



ARNOLD SOMMERFELD
CENTER FOR THEORETICAL PHYSICS

Emergent Cosmology from (T)GFT Condensates

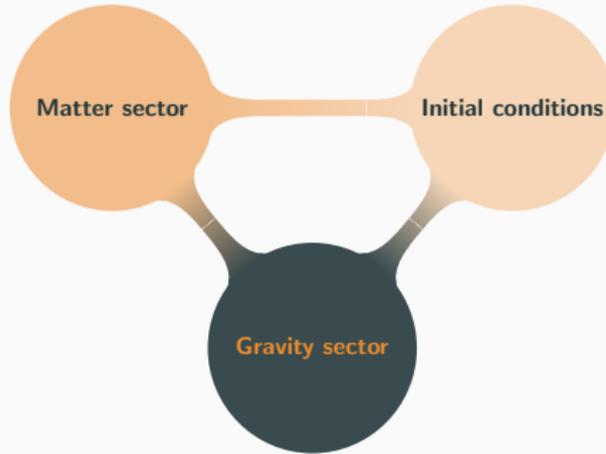
Luca Marchetti

Quantum gravity, Hydrodynamics and Emergent Cosmology
Munich, 9 December 2022

Arnold Sommerfeld Center
LMU Munich

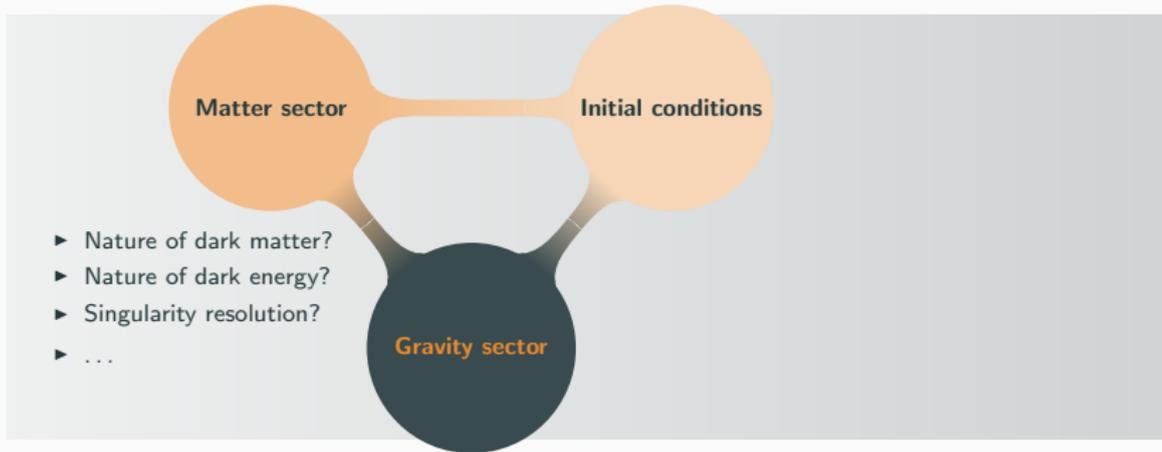
The QG perspective on Cosmology

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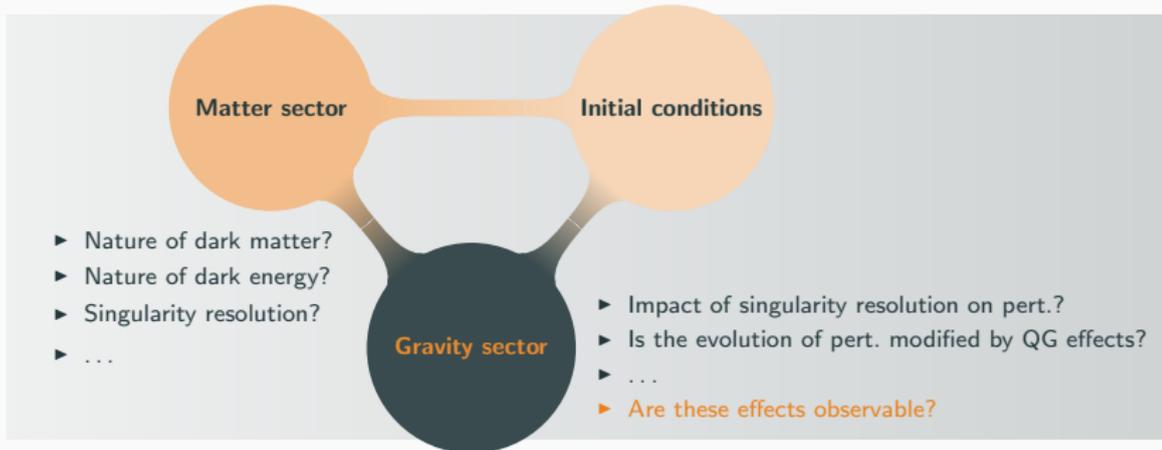


Ashtekar, Kaminski, Lewandowski 0901.0933; Agullo, Ashtekar, Nelson 1302.0254; Gielen, Oriti 1709.01095; Gerhart, Oriti, Wilson-Ewing 1805.03099; ...

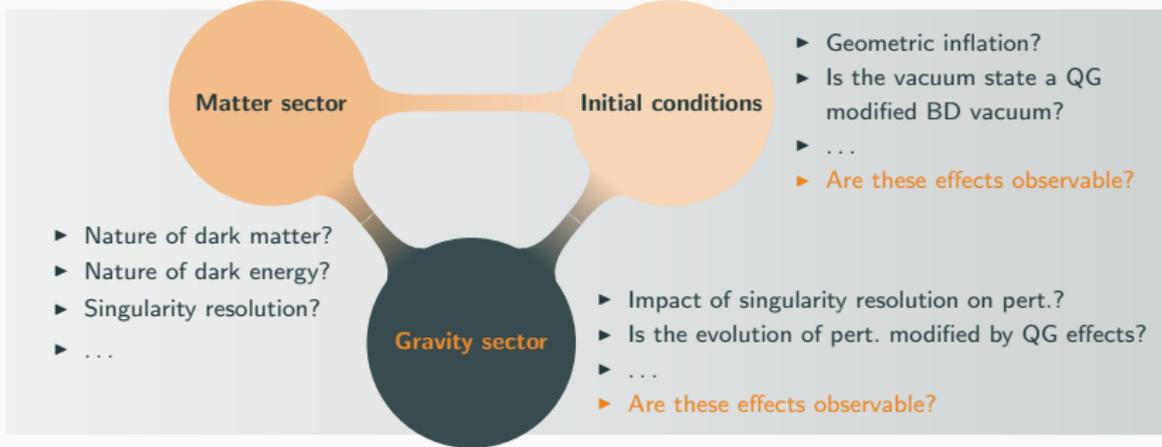
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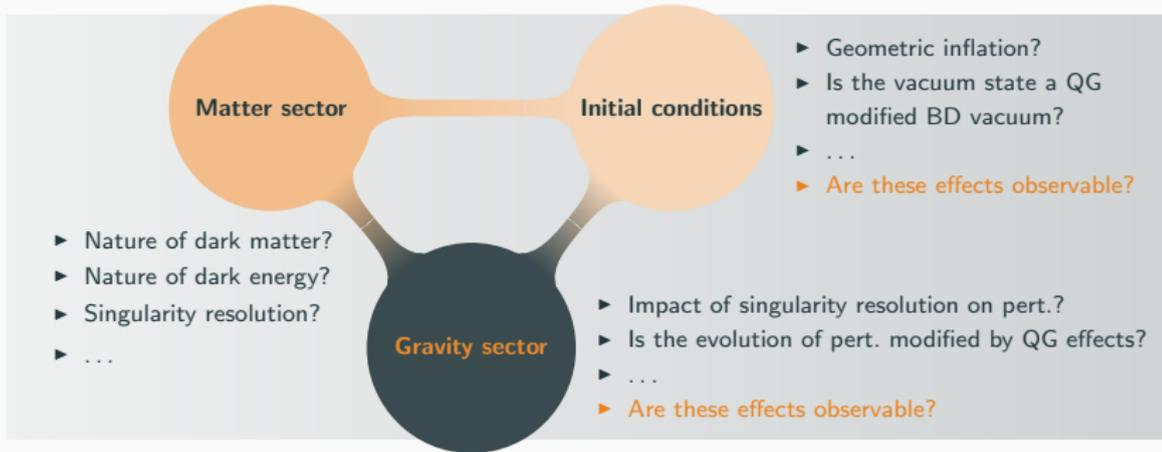
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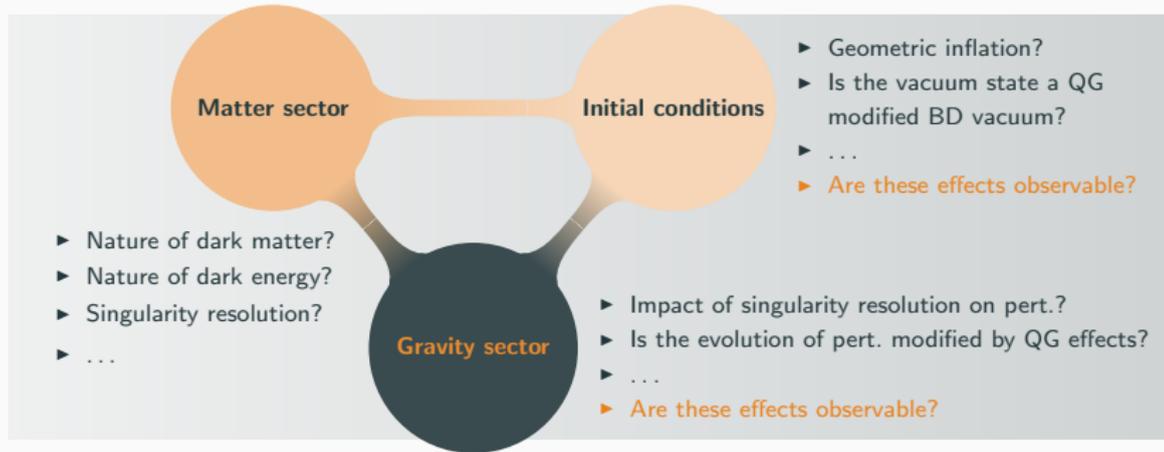
The QG perspective on Cosmology



