e^{i\omega t}\langle[d^\dagger(t)d(t) - \bar{n}][d^\dagger(0)d(0) - \bar{n}]\rangle_{\rm FS} = 4\hbar\lambda^2\int d\epsilon A(\epsilon)A(\epsilon - \hbar\omega)f(\epsilon - \hbar\omega)[1 - f(\epsilon)]. \tag{8}$$

Here, the d, $d^\dagger$ are taken in the interaction representation and $\langle\,\rangle_{\rm FS}$ denotes averaging over the free Fermi sea. A similar expression applies for $\tilde{S}_{V'}$, with the operator $d^\dagger d$ in Eq. (8) substituted by $\sum_k V'_{dk} d^\dagger c_k$ + H.c. Because we assume the TC is in thermal equilibrium, Eq. (8) satisfies the detailed balance condition $\tilde{S}(-\omega) = \exp(-\hbar\omega/k_BT)\tilde{S}(\omega)$. Moreover, the total noise $\int \tilde{S}_\lambda(\omega)d\omega$ is equal to the mean square fluctuation $S_\lambda(0) = 4\lambda^2(\bar{n} - \bar{n}^2)$, which is appreciable only if $\bar{n}$ is not close to zero or one. This happens only if $|\tilde{\epsilon} - \epsilon_{\rm F}| \lesssim \max\{k_BT, \gamma\}$, so Eq. (8) is only of appreciable size if $\tilde{\epsilon}$ is close enough to the Fermi energy. For $\tilde{\epsilon} = \epsilon_{\rm F}$ we have $\bar{n} = 1/2$ and $S_\lambda(0) = \lambda^2$ is maximum. Consider the noise spectrum at high temperature, $k_BT \gg \gamma$. Because the integral in Eq. (8) is appreciable only within the range of $A(\epsilon)$, the Fermi functions can be approximated by 1/2 and the resulting integral easily integrated by the residue method. Therefore the high-temperature noise is determined solely by the pole structure of $A(\epsilon)$. Using Eq. (4), we recognize the spectrum for random telegraph noise [Eq. (1)] with correlation time $\tau_c = \hbar/(2\gamma)$.

Now consider the opposite limit of $k_BT \ll \gamma$. For high frequencies ($\hbar\omega \gg \max\{|\tilde{\epsilon} - \epsilon_{\rm F}|, \gamma\}$) we have the asymptotic behavior $\tilde{S}_\lambda(\omega) \approx A(\epsilon_{\rm F} - \hbar\omega)(1 - \bar{n}) \sim 1/\omega^2$. For positive low frequencies ($0 \le \hbar\omega \ll \gamma$) but arbitrary $\hbar\omega/k_BT$, we may approximate Eq. (8) by

$$\tilde{S}_\lambda(\omega) \approx 4\hbar\lambda^2 A(\epsilon_{\rm F})^2\int d\epsilon f(\epsilon - \hbar\omega)[1 - f(\epsilon)] = 4\hbar^2\lambda^2 A(\epsilon_{\rm F})^2\omega[n(\omega) + 1], \tag{9}$$

where $n(\omega) = 1/[e^{\hbar\omega/k_BT} - 1]$ is the Bose function. Using detailed balance we get

$$\frac{\tilde{S}_\lambda(\omega)+\tilde{S}_\lambda(-\omega)}{2} \approx 2\hbar^2\lambda^2 A(\epsilon_F)^2\omega\coth\left(\frac{\hbar\omega}{2k_BT}\right), \quad (10)$$

which makes evident the universal behavior of low frequency noise arising due to a TC interacting with a Fermi sea; i.e., $\tilde{S}$ has the Johnson-Nyquist form for small ω, independent of the functional form of $A(\epsilon)$, provided that this is analytic at ϵ_F. The transition between Eqs. (1) and (9) is shown in Fig. 1. This behavior is a signature of the crossover from zero-point quantum to classical fluctuations for the electron-hole excitations whose energy $\hbar\omega$ lies within the hybridization bandwidth γ. In other words, the TC acts as a filter of bandwidth γ, allowing only electron-hole excitations of energy $|\hbar\omega| \lesssim \gamma$ to affect the TLS. When $k_BT < \gamma$, the environment behaves effectively like a flat band Fermi gas, with bosonic electron-hole pair excitations leading to Johnson-Nyquist noise. However, when $k_BT > \gamma$ the frequency of fluctuations with $|\hbar\omega| \lesssim \gamma$ are always within the semiclassical regime ($\hbar\omega < k_BT$), which leads to incoherent hopping of the TC charge into and out of the Fermi sea, giving rise to random telegraph noise.

