A tale of Background-Independent Coarse-Graining and the role of matter

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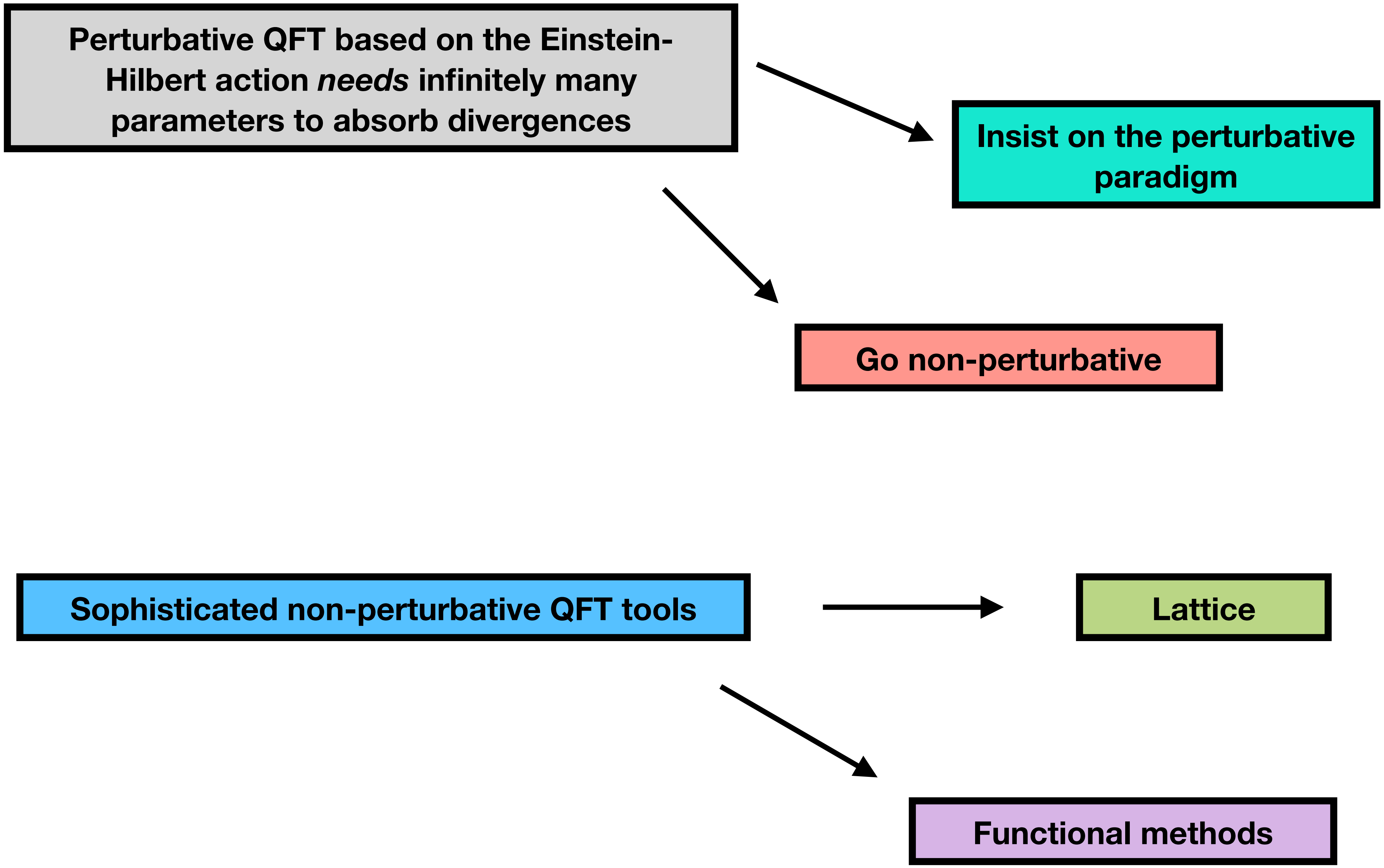
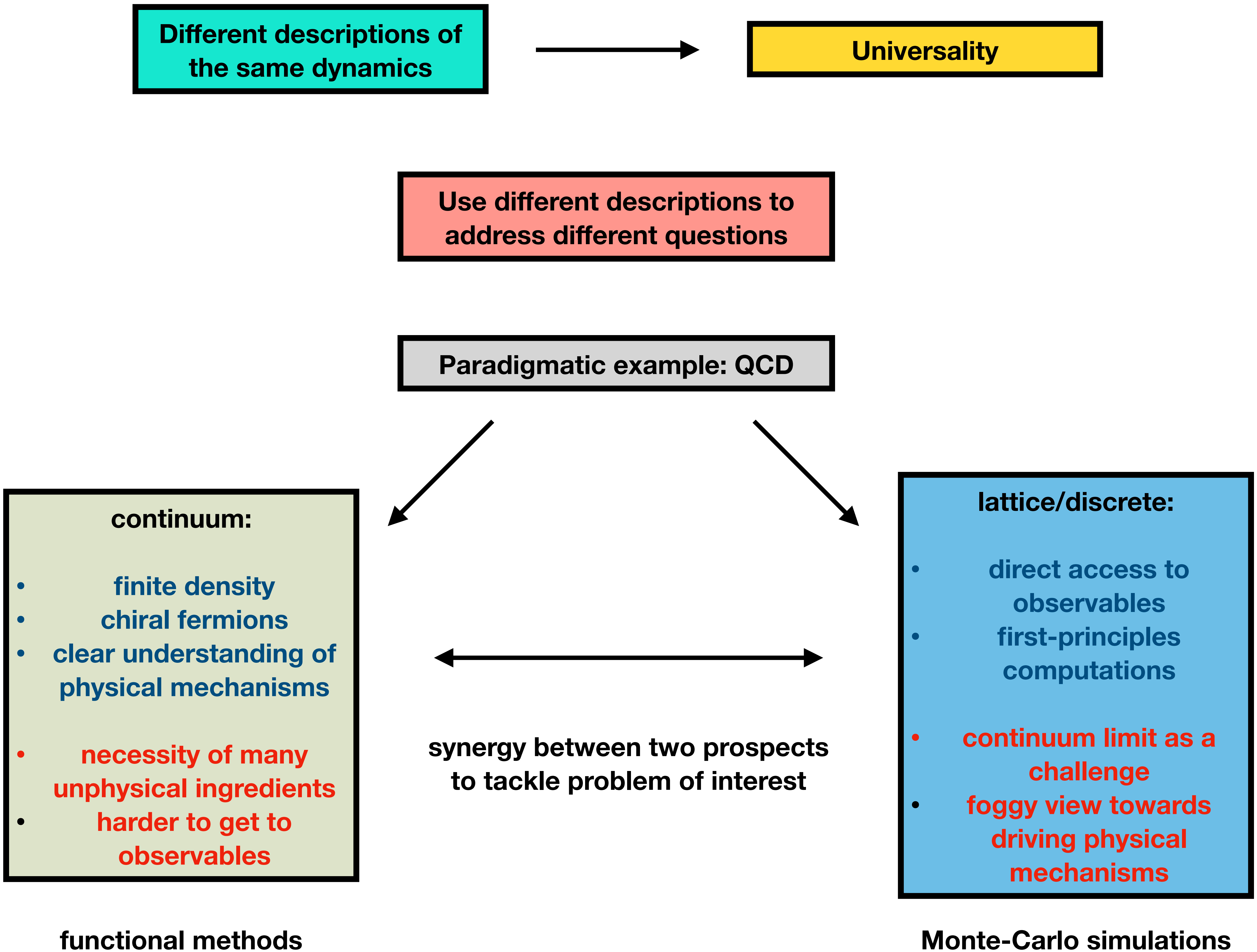
Outline