Challenges from the QG perspective:

- ▶ How to define (in)homogeneity?
- ▶ How to extract macroscopic dynamics?
- ▶ How to construct cosmological geometries?
- ▶ ...

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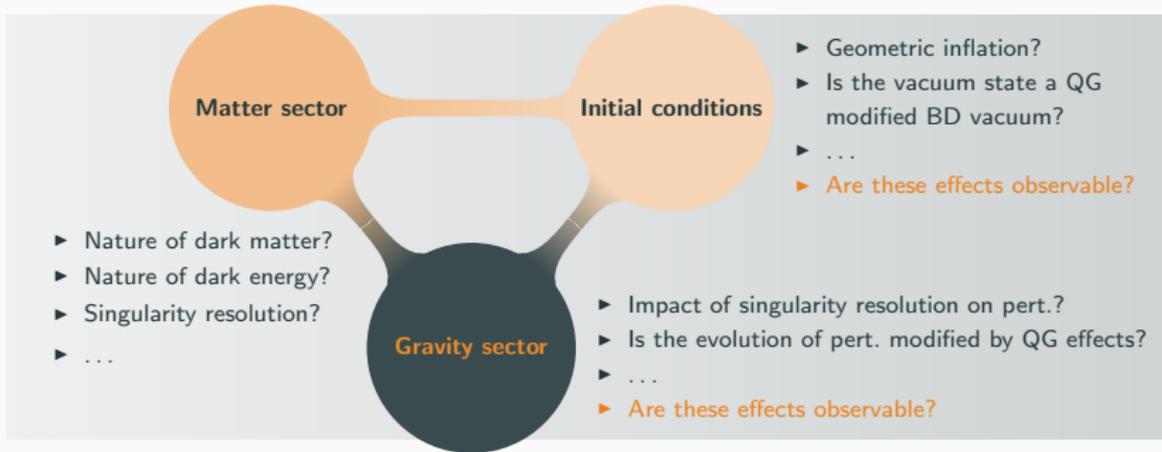


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Relational description

The QG perspective on Cosmology



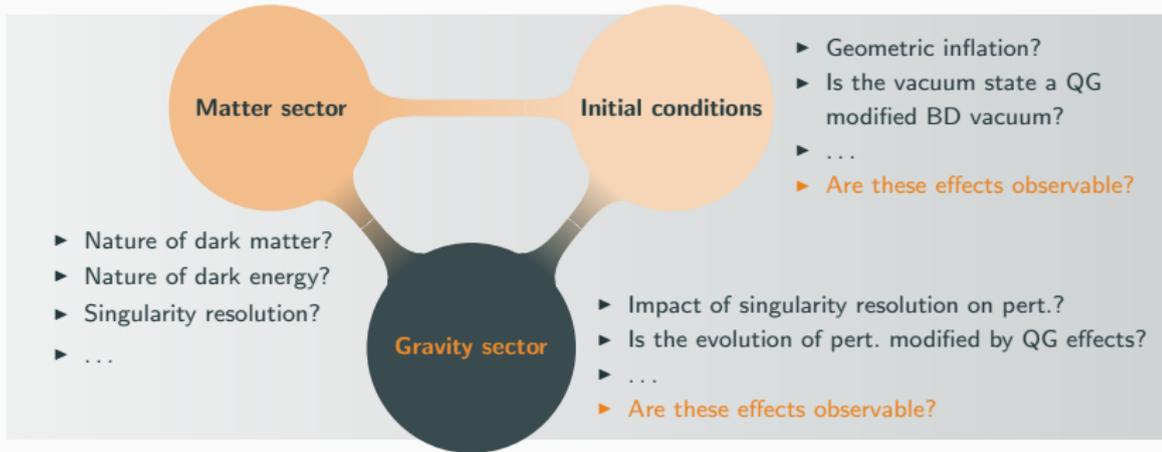
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Relational description

Coarse-graining/
collective behavior

The QG perspective on Cosmology



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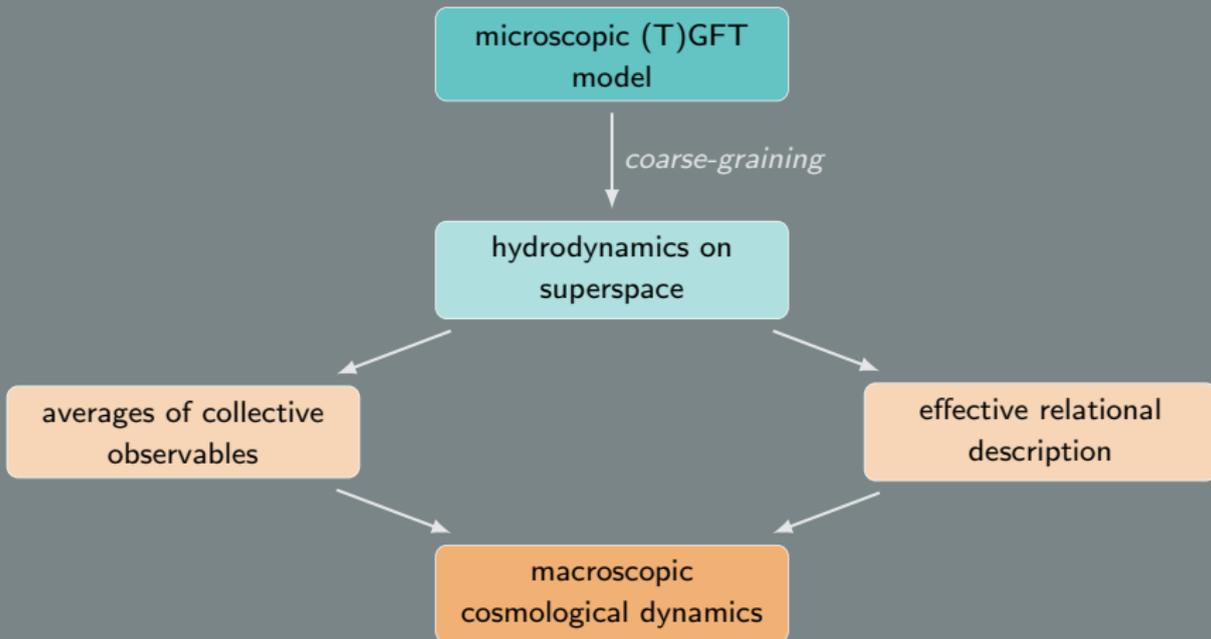
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Approximate only