Staircase noise.—We now turn to the effects of the V'_{dk} interaction. In the original Eq. (2) this term changes the hybridization coupling of the TC and Fermi sea, which transforms into a TLS-dependent modification in the dressed trap energy $\tilde{\epsilon}$. A calculation similar to Eq. (8) yields

$$\tilde{S}_{V'}(\omega) = 4\hbar\int d\epsilon[V'^2_{\epsilon-\hbar\omega}g'(\epsilon-\hbar\omega)A(\epsilon) + V'^2_{\epsilon}g'(\epsilon) \times A(\epsilon-\hbar\omega)]f(\epsilon-\hbar\omega)[1-f(\epsilon)]. \quad (11)$$

In contrast to the λ processes, this rate is finite for trap energies $\tilde{\epsilon}$ far from the Fermi level, i.e., $|\tilde{\epsilon}-\epsilon_F| \gg \{\gamma, k_BT\}$: The first term dominates if $\tilde{\epsilon} > \epsilon_F$, while the second dominates if $\tilde{\epsilon} < \epsilon_F$. For realistic parameters $g'(\epsilon) \approx g(\epsilon_F)$ (since $\hbar\omega \ll \epsilon_F$), we may approximate Eq. (11) by

$$\tilde{S}_{V'}(\omega) \approx \hbar\gamma'\int d\epsilon[A(\epsilon)+A(\epsilon-\hbar\omega)] \times f(\epsilon-\hbar\omega)[1-f(\epsilon)], \quad (12)$$

where $\gamma' = \pi V'^2_{\epsilon_F}g(\epsilon_F)$ is a linewidth associated with V'_{dk}. $\tilde{S}_{V'}(\omega)$ has a step at $\omega = |\tilde{\epsilon}-\epsilon_F|/\hbar$, with step-width given approximately by $\max\{\gamma, k_BT\}$. For low temperatures ($k_BT \ll \gamma$) Eq. (12) is controlled through the phases of electrons scattering off the local level and reads

$$\tilde{S}_{V'}(|\omega|) = \frac{\hbar\gamma'}{\pi}\left[\arctan\left(\frac{\tilde{\epsilon}-\epsilon_F+|\hbar\omega|}{\gamma}\right) - \arctan\left(\frac{\tilde{\epsilon}-\epsilon_F-|\hbar\omega|}{\gamma}\right)\right]. \quad (13)$$

At high frequencies ($\omega \gg |\tilde{\epsilon}-\epsilon_F|/\hbar$) the noise spectrum saturates at $\simeq \hbar\gamma'$ for all temperatures, as long as ω is less than a cutoff frequency, arising due to either a cutoff in the matrix element V' or the bandwidth.

It is straightforward to generalize Eq. (2) to the case of several TC's tunneling to different gate electrodes [6,14], each with energy $\tilde{\epsilon}_i$ and linewidth γ_i. If more than one TC is located in the same gate, their correlation energy E_{ij} (arising due to Coulomb interaction or to intertrap tunneling) can be neglected provided $E_{ij} \ll |\tilde{\epsilon}_i - \tilde{\epsilon}_j|$. In this case V' noise allows spectroscopy of the energy levels $\tilde{\epsilon}_i$ with efficiency proportional to γ'_i. The resulting spectrum will be a staircase with each step located at $\tilde{\epsilon}_i$ [see Fig. 2(b)]. This can be used for experimental determination of the number of trap levels within a specified range from the island. If the separation between levels is approximately equal, and the experiment does not have enough resolution to resolve the steps, the noise spectrum will look Ohmic. In contrast to the previous case of just V coupling, approximate Ohmic behavior due to the V' couplings now arises even when the high-temperature condition is satisfied ($k_BT \gg \gamma_i$).

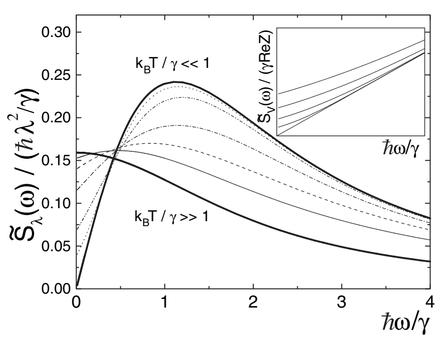

FIG. 1. Noise spectral density describing energy transfer from a TLS to a single trap level close to the Fermi sea for $k_BT/\gamma = 0, 0.1, 0.2, 0.5, 1, 2, \infty$. At high temperatures we have the standard random telegraph noise spectrum. As the temperature is lowered below the trapping center linewidth γ, Ohmic dispersion can be observed in the low frequency regime. This behavior is very similar to the Johnson-Nyquist spectrum (inset).