- Introduction
- Functional Renormalization Group
- Asymptotically Safe Quantum Gravity - the technical side
- Coupling Scalars
- Questions

Introduction




Different descriptions of the same dynamics
Universality
Use different descriptions to address different questions
Paradigmatic example: QCD
continuum:
finite density
chiral fermions
clear understanding of physical mechanisms
necessity of many unphysical ingredients
harder to get to observables
synergy between two prospects to tackle problem of interest
lattice/discrete:
direct access to observables
first-principles computations
continuum limit as a challenge
foggy view towards driving physical mechanisms
functional methods
Monte-Carlo simulations

… what about Quantum Gravity?

perturbative renormalizability is neither necessary nor sufficient to define a *fundamental* theory

QED

- perturbatively renormalizable
- features a Landau pole

Quantum GR as an effective field theory

- perfectly valid for computations up to UV cutoff
- physical predictions

→ Both QFTs are well-defined in the presence of a UV cutoff

↓

Those are "effective field theories" as opposed to "fundamental field theories".

Can one take the continuum limit?

Naively…

$$\int_{\Lambda_{\mathrm{UV}}} \frac{[\mathscr{D} g_{\mu\nu}]}{V_{\mathrm{Diff}}} \, \mathrm{e}^{iS_{\mathrm{EH}}} \longrightarrow \Lambda_{\mathrm{UV}} \to \infty \quad ?$$

$$\sum_{\Delta} \frac{1}{C(\Delta)} \, \mathrm{e}^{iS_{\mathrm{Regge}}} \longrightarrow \begin{array}{c} \mathrm{Volume}\,(\Delta) \to 0 \\ \#\,(\Delta) \to \infty \end{array} \quad ?$$

Both in the "continuum" as well as in the discrete descriptions, one looks for a well-defined continuum limit

Technically, a well-defined continuum limit is achieved by a scale-invariant regime → Couplings freeze and the cutoff can be safely removed

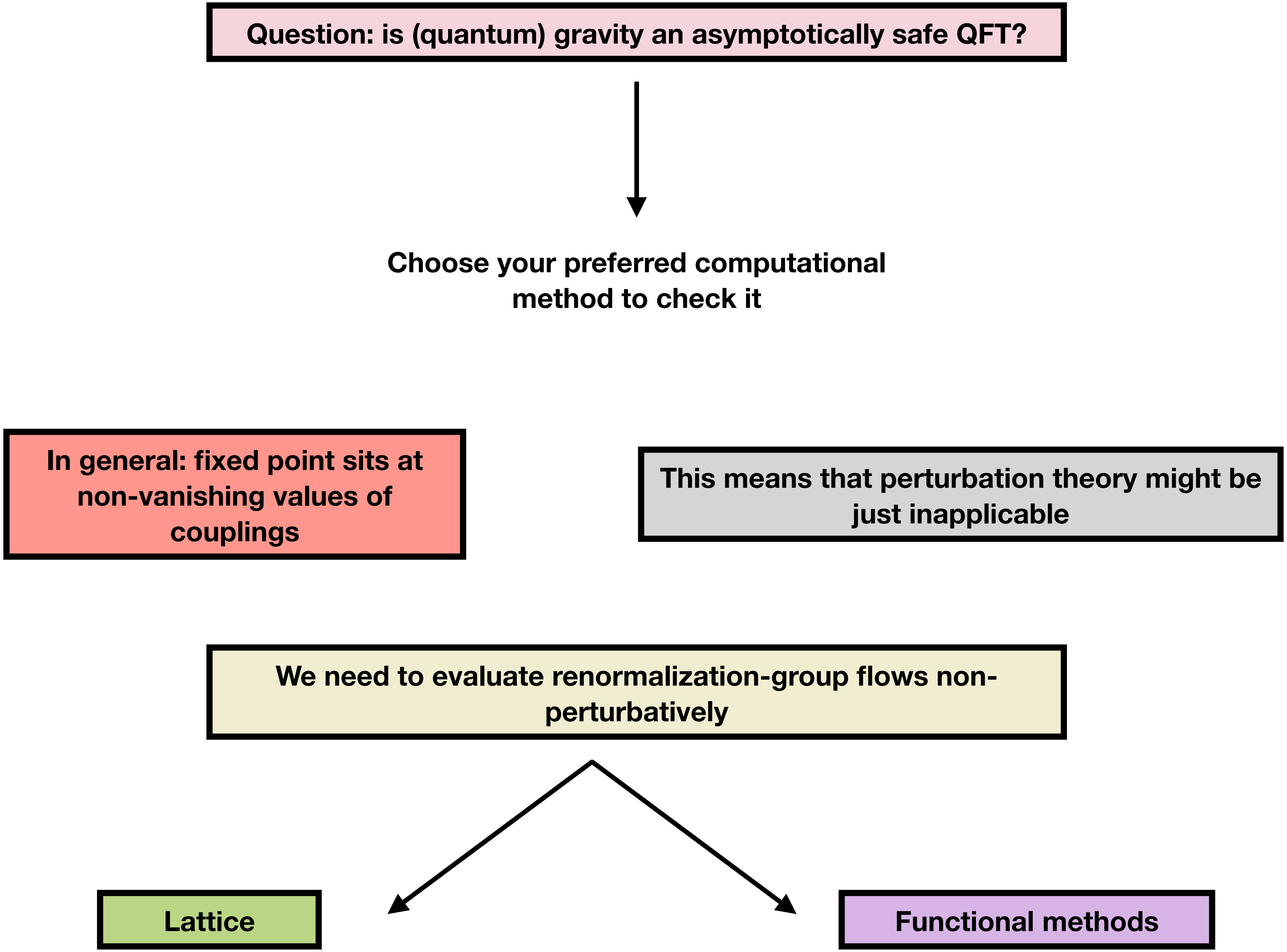
The continuum limit must be universal, i.e., should not depend on the details of the regularization scheme

Renormalization Group Fixed Point

The existence of a suitable fixed point which requires the tunning of finitely many free parameters to adjust a renormalization group trajectory defines a predictive UV-completion of the underlying QFT

Asymptotic Safety

This is a concept — not a method!