Homogeneous cosmologies from (T)GFT condensates



microscopic (T)GFT
model

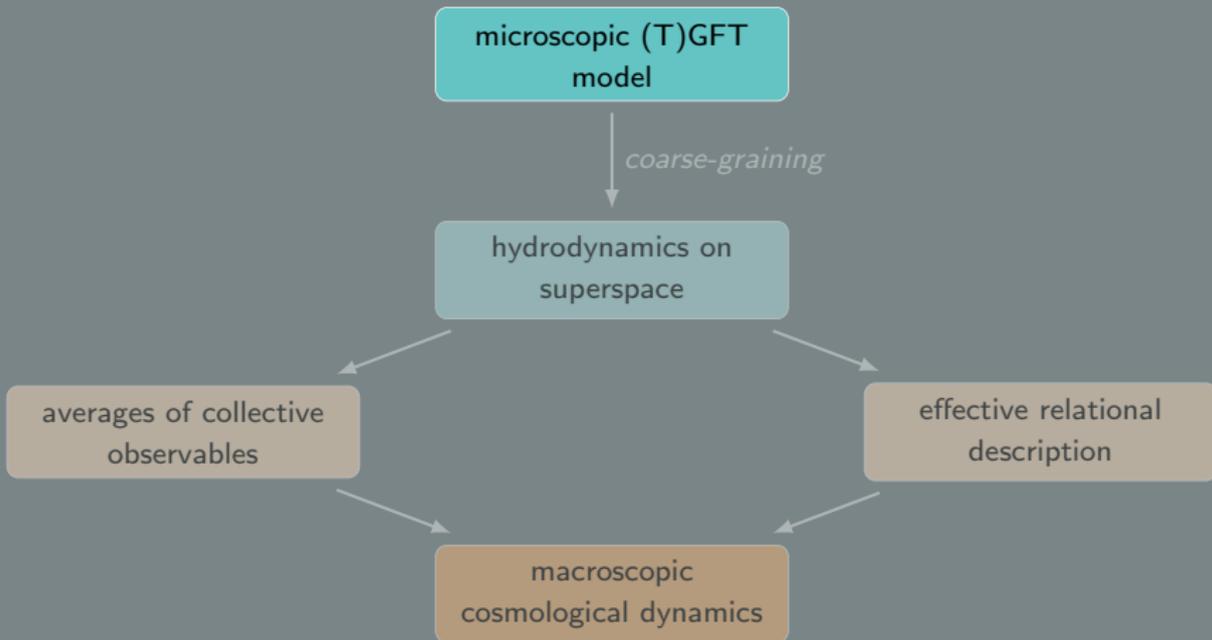
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The (T)GFT approach to QG

(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined
on d copies of a group manifold G .

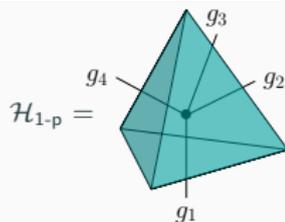
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Kinematical states: $d - 1$ -simplices decorated with discretized field data



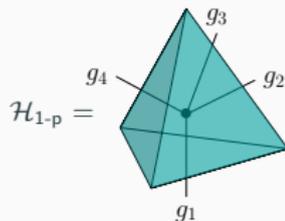
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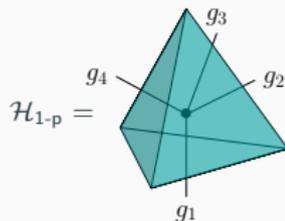
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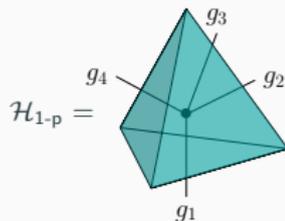
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Dynamics: connection with simplicial/spinfoam models

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity path integral.

$$Z_{\text{GFT}} = \sum_{\Gamma} \frac{\prod_i \lambda_i^{n_i(\Gamma)}}{\text{sym}(\Gamma)} Z_{\text{GFT}}(\Gamma)$$

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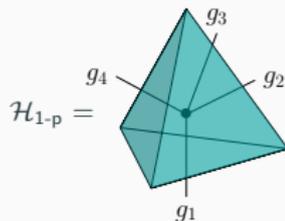
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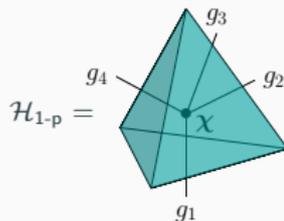
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- ▶ Group (Lie algebra) variables associated to discretized gravitational quantities.
- ▶ Appropriate (**geometricity**) constraints allow the simplicial interpretation.
- ▶ Scalar field discretized on each d -simplex: each $d - 1$ -simplex composing it carries values $\chi \in \mathbb{R}^d$.



Dynamics: connection with simplicial/spinfoam models

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + matter path integral.

- ▶ **Non-local and combinatorial** interactions guarantee the gluing of $d - 1$ -simplices into d -simplices.
- ▶ Γ are **dual to spacetime triangulations**.
- ▶ Scalar field data are **local** in interactions.

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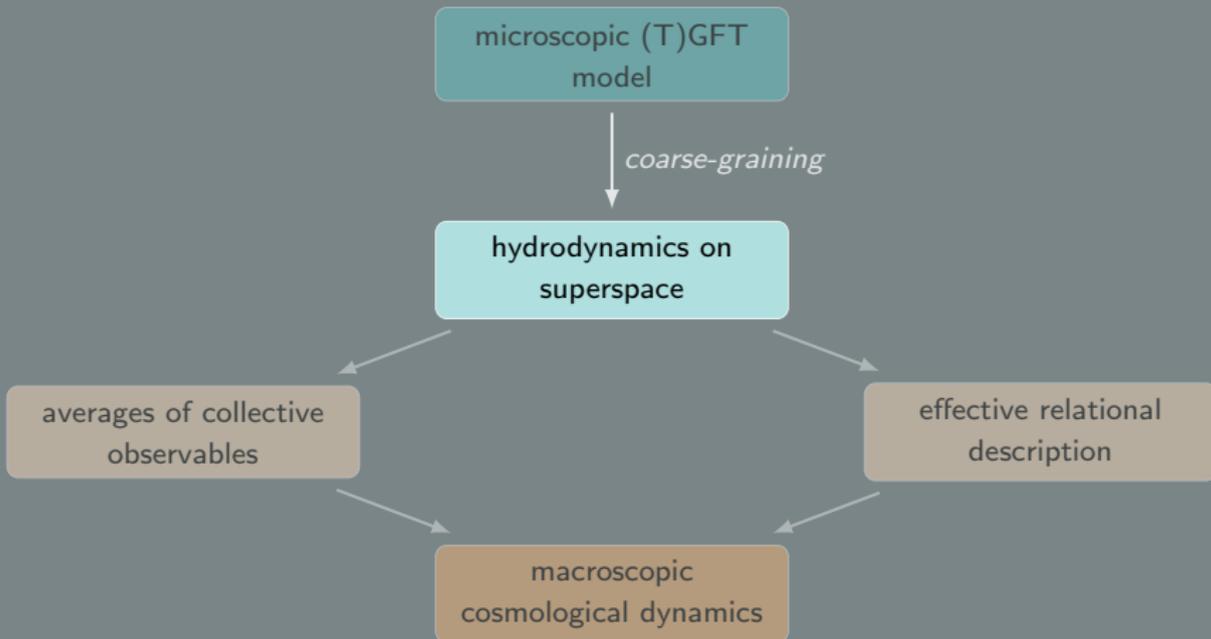
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(T)GFT condensates

Simplest collective behavior: macroscopic σ dynamics well described in the mean-field approx.