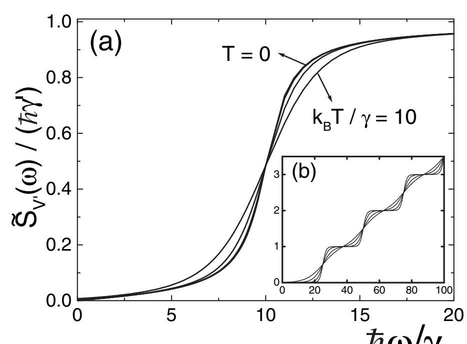

FIG. 2. (a) Noise spectrum from a single trapping center away from the Fermi sea. The step feature occurs at $\omega = |\tilde{\epsilon}-\epsilon_F|/\hbar$ with step-width given by $\max\{\gamma, k_BT\}$. (b) Staircase noise arising from several traps with energy levels equally spaced. Here as the temperature increases (with respect to the level separation), the noise appears to be linear in frequency.

Estimated parameters.—To obtain order of magnitude estimates of λ, γ, and γ', consider a hydrogenic model for the trapping center wave function, $\Psi_d(r) = \exp(-r/a_B)/\sqrt{\pi a_B^3}$, with Bohr radius $a_B \sim 15$ Å. The linewidth γ is calculated from the overlap integral with conduction electrons in the gate electrode, leading to $\gamma \approx 96\pi^2 \epsilon_F n a_B^3/(k_F a_B)^8$. Using an electron density $n = 10^{22}$ cm^{-3} and $\epsilon_F \sim 1$ eV, we obtain $\gamma \sim 10\ \mu$eV $\sim k_B \times 100$ mK $\sim h \times 3$ GHz. λ is given by the dipolar coupling between the TLS (dipole er_{12} with $r_{12} \sim 0.1\ \mu$m) and the TC (dipole ea_B due to the image charge produced in the gate), leading to $\lambda \sim 0.03\ \mu$eV when the TLS-trap distance is $r \sim 0.5\ \mu$m. This gives a contribution to TLS relaxation of the order of $\frac{1}{T_1} \sim 10^5$ s^{-1} at low frequencies. The step amplitude γ' [Eq. (12)] derives from the distortion of the TC wave function (and, consequently, the overlap integral), by the electric field $\mathcal{E}$ produced by the TLS, $V'_{dk}/V_{dk} \sim \delta\Psi_d/\Psi_d \sim e\mathcal{E}a_B/(e^2/a_B)$. If $r \sim 1\ \mu$m, this effect is very small, $\gamma'/\gamma \sim a_B^4 r_{12}^2/(\kappa^2 r^6) \sim 10^{-13}$ (here the dielectric constant $\kappa = 10$). However, TC's close to the TLS ($r \sim 0.01\ \mu$m) are affected by a monopolar electric field, and hence are given by $\gamma'/\gamma \sim (a_B/r)^4/\kappa^2 \sim 10^{-5}$. This leads to a step amplitude of the order of $\frac{1}{T_1} \sim 10^5$ s^{-1} at high frequencies, which is high enough for the number of TC's to be determined. Note that γ, γ', λ are strongly dependent on the TC Bohr radius.

The above results are valid in the weak-coupling regime, when $\lambda \ll \gamma$ and $\gamma' \ll \epsilon_F$. For $\lambda > \gamma$ the TLS dynamics acts back on the TC; for example, Eq. (4) splits into two peaks at energies $\tilde{\epsilon}_\pm \approx \tilde{\epsilon} \pm \lambda$ (zero-frequency noise was studied recently at the strong coupling regime; see [17]). n this regime backaction effects are important, and assuming the TC is in thermal equilibrium may not be appropriate.

Remarkably, the ultraviolet cutoff of the Ohmic spectrum, as mentioned above, is a power law, similar to the Drude cutoff of the standard Ohmic model [12]. On the other hand, the high-temperature regime of the Ohmic bath leads to a Lorentzian power spectrum as in Eq. (1).

Relevance to experiments.—Our theoretical result that even a single TC hybridized with a Fermi sea can give rise to Ohmic noise contrasts with previous studies that have assumed a distribution of two-level fluctuators [18]. The current study was motivated by measurements of f noise in a Cooper-pair box [15] over the frequency range $\omega =$ 3–100 GHz at $T = 50$ mK. Depending on the value of the TC Bohr radius, we find that at low temperatures we will have either Ohmic noise deriving from a TC with energy close to the Fermi energy or a step noise spectrum for a TC well above or below the Fermi energy. The latter becomes a staircase function for multiple traps. A more detailed experimental analysis with greater resolution than [15] is required to differentiate between these two behaviors. One additional possibility for measuring the predicted signature of individual fluctuators is to vary the gate voltage configurations as in Ref. [8].

Conclusions.—We describe a scenario for f noise that is based on the interaction of a single electron tunneling device with one or only a few trapping centers. At the lowest temperatures ($T < \gamma/k_B \lesssim 100$ mK) we find that the low frequency noise can be surprisingly different from the standard random telegraph spectrum. At high frequencies, we predict that multiple traps can give rise to a staircase signal arising from traps that are located nearby the device but have energies far away from the Fermi level. These could be a source of f noise even at high temperatures and can be used to measure the number of traps within a certain distance from the device.

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