Question: is (quantum) gravity an asymptotically safe QFT?
Choose your preferred computational method to check it
In general: fixed point sits at non-vanishing values of couplings
This means that perturbation theory might be just inapplicable
We need to evaluate renormalization-group flows non-perturbatively
Lattice
Functional methods

Functional Renormalization Group

Starting point: single scalar field

We go Euclidean from now on [sorry Andreas!]

$$Z[J] = \int_\Lambda \mathscr{D}\phi \, \mathrm{e}^{-S[\phi] + \int \mathrm{d}^d x \, J(x)\phi(x)}$$

generating functional of correlation functions

$$W[J] = \ln Z[J]$$

generating functional of *connected* correlation functions

$$\langle \phi(x_1)\ldots\phi(x_n)\rangle = \left.\frac{\delta^n W[J]}{\delta J(x_1)\ldots\delta J(x_n)}\right|_{J=0}$$

$$\varphi(x) \equiv \langle \phi(x) \rangle_J = \frac{\delta W[J]}{\delta J(x)}$$

$$\Gamma[\varphi] = -W[J_\varphi] + \int \mathrm{d}^d x \; J_\varphi \; \varphi$$

generating functional of one-particle irreducible correlation functions

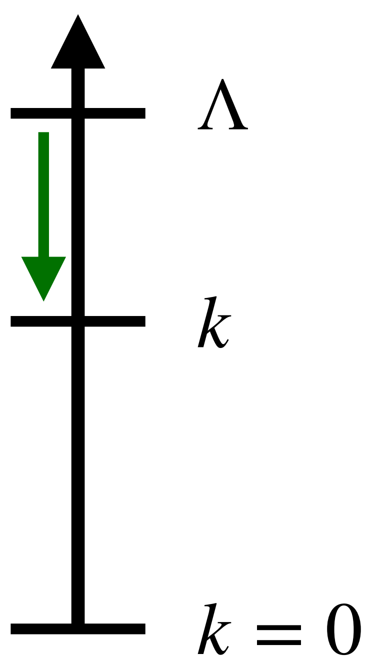
introduction of regulator function

$$Z[J] = \int_\Lambda \mathscr{D}\phi \, \mathrm{e}^{-S[\phi] + \int \mathrm{d}^d x \, J(x)\phi(x)} \longrightarrow Z_k[J] = \int_\Lambda \mathscr{D}\phi \, \mathrm{e}^{-S[\phi] - \Delta S_k[\phi] + \int \mathrm{d}^d x \, J(x)\phi(x)}$$

regulator "action":

$$\Delta S_k = \frac{1}{2} \int \mathrm{d}^d x \, \phi(x) \, \mathscr{R}_k(-\partial^2) \phi(x)$$

gives a (large) mass to field modes with momentum lower than *k*

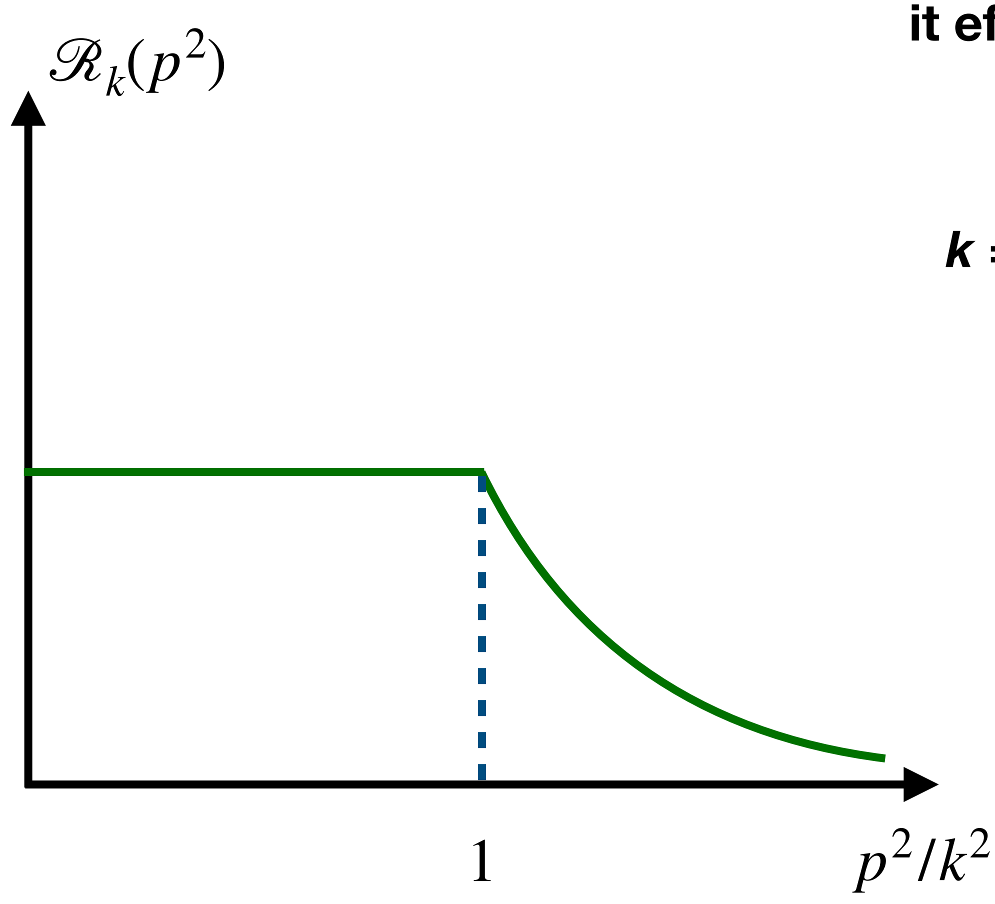


in flat space: Fourier modes

$$\phi(x) = \int_p \mathrm{e}^{ix\cdot p} \, \tilde{\phi}(p)$$

essentially

$$\mathscr{R}_k(p^2) \to \begin{cases} k^2 \,, & p^2 < k^2 \\ 0 \,, & p^2 > k^2 \end{cases}$$



it effectively implements the suppression of "slow modes"

***k = 0 : complete* functional integration**

$$W_k[J] = \ln Z_k[J]$$

next to that:

$$\bar{\Gamma}_k[\varphi] = -\,W_k[J_\varphi] + \int \mathrm{d}^d x \; J_\varphi \; \varphi$$

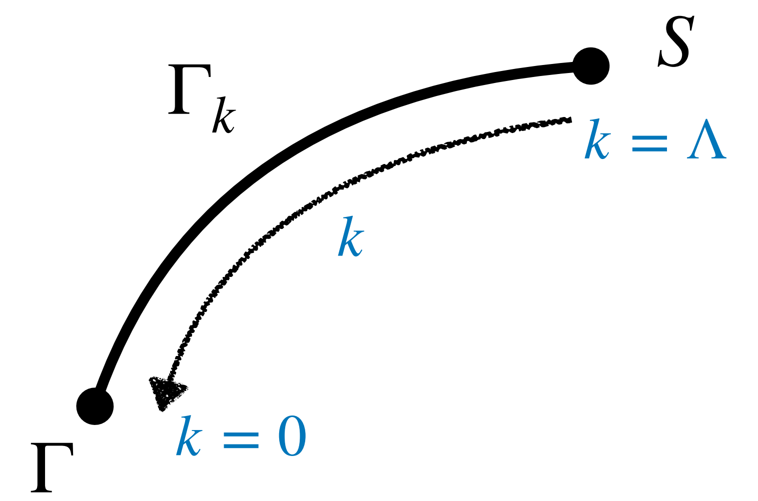
finally

$$\Gamma_k[\varphi] \equiv \bar{\Gamma}_k[\varphi] - \frac{1}{2}\int \mathrm{d}^d x \; \varphi(x)\mathscr{R}_k(-\partial^2)\varphi(x)$$

effective average action (EAA)