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d_l^d \chi \int d\mathbf{g}_l \sigma(\mathbf{g}_l, \chi^\mu) \hat{\varphi}^\dagger(\mathbf{g}_l, \chi^\mu) \right] |0\rangle$$

QG condensates and peaked states

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QG condensates and peaked states

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Relationality

Condensate Peaked States

- ▶ If σ is peaked on $\chi^\mu \simeq x^\mu$, $|\sigma\rangle_x$ encodes relational info. about spatial geometry + matter at x^μ .
 $\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$
- ▶ Relational strategy implemented at an effective level on “hydrodynamic” (averaged) quantities.

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Mean-field approximation and QG hydrodynamics

Mean-field approximation: A non-linear and non-local extension of QC

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(\mathbf{g}_I, x^0)} \right\rangle_{\sigma_{x^0}} = \int d\mathbf{h}_I \int d\chi \mathcal{K}(\mathbf{g}_I, \mathbf{h}_I, (x^0 - \chi)^2) \sigma_{x^0}(\mathbf{h}_I, \chi) + \lambda \left. \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(\mathbf{g}_I, x^0)} \right|_{\varphi=\sigma_{x^0}} = 0.$$

- ▶ **Non-localities** present in geometric (\mathbf{g}_I) and pre-matter (χ) variables.
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- ▶ Isotropy: $\tilde{\sigma}_v \equiv \rho_v e^{i\theta_v}$ fundamental variables, with
 - $v = j \in \mathbb{N}/2$ for SU(2) (EPRL-like);
 - $v = \rho \in \mathbb{R}$ for SL(2, \mathbb{C}) (extended BC).
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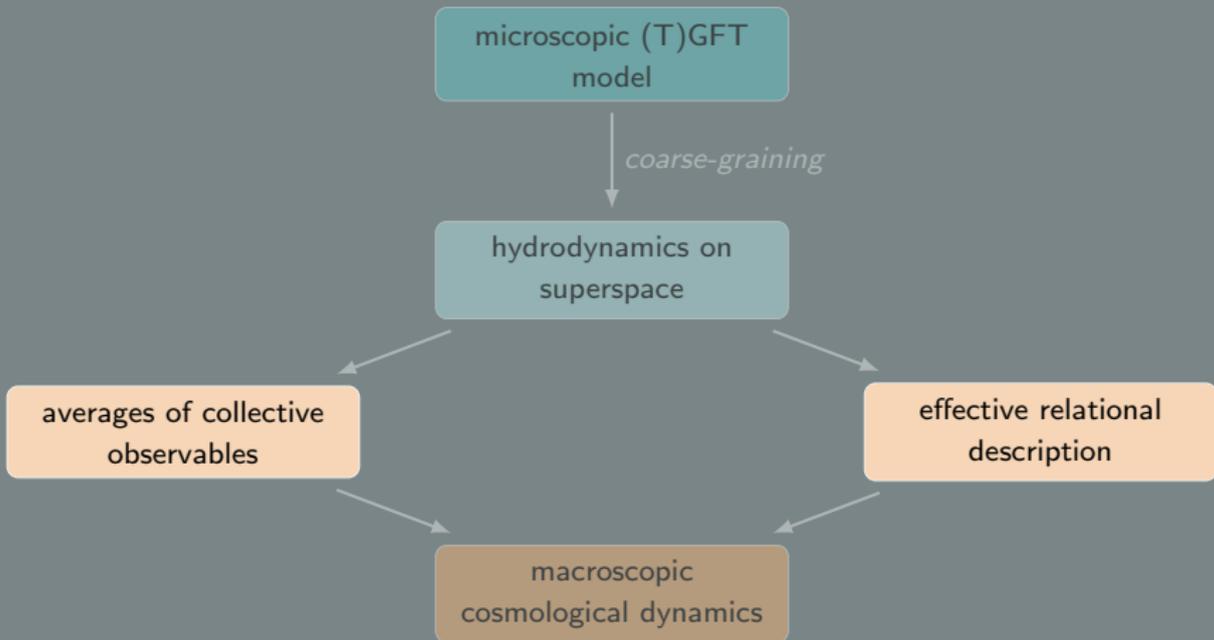
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$$\tilde{\sigma}_v'' - 2i\tilde{\pi}_0 \tilde{\sigma}_v' - E_v^2 \tilde{\sigma}_v = 0$$



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Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_I$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

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► Observables \leftrightarrow collective operators on Fock space.

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- $\langle \hat{O} \rangle_{\sigma, x^0} = O[\tilde{\sigma}]|_{\chi^0=x^0}$ **hydrodynamic**
variables: functionals of $\tilde{\sigma}$ localized at x^0 .

Cosmology from QG condensates: observables and relationality

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Relationality

► Averaged evolution wrt x^0 is physical:

Intensive $\longleftarrow \langle \hat{X} \rangle_{\sigma_{x^0}} \equiv \langle \hat{X} \rangle_{\sigma_{x^0}} / \langle \hat{N} \rangle_{\sigma_{x^0}} \simeq x^0$

► Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian $H_{\sigma_{x^0}} \simeq \langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}}$.

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(\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_I$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

- $\langle \hat{O} \rangle_{\sigma_{x^0}} = O[\tilde{\sigma}]|_{\chi^0=x^0}$ **hydrodynamic**
variables: functionals of $\tilde{\sigma}$ localized at x^0 .

Relationality

► Averaged evolution wrt x^0 is physical:

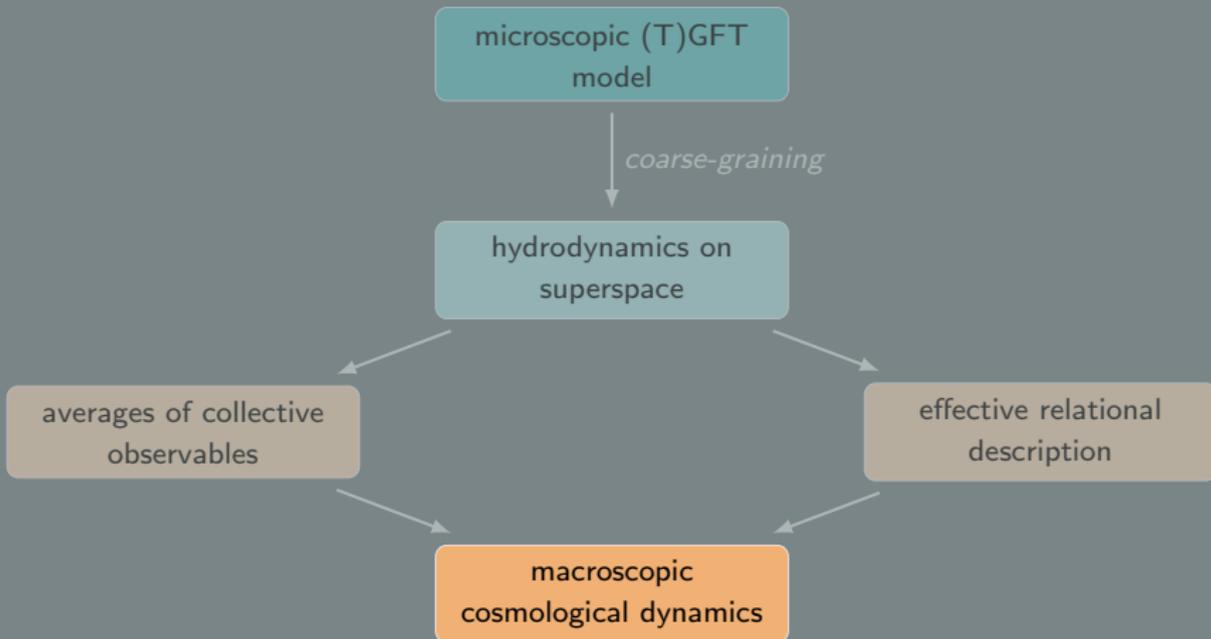
$$\langle \hat{X} \rangle_{\sigma_{x^0}} \equiv \langle \hat{X} \rangle_{\sigma_{x^0}} / \langle \hat{N} \rangle_{\sigma_{x^0}} \simeq x^0$$

► Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian $H_{\sigma_{x^0}} \simeq \langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}}$.