Properties:

interpolates between full effective action and the “classical” one



satisfies an exact flow equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\left(\Gamma_k^{(2)} + \mathscr{R}_k \right)^{-1} \partial_t \mathscr{R}_k \right]$$

$$\partial_t \equiv k \partial_k$$

(exact) flow equation

Wetterich equation

conversion of functional integral into functional differential equation → solving the flow equation = solving the functional integral

Theory Space

(Infinitely many)
space of all functionals of the field which are compatible with the symmetries of the theory

the effective average action is expanded as

$$\Gamma_k[\varphi] = \sum_i \bar{g}_i(k)\, \mathcal{O}_i[\varphi]$$

$$\partial_t \Gamma_k[\varphi] = \sum_i (\partial_t \bar{g}_i(k))\, \mathcal{O}_i[\varphi]$$

$$\bar{g}_i = k^{d_i} g_i$$

$$\partial_t \bar{g}_i = k^{d_i} (d_i\, g_i + \beta_i)$$

$$\beta_i = -\, d_i\, g_i + k^{-d_i} \partial_t \bar{g}_i$$

$$\partial_t \Gamma_k[\varphi] = \sum_i k^{d_i} (d_i\, g_i + \beta_i)\, \mathcal{O}_i[\varphi]$$

→ extraction of beta functions

suitable projection rule for the Wetterich equation

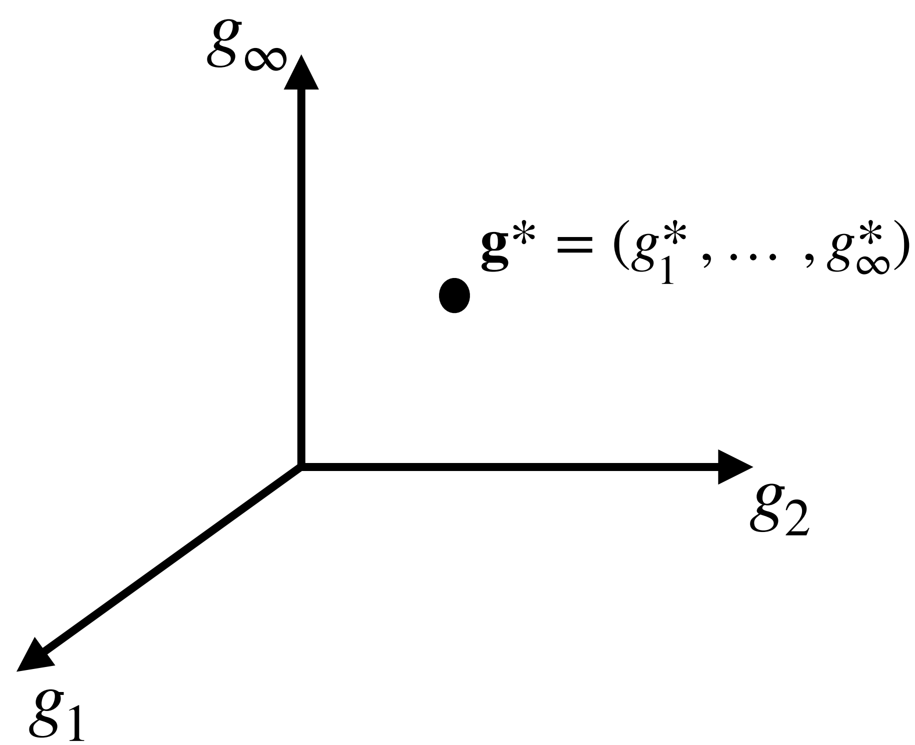
Approximations are necessary, but we don't need to use a perturbative scheme!

Theory Space

Looking for fixed points:

$$\beta_i(\mathbf{g}^*) = 0\,, \quad i = 1, \ldots, \infty$$

$$\mathbf{g}^* = (g_1^*, \ldots, g_\infty^*)$$



Linearized flow around the fixed point:

$$\partial_t (g_i - g_i^*) = \sum_j \frac{\partial \beta_i}{\partial g_j}(g_j - g_j^*)$$

diagonalize

$$\partial_t z_i = \lambda_i \, z_i$$

$$z_i(t) = C_i \left(\frac{k}{k_0}\right)^{-\theta_i} \qquad \textbf{w/} \qquad \theta_i = -\lambda_i$$

In order to hit the fixed point:

$\theta_i < 0$ → z_i **grows towards de UV**

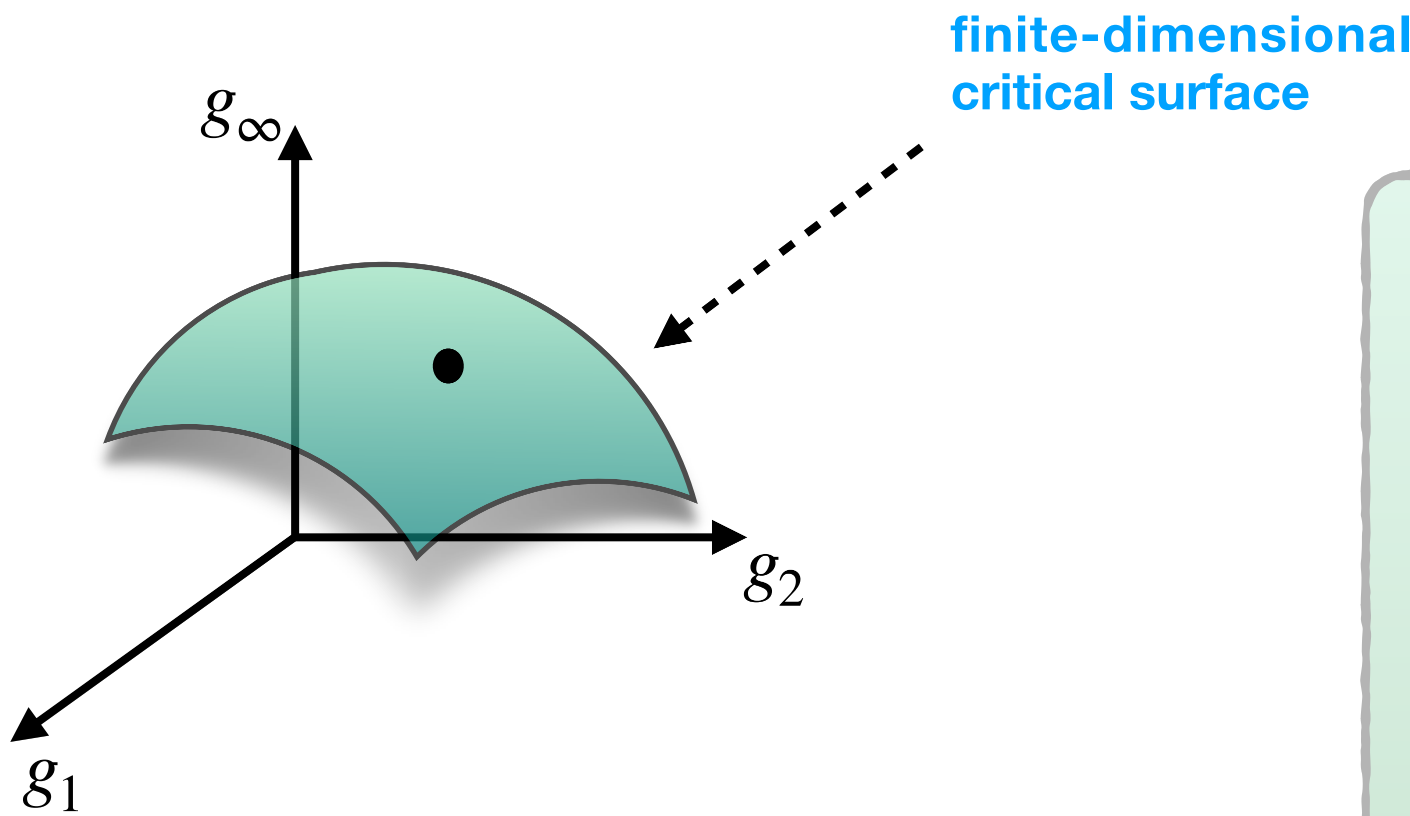
$C_i = 0$ **irrelevant direction**

$\theta_i > 0$ → z_i **decreases towards de UV**

C_i **free parameter**

relevant direction

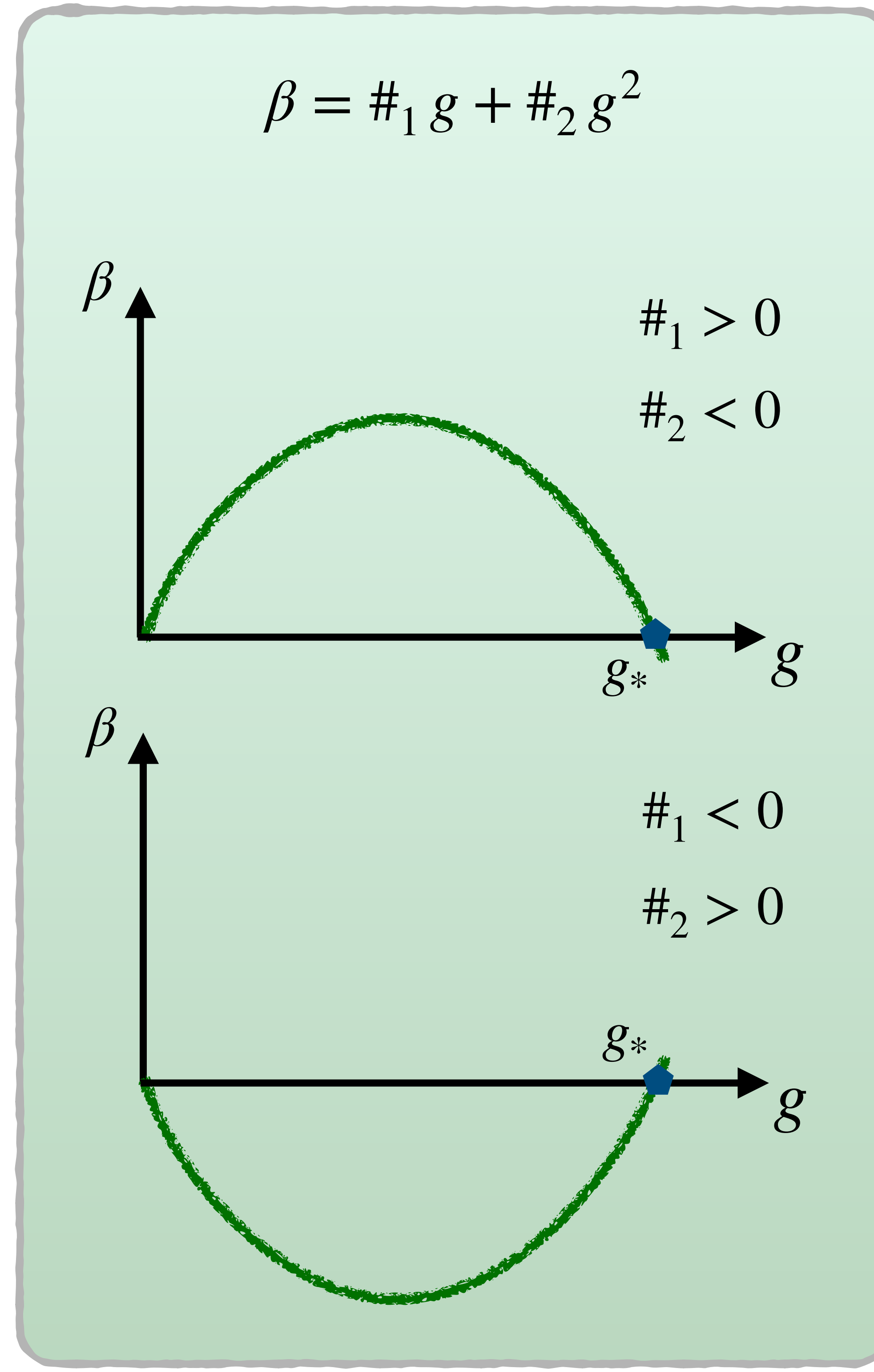
Predictivity requires that the number of relevant directions is finite



Asymptotic Safety:

Existence of a renormalization-group fixed point;

Fixed point features finitely many relevant directions;



Asymptotically Safe Quantum Gravity

- the technical side -

$$Z = \int \mathcal{D} g_{\mu\nu}\, \mathrm{e}^{-S[g_{\mu\nu}]}$$

No background to set a scale: *background field method*

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

The spectrum of the Laplacian of the background metric defines a scale

background independence is encoded in split symmetry

$$\bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu} + \epsilon_{\mu\nu}$$
$$h_{\mu\nu} \to h_{\mu\nu} - \epsilon_{\mu\nu}$$

The gravitational action is invariant under general coordinate transformations: *gauge invariance*

→

Introduction of a gauge fixing term:

Faddeev-Popov procedure

Gauge fixing

$$Z = \int \mathcal{D}h_{\mu\nu}\mathcal{D}\bar{C}_\alpha\mathcal{D}C^\beta e^{-S[\bar{g}+h]-S_{\rm gf}[\bar{g};h]-S_{\rm gh}[\bar{g};h,\bar{C},C]}$$

Faddeev-Popov ghosts

$$S_{\rm gf}[\bar{g};h] = \frac{1}{2\alpha}\int {\rm d}^dx\sqrt{\bar{g}}\,\bar{g}^{\mu\nu}F_\mu[\bar{g};h]F_\nu[\bar{g};h]$$

$$F_\mu[\bar{g};h] = \bar{\nabla}_\nu h^\nu{}_\mu - \frac{1+\beta}{d}\bar{\nabla}_\mu h$$

$$S_{\rm gh}[\bar{g};h,\bar{C},C] = \int {\rm d}^dx\sqrt{\bar{g}}\,\bar{C}_\alpha\mathcal{M}^\alpha{}_\beta[\bar{g};h]C^\beta$$

Gauge-fixing term breaks split symmetry!