Wavefunction
isotropy \rightarrow

$$\langle \hat{V} \rangle_{\sigma_x^0} = \sum_{\nu} V_{\nu} |\tilde{\sigma}_{\nu}|^2(x^0)$$



Mean-field approximation

- ▶ Mesoscopic regime: large N but negligible interactions.
- ▶ Hydrodynamic approx. of kinetic kernel.
- ▶ Isotropy: $\tilde{\sigma}_\nu \equiv \rho_\nu e^{i\theta_\nu}$ fundamental variables.

$$\tilde{\sigma}_\nu'' - 2i\tilde{\pi}_0\tilde{\sigma}_\nu' - E_\nu^2\tilde{\sigma} = 0.$$

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Effective relational Friedmann equations

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\mathcal{F}_v V_v \rho_v \text{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3\mathcal{F}_v V_v \rho_v^2}\right)^2 \quad \frac{V''}{V} \simeq \frac{2\mathcal{F}_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\mathcal{F}_v V_v \rho_v^2}$$

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Classical limit (large ρ_v s, late times)

- ▶ If μ_v^2 is mildly dependent on v (or one v is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

- ▶ **Quantum fluctuations** on clock and geometric variables are **under control**.

Effective relational volume dynamics

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Bounce

- ▶ A **non-zero volume bounce** happens for a large range of initial conditions (at least one $Q_v \neq 0$ or one $\mathcal{E}_v < 0$).
- ▶ The average singularity resolution may still be spoiled by quantum effects on geometric and clock variables.

Exploring the physics of (T)GFT condensates

Geometric acceleration from interactions

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.

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- ▶ Classical system: gravity + 5 m.c.m.f. scalar fields, 4 of which constitute the relational frame.
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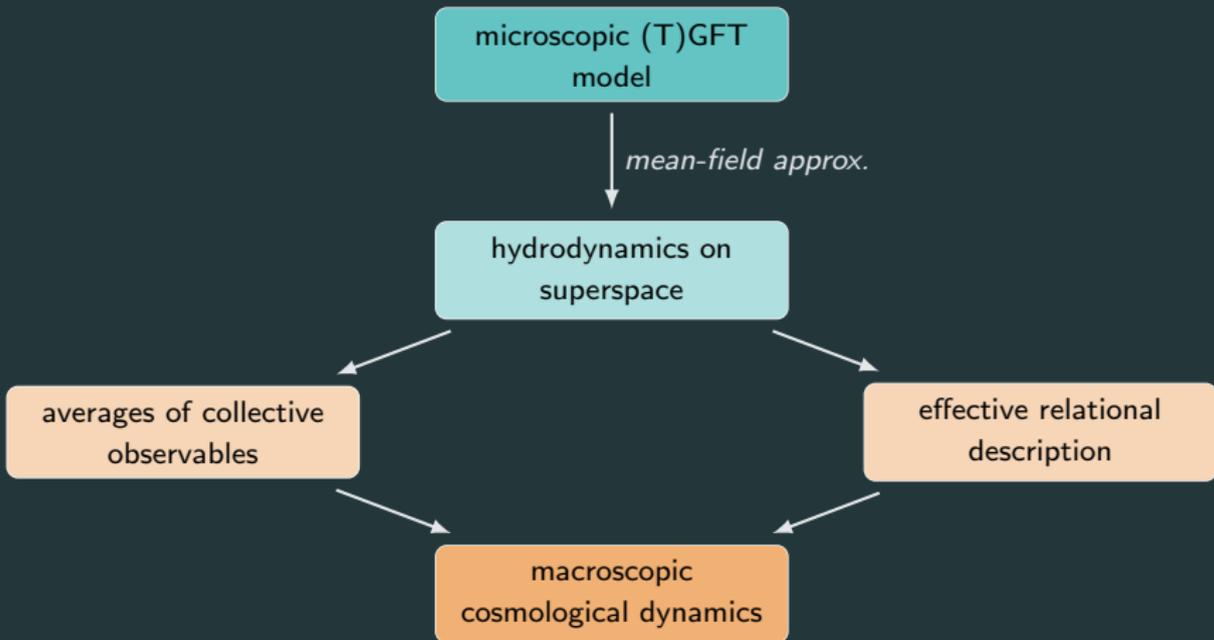
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Why the mismatch?

Geometry from quantum correlations!



► Quanta: simplices decorated with discretized fields.

microscopic (T)GFT model

► Models related to simplicial gravity, spinfoams and LQG.

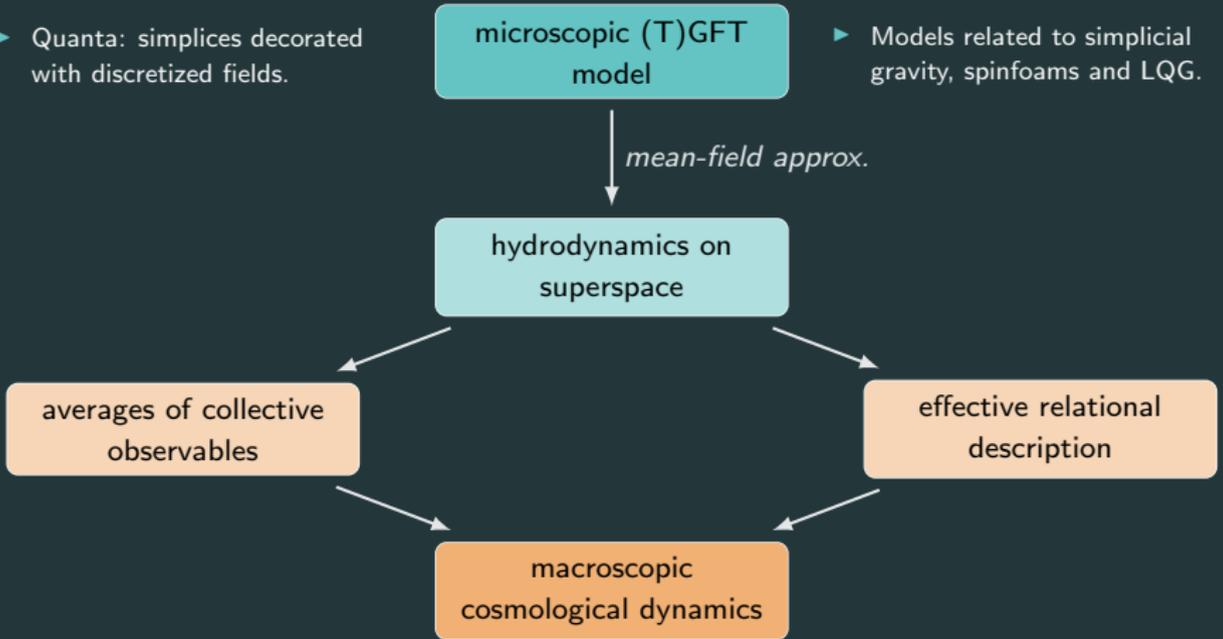
mean-field approx.