Harmless breaking

Introducing regulators

$$\Delta S_k[\bar{\Phi};\Phi] = \Delta S_k^h[\bar{g};h] + \Delta S_k^{\bar{C}C}[\bar{g};\bar{C},C]$$

Regulators break split symmetry and (quantum) gauge invariance!

$$\Delta S_k^h[\bar{g};h] = \frac{1}{2}\int {\rm d}^dx\sqrt{\bar{g}}\,h_{\mu\nu}\,R_k^{\mu\nu,\alpha\beta}(-\bar{\nabla}^2)\,h_{\alpha\beta}$$

$$\Delta S_k^h[\bar{g};h] = \int {\rm d}^dx\sqrt{\bar{g}}\,\bar{C}_\alpha\,R^\alpha_{k,\beta}(-\bar{\nabla}^2)\,C^\beta$$

$$Z_k[\mathscr{J}] = \int \mathscr{D}h_{\mu\nu}\mathscr{D}\bar{C}_\alpha \mathscr{D}C^\beta \, \mathrm{e}^{-S[\bar{g}+h]-S_{\mathrm{gf}}[\bar{g};h]-S_{\mathrm{gh}}[\bar{g};h,\bar{C},C]-\Delta S_k[\bar{\Phi};\Phi]+\int \mathrm{d}^d x\sqrt{\bar{g}}\mathscr{J}\cdot\Phi} \equiv \mathrm{e}^{W_k[\mathscr{J}]}$$

In complete analogy: construction of effective average action $\Gamma_k = \Gamma_k[\bar{\Phi};\Phi]$

$$\partial_t \Gamma_k[\bar{\Phi},\Phi] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(0,2)}[\bar{\Phi},\Phi] + \mathbb{R}_k\right)^{-1}\partial_t \mathbb{R}_k\right]$$

The effective average action is a functional of two fields;

Integrating the flow and taking *k=0* leads to an effective action that depends on two fields, but background independence is guaranteed by BRST symmetry;

WARNING!

very little is known about the Gribov problem in quantum gravity and how it can affect background independence

Choices…

Our starting point was a path integral over Riemannian metrics

However…

$h_{\mu\nu}$ **can fluctuate widely**

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

metric can be degenerate

metric can change signature

Such a linear split of the metric might introduce many spurious configurations in the non-perturbative realm!

Alternative: $$g_{\mu\nu} = \bar{g}_{\mu\alpha}\left(\mathrm{e}^{\bar{g}^{-1}h}\right)^{\alpha}{}_{\nu}$$ → **avoid the previous problems + cover the space of Riemannian metrics**

In the path integral, should we adopt different variables that lead to the same field equations in the case of GR?

Palatini $(g_{\mu\nu}, \Gamma^{\alpha}_{\beta\sigma})$ **or pure** e^{a}_{μ} **or** $(e^{a}_{\mu}, \omega^{bc}_{\nu})$

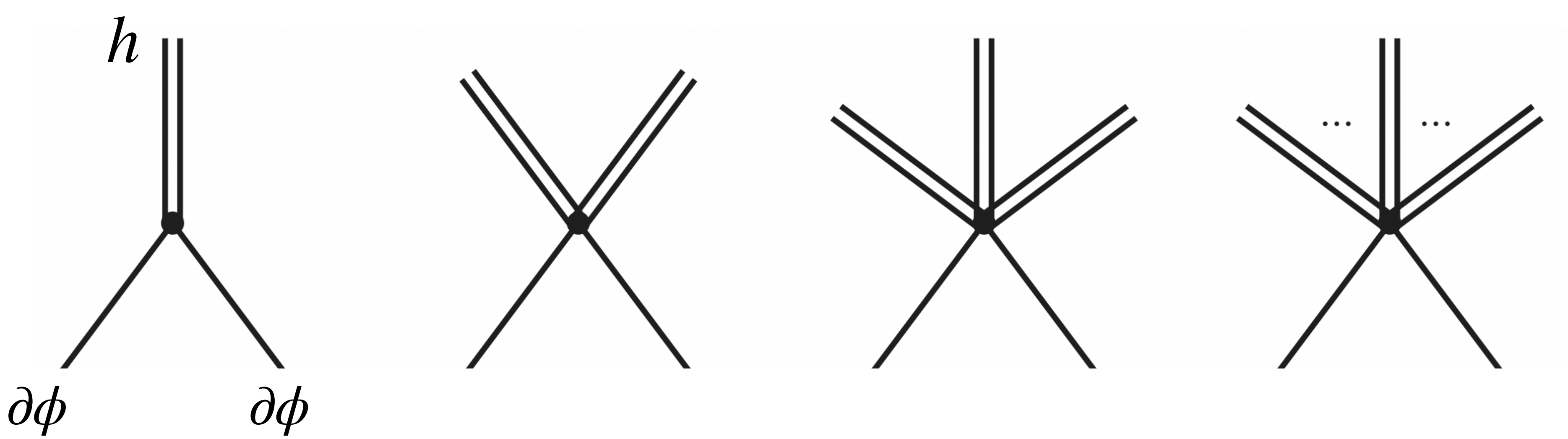
No *a priori* reason to choose one formulation instead of the other

Coupling scalars

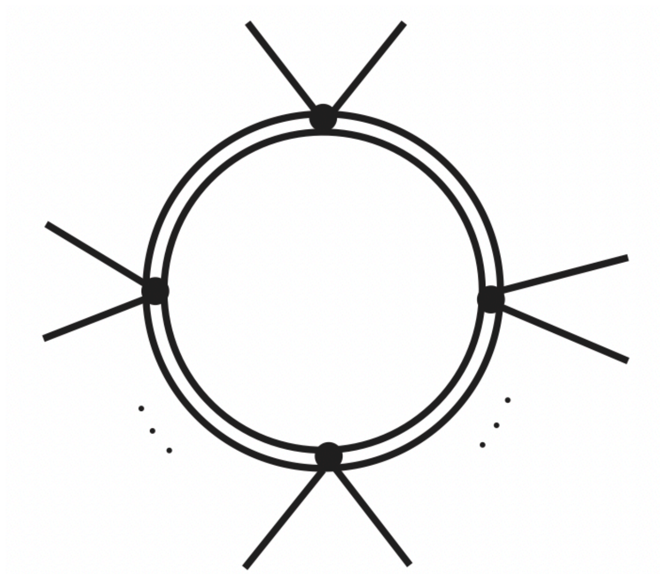
- **Basic question in this program: Does the would-be QG fixed point extends to gravity-matter systems?**
- **Coupling matter to gravity: generation of infinitely many matter-graviton vertices already from the matter-field kinetic term.**

$$S_\phi = \frac{1}{2}\int \mathrm{d}^d x \sqrt{g}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$g^{-1} = \bar{g}^{-1} - h + hh + \dots$$



This generates induced matter-self-interactions as well as non-minimal interactions.



induced self-interactions

The induced interactions share the same symmetries of the kinetic term. In particular,

$$\phi \to \phi + c \qquad\qquad \phi \to -\phi$$

shift symmetry — Z_2 - symmetry

$$\mathcal{I}_1 = \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

$$\mathcal{I}_2 = \int \mathrm{d}^d x \sqrt{g}\, R\, g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$\mathcal{I}_3 = \int \mathrm{d}^d x \sqrt{g}\, R^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$\mathcal{I}_\# = \int \mathrm{d}^d x \sqrt{g}\, G(R, \mathbf{Ric}, \mathbf{Riem})^{\mu\nu} K_{\mu\nu}(X) \qquad\qquad X_{\alpha\beta} \sim (\partial_\alpha \phi)(\partial_\beta \phi)$$

Apart from the induced interactions, one can still introduce a "standard" potential for the scalar fields in the effective action.

$$V(\phi^2) = \sum_i \bar{g}_i \phi^{2i}$$

breaks shift symmetry

$$\phi \to \phi + c$$

$$\Gamma_k = \Gamma_k^{\text{grav}} + \boxed{\Gamma_k^{\phi}}$$

$$\Gamma_k^{\phi} = \Gamma_k^{\text{ss}} + \Gamma_k^{\text{nss}}$$

Several results in the literature support that *nss* interactions feature just a Gaussian fixed point - the so-called Gaussian-Matter fixed point

$$\text{NGFP}^{\text{ASQGM}} = \text{NGFP}^{\text{ASQG}} \otimes \text{GFP}^{\text{matter}}$$

[see, e.g., Percacci & Narain '09]

$$\Gamma_k^{\text{ss}} \sim \frac{Z_\phi}{2} \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$+ Z_\phi^2\, C_k \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

However…

shift-symmetric interactions *cannot* feature a GFP

[Eichhorn' 12, …]

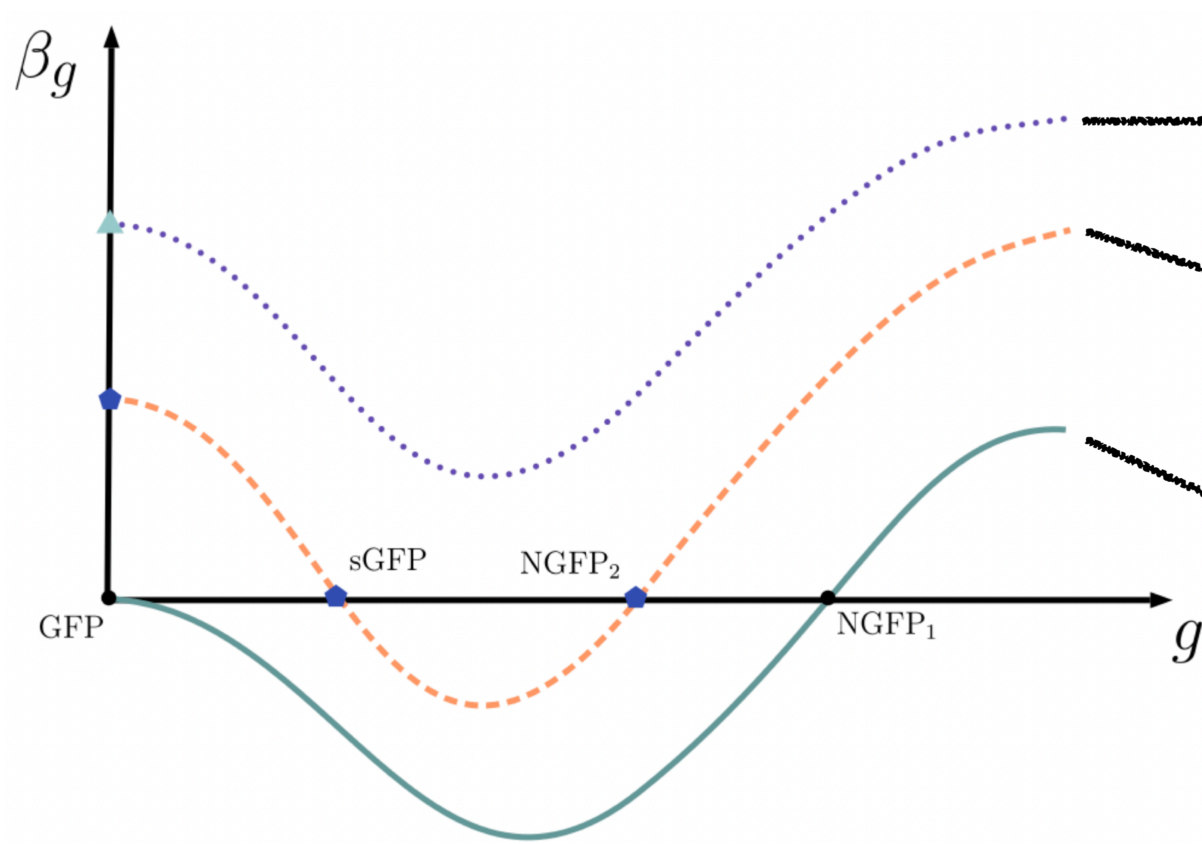
$$\beta_c = a_0 + a_1\, c + a_2\, c^2$$

contributions arising from the kinetic term - they do not depend on *c*

associated to canonical/ anomalous dimensionality of *c*

contributions from two-vertex diagrams

Setting *c* = *0* shows that, in general, this does not correspond to a fixed point → **QG fluctuations shift the GFP**