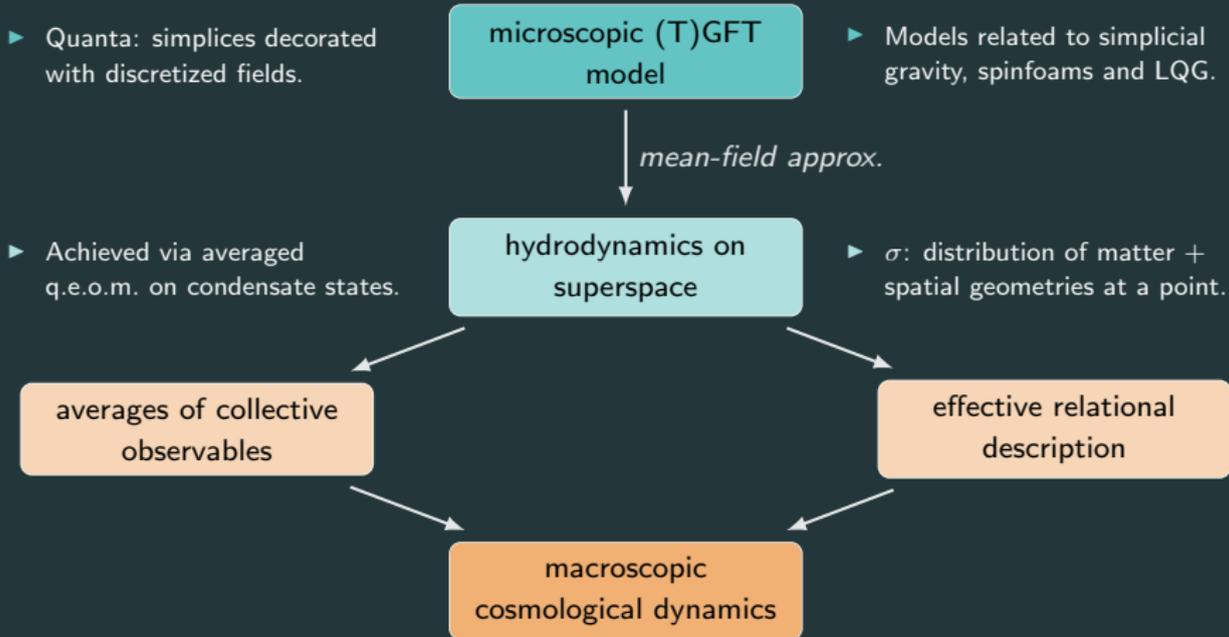
hydrodynamics on superspace

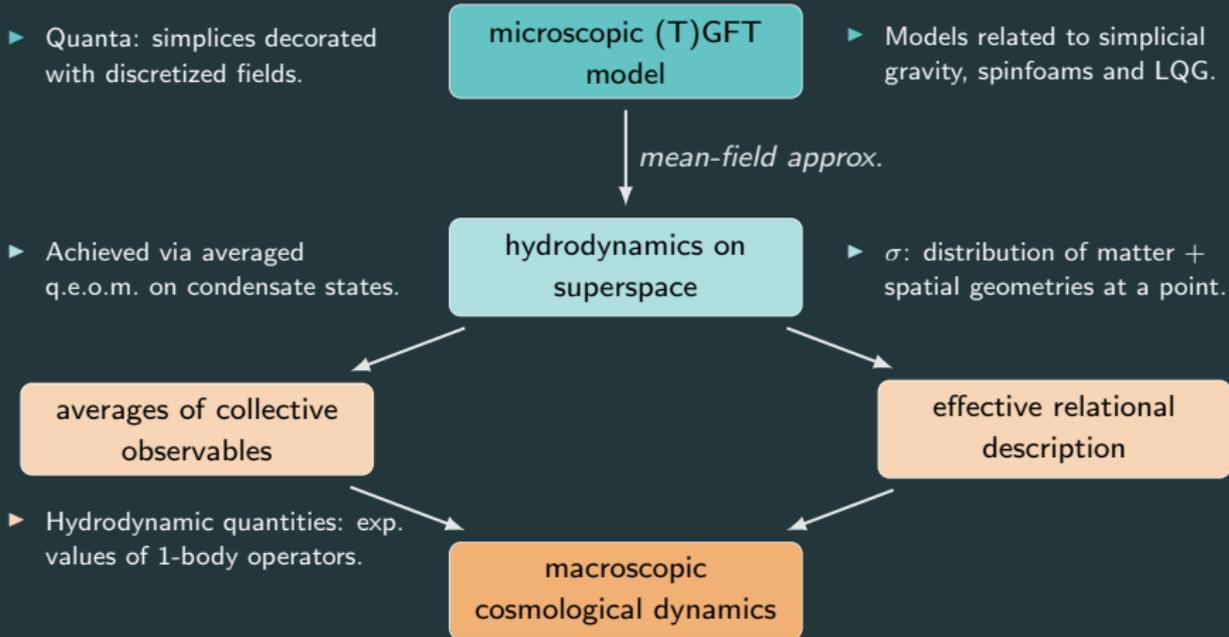
averages of collective observables

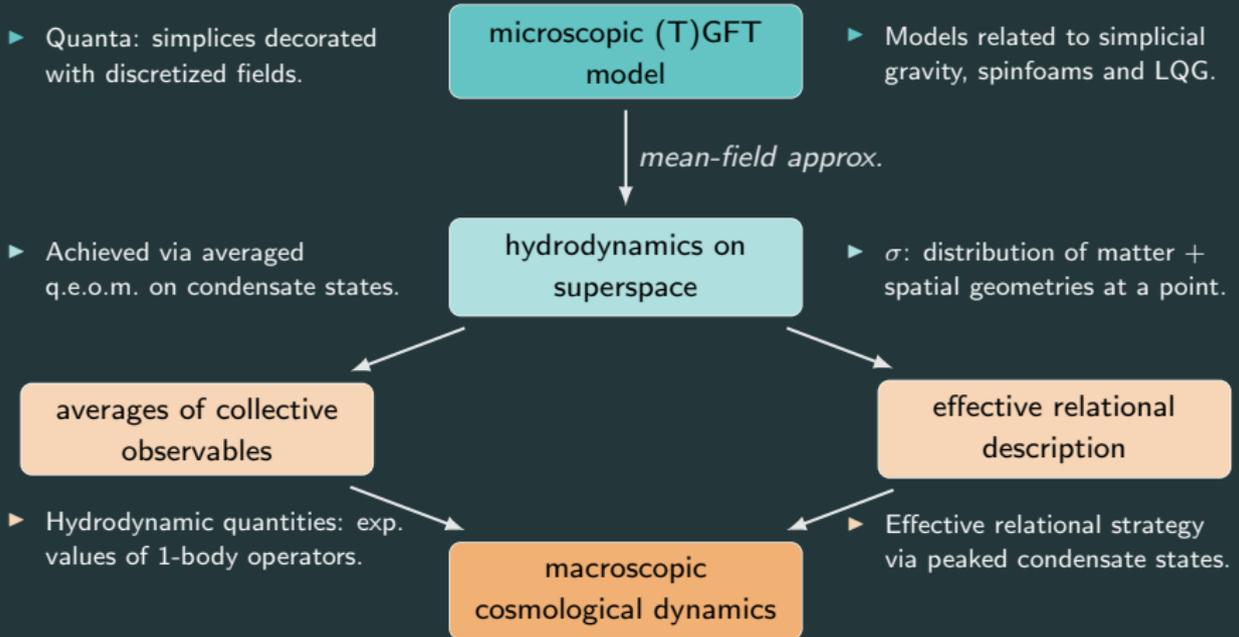
effective relational description

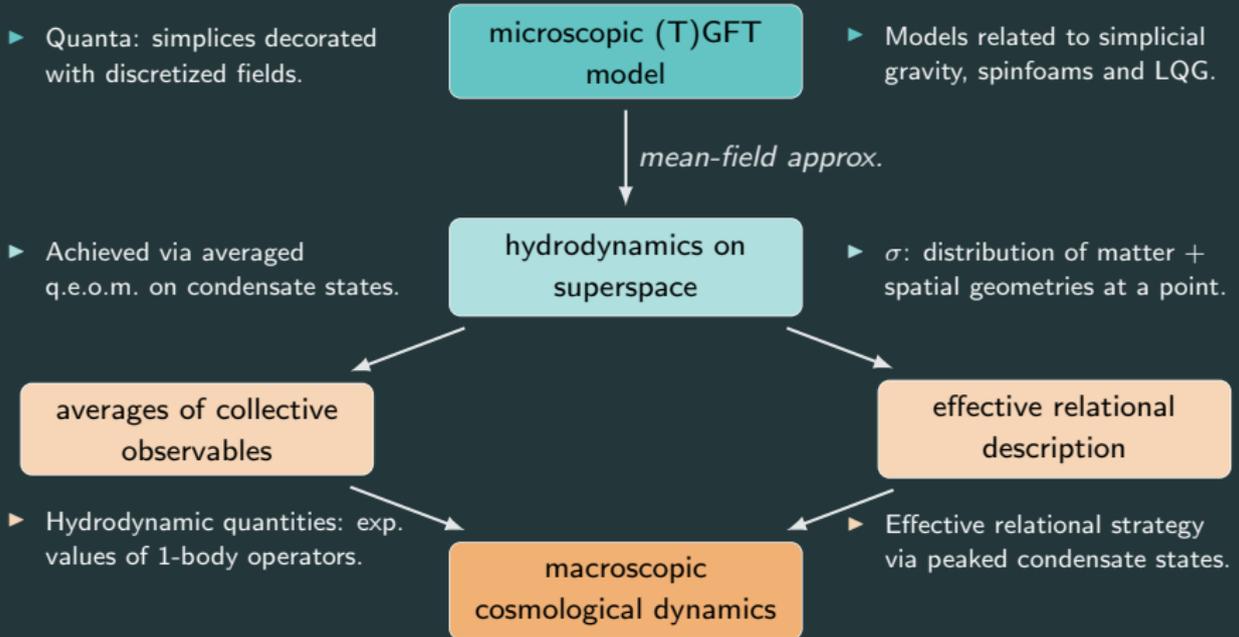
macroscopic cosmological dynamics





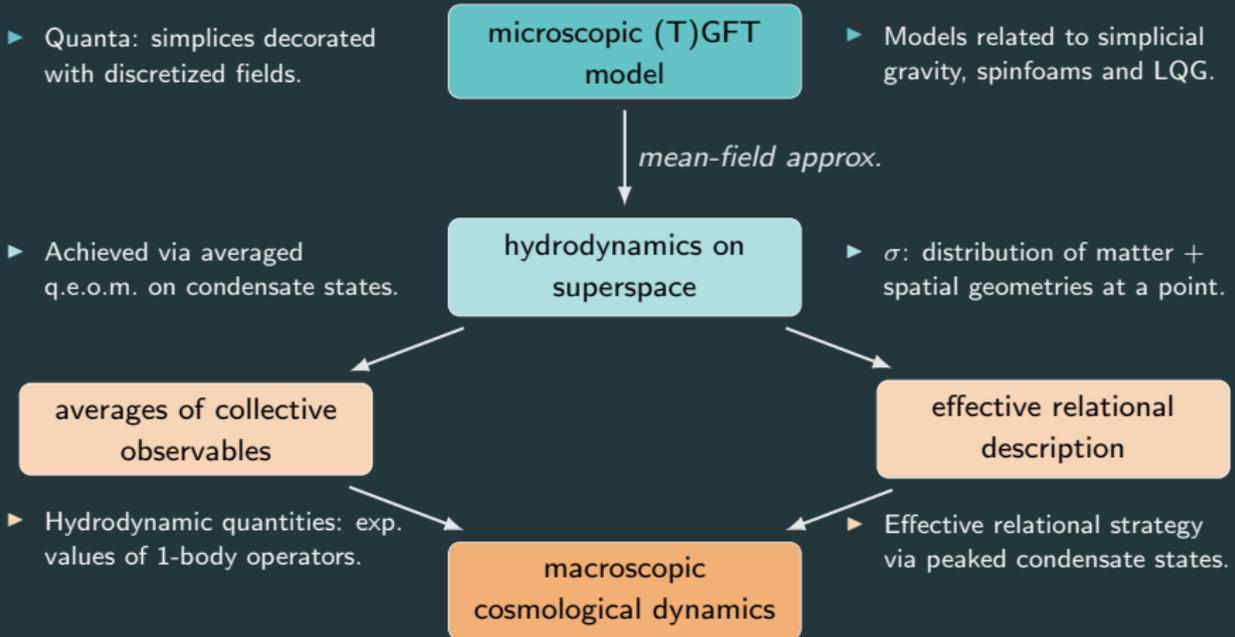






Homogeneous, free

- ▶ Late times: FRLW flat classical dynamics.
- ▶ Early times: averaged quantum bounce.

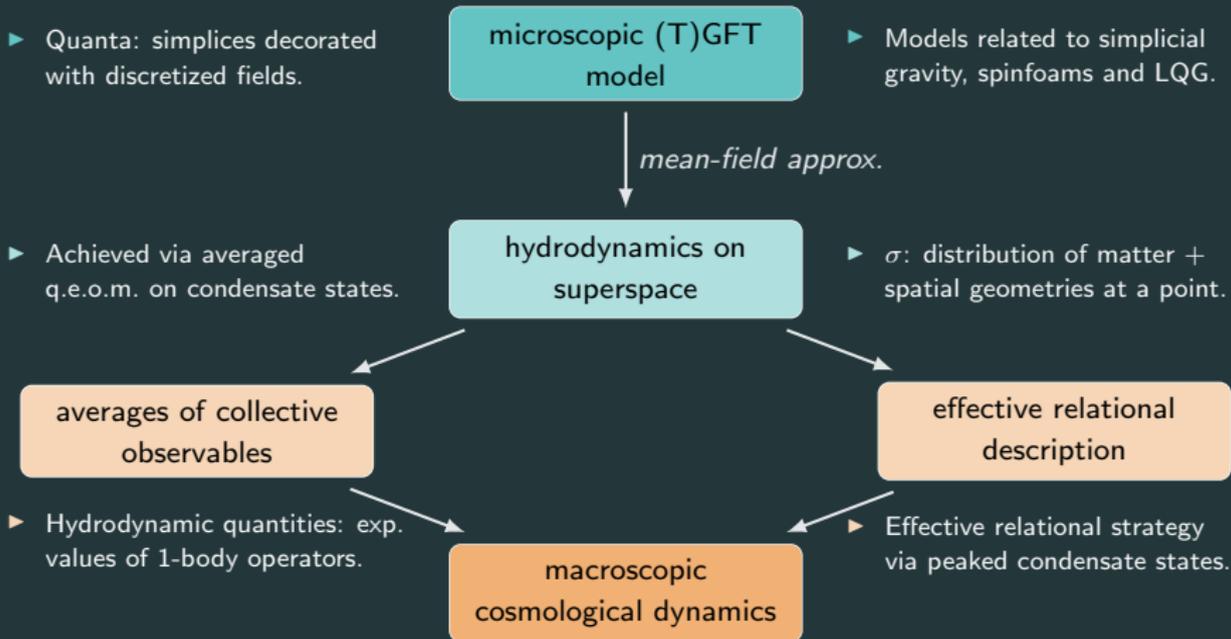


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Homogeneous, interacting

- ▶ Late times: emergence of phantom dark energy.
- ▶ Early times: possible geometric slow-roll inflation.



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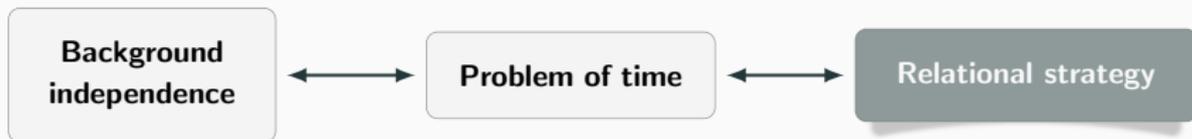
- ▶ Late times: emergence of phantom dark energy.
- ▶ Early times: possible geometric slow-roll inflation.

(Scalar) Inhomogeneities, free

- ▶ Late times: strong indications for GR matching.
- ▶ Early times: deviations from classical gravity.

Backup

Relational strategy: the classical and quantum GR perspective



Quite well understood from a classical perspective, less from a quantum perspective.

Classical

Notion of relationality can be classically encoded in **relational observables**:

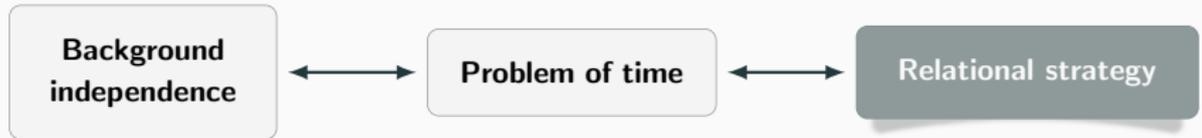
- ▶ Take two phase space functions, f and T with $\{T, C_H\} \neq 0$ (T relational clock).
- ▶ The relational extension $F_{f,T}(\tau)$ of f encodes the value of f when T reads τ .
- ▶ Evolution in τ is relational.
- ▶ $F_{f,T}(\tau)$ is a very complicated function, often written in series form.
- ▶ Applications only for (almost) deparametrizable systems, such as GR plus pressureless dust or massless scalar fields.

Quantum GR

Dirac approach: first quantize, then implement relationality

- ▶ Perspective neutral approach: all variables are treated on the same footing.
 - ▶ Poor control of the physical Hilbert space.
- Reduced phase space approach**: first implmt relationality, then quantize
- ▶ No quantum constraint to solve.
 - ▶ Led to quantization of simple deparametrizable models (LQG).
 - ▶ Not perspective neutral. Too complicated to implement in most of the cases.

Relational strategy and emergent quantum gravity theories



- ▶ Well understood from a classical perspective, less from a quantum perspective.
- ▶ Difficulties especially relevant for **emergent** QG theories.

Microscopic pre-geo

- ▶ Fundamental d.o.f. are weakly related to spacetime quantities;
- ▶ The latter expected to emerge from the former when a continuum limit is taken.

Macroscopic proto-geo

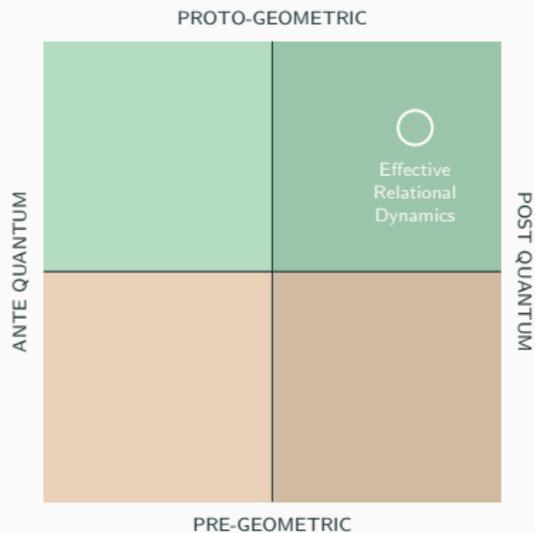
- ▶ Set of collective observables;
- ▶ Coarse grained states or probability distributions.

The quantities whose evolution we want to describe relationally are the result of a coarse-graining of some fundamental d.o.f.

Effective approaches:

- ▶ Bypass most conceptual and technical difficulties;
- ▶ Relevant for observative purposes.

Emergent effective relational dynamics



Basic principles

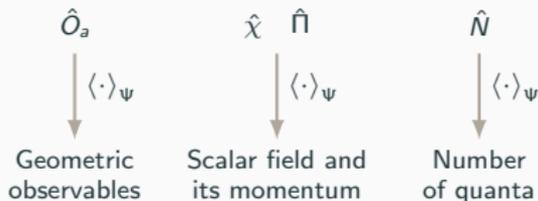
Emergence Rel. dynamics formulated in terms of collective observables and states defined in the microscopic theory.

Effectiveness Rel. evolution intended to hold on average. Internal clock not too quantum.

Concrete example: scalar field clock

Emergence

- ▶ Identify a class of states $|\Psi\rangle$ which encode **collective behavior** and admit a **continuum** proto-geometric **interpretation**.
- ▶ Identify a set of collective observables:



Effectiveness

- ▶ It exists a “Hamiltonian” \hat{H} such that

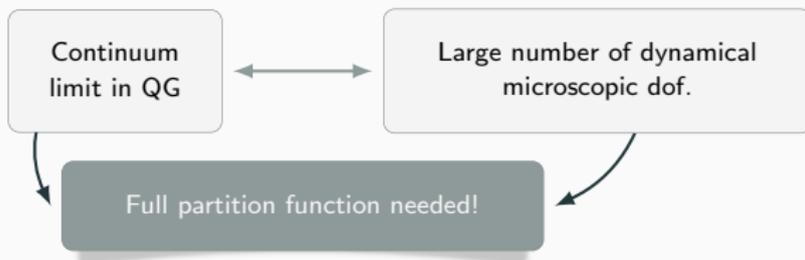
$$i \frac{d}{d \langle \hat{\chi} \rangle_\Psi} \langle \hat{O}_a \rangle_\Psi = \langle [\hat{H}, \hat{O}_a] \rangle_\Psi,$$

and whose moments coincide with those of $\hat{\Pi}$.

- ▶ Relative variance of $\hat{\chi}$ on $|\Psi\rangle$ should be $\ll 1$ and have the characteristic $\langle \hat{N} \rangle_\Psi^{-1}$ behavior:

$$\sigma_{\hat{\chi}}^2 \ll 1, \quad \sigma_{\hat{\chi}}^2 \sim \langle \hat{N} \rangle_\Psi^{-1}.$$

Continuum physics and QG: the general perspective



The (F)RG perspective

Spacetime QFT

- ▶ Energy **scale** defines the flow from IR and UV.

UV and IR have different meaning in QG!

- ▶ Theory space constrained by **symmetries**.

Little control over QG theory space!

QG theory

- ▶ Only **internal** “timeless” scales available.

- ▶ **Symmetries** of QG models hard to classify.

Approximate methods: mean-field theory

- ▶ Based on collective quantity: **order parameter**.
- ▶ **Mean-field** (saddle-point) approx. of Z: “simple” computations!
- ▶ Good description of quantum **condensate** phase transitions.
- ▶ QG counterpart of the Gross-Pitaevskii approximation in the **hydrodynamics** of quantum fluids.

Scalar perturbations from (T)GFT condensates

Simplest (slightly) relationally inhomogeneous system

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Simplest (slightly) relationally inhomogeneous system

Classical

- ▶ 4 MCM **reference** fields (χ^0, χ^i) , with Lorentz/Euclidean invariant S_χ in field space.
- ▶ 1 MCM **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

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Quantum

- ▶ (T)GFT field: $\varphi(g_I, \chi^\mu, \phi)$, depends on 5 discretized scalar variables.
- ▶ EPRL-like or extended BC model with S_{GFT} respecting the classical matter symmetries.

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States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):
 $\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu)$, $\theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu)$.

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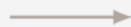
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Late times volume and matter dynamics

- ▶ Averaged q.e.o.m. \rightarrow coupled differential equations for ρ and θ .
- ▶ Decoupling for a range of values of CPSs and large N (late times).



Dynamic equations
for $\langle \hat{V} \rangle_\sigma$, $\langle \hat{\Phi} \rangle_\sigma$

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Perturbations

- ▶ Super-horizon GR matching.

Scalar perturbations from (T)GFT condensates

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\mathbf{g}_I$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):
 $\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu)$, $\theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu)$.

Late times volume and matter dynamics

- ▶ Averaged q.e.o.m. \rightarrow coupled differential equations for ρ and θ .
- ▶ Decoupling for a range of values of CPSs and large N (late times).

Dynamic equations
for $\langle \hat{V} \rangle_\sigma$, $\langle \hat{\Phi} \rangle_\sigma$

Background

- ▶ Matching with GR (assuming peaking on matter momenta).
- ▶ Emergent matter and G defined in terms of microscopic parameters.

Perturbations

- ▶ Super-horizon GR matching.
- ▶ **No matching** for intermediate modes (because of different coupling with bkg effective metric)!
- ▶ Effective metric signature determined by CPSs.

Volume at late times

Background

Classical

- ▶ Harmonic gauge: $N = a^3$.
- ▶ Negligible contribution of reference matter.

$$(\bar{V}'/\bar{V})^2 = 12\pi G\pi_\phi^{(c)}$$

$$(\bar{V}'/\bar{V})' = 0$$

Quantum

- ▶ Wavefunction peaked on $\pi_\phi = \tilde{\pi}_\phi$.
- ▶ Domination of single spin v