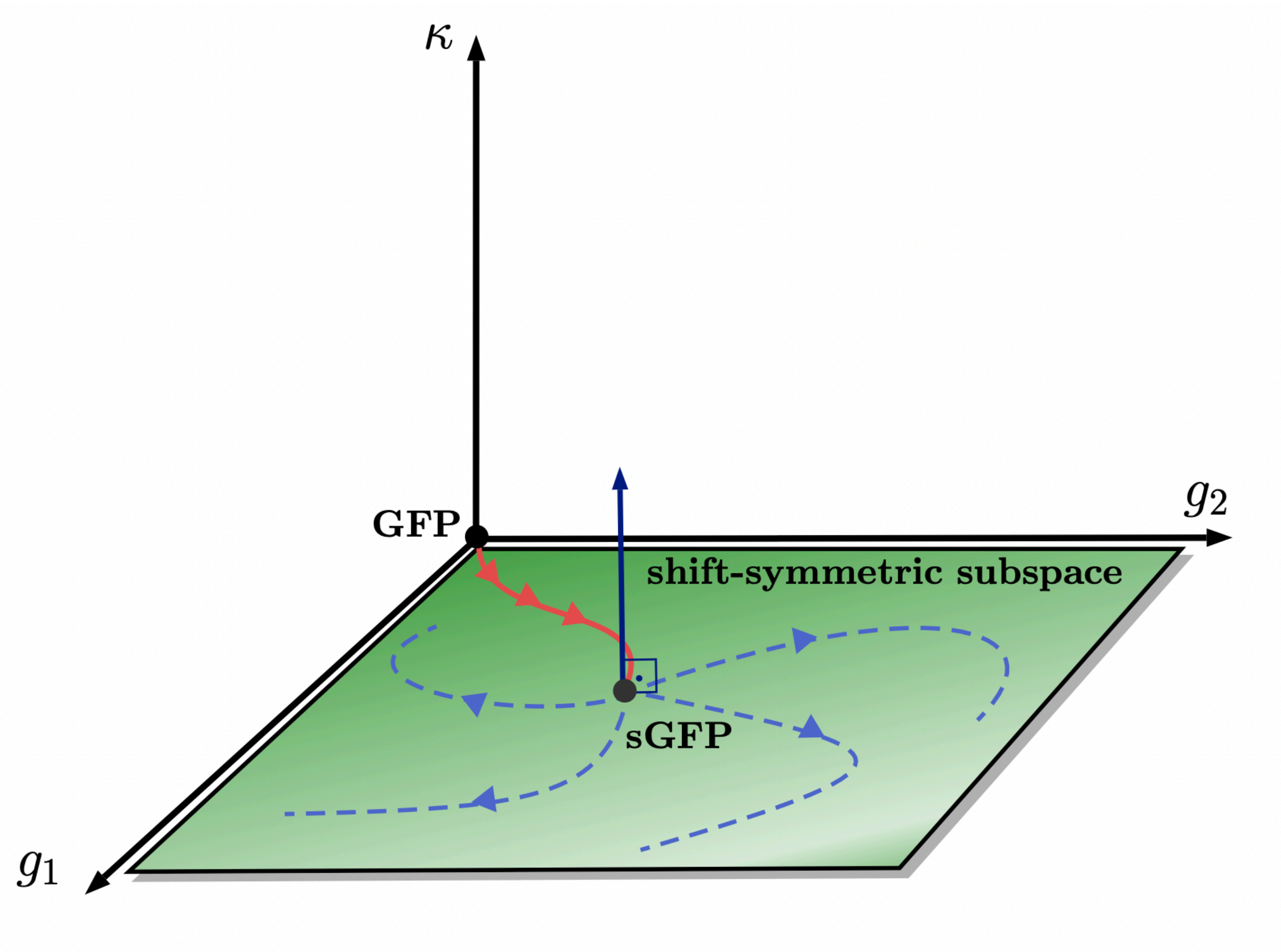
QG fluctuations are too strong

sufficiently “small” QG fluctuations

no QG fluctuations

Hence, tentatively, the fixed-point structure should have the form

$$\mathrm{NGFP}^{\mathrm{ASQGM}} = \mathrm{NGFP}^{\mathrm{shift-symmetric}} \otimes \mathrm{GFP}^{\mathrm{non\ shift-symmetric}}$$



$$\int_x \frac{\delta \Gamma_k}{\delta \phi(x)} = 0$$

Shift-symmetry is preserved along the flow: shift-symmetric subspace is closed under the RG

Shift-symmetric Scalar-Tensor theories: fixed-point structure

Laporte, ADP, Saueressig, Wang, *JHEP* 12 (2021) 001

$$\Gamma_k = \Gamma_k^{\rm grav} + \Gamma_k^{\rm ss} + \Gamma_k^{\rm gf} + \Gamma_k^{\rm ghost}$$

$$\Gamma_k^{\rm grav} = \frac{1}{16\pi G_k}\int \mathrm{d}^d x\sqrt{g}\left(2\Lambda_k - R\right)$$

$$\Gamma_k^{\rm ss} \sim \frac{Z_\phi}{2}\int \mathrm{d}^d x\sqrt{g}\, g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)$$

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Eichhorn '12; de Brito, Eichhorn, dos Santos '21

$$\Gamma_k^{\text{ss}} \sim \frac{Z_\phi}{2} \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + Z_\phi^2 C_k \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

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$$+ Z_\phi\, \tilde{C}_k \int \mathrm{d}^d x \sqrt{g}\, R^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

Eichhorn, Lippoldt, Skrinjar ’17

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& Feynman-de Donder gauge

$$\Gamma_k^{\text{ss}} \sim \frac{Z_\phi}{2}\int \mathrm{d}^d x\sqrt{g}\, g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) \;+ Z_\phi^2\, C_k\int \mathrm{d}^d x\sqrt{g}\, g^{\mu\alpha}g^{\nu\beta}(\partial_\mu\phi)(\partial_\alpha\phi)(\partial_\nu\phi)(\partial_\beta\phi)$$

$$+ Z_\phi\, \tilde{C}_k\int \mathrm{d}^d x\sqrt{g}\, R^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) \;+ Z_\phi\, D_k\int \mathrm{d}^d x\sqrt{g}\, R g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)$$

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& Feynman-de Donder gauge

$$\Gamma_k^{\rm ss} \sim \frac{Z_\phi}{2}\int \mathrm{d}^d x\sqrt{g}\, g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) \;+ Z_\phi^2\, C_k\int \mathrm{d}^d x\sqrt{g}\, g^{\mu\alpha}g^{\nu\beta}(\partial_\mu\phi)(\partial_\alpha\phi)(\partial_\nu\phi)(\partial_\beta\phi)$$

$$+ Z_\phi\,\tilde{C}_k\int \mathrm{d}^d x\sqrt{g}\, R^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) \;+ Z_\phi\, D_k\int \mathrm{d}^d x\sqrt{g}\, R g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)$$

Employ a generic background to disentangle the tensor structures

We can get some intuition about the behavior of the system of beta functions when the anomalous dimension of the scalar field is set to zero

The beta function of the induced quartic matter self-interaction has a structure of a polynomial of even degree on the coupling *c*

↓

It is possible to have multiple, 1 or no real solution!

↓

QG fluctuations might just be too strong and remove real fixed point solutions

On the other hand, the beta functions associated to the non-minimal interactions have the structure of polynomials which are odd on the non-minimal couplings

↓

There is always a real solution

↓

QG fluctuations are compatible with NGFP

$$Z_\phi^2\, C_k \int \mathrm{d}^d x \sqrt{g}\; g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

No fixed point!

$$Z_\phi\, \tilde{C}_k \int \mathrm{d}^d x \sqrt{g}\; R^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$Z_\phi\, D_k \int \mathrm{d}^d x \sqrt{g}\; R g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$Z_\phi^2 C_k \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

$$Z_\phi \tilde{C}_k \int \mathrm{d}^d x \sqrt{g}\, R^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

→ **fixed point!**

$$Z_\phi D_k \int \mathrm{d}^d x \sqrt{g}\, R g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$Z_\phi^2 \, C_k \int \mathrm{d}^d x \sqrt{g} \, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

$$Z_\phi \, \tilde{C}_k \int \mathrm{d}^d x \sqrt{g} \, R^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

→ **No fixed point!**

$$Z_\phi \, D_k \int \mathrm{d}^d x \sqrt{g} \, R g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$Z_\phi^2\, C_k \int \mathrm{d}^d x \sqrt{g}\; g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

$$Z_\phi\, \tilde{C}_k \int \mathrm{d}^d x \sqrt{g}\; R^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$Z_\phi\, D_k \int \mathrm{d}^d x \sqrt{g}\; R g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

→ **Suitable fixed point!**

2 relevant directions

+

3 irrelevant ones

Non-minimal interactions play a crucial role in the fixed-point structure

$$Z_\phi^2 C_k \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

$$Z_\phi \tilde{C}_k \int \mathrm{d}^d x \sqrt{g}\, R^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$Z_\phi D_k \int \mathrm{d}^d x \sqrt{g}\, R g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

→ **Suitable fixed point!**

rections

ones

Upon inclusion of the anomalous dimension, two fixed points are found. The second one has another relevant direction.

Non-minimal interactions play a crucial role in the fixed-point structure

Questions:

In the case of starting with a discrete formulation of gravity (either in a pre-geometric setting or not) + scalar fields, what is the corresponding analogue of those induced interactions?

Quantum gravity directly impacts the structure of effective matter vertices: how does this affect the use of scalar fields as “clocks” is a quantum-cosmological framework?

Coarse-graining of GFTs with local coordinates: any hint of this type of induced interactions?

The present results suggest that one should include “non-minimal” interactions in truncations for the stabilization of the fixed-point solution. What is the status of that in discrete approaches?

What does GFT+Scalars fixed-point structure tell us?

Thank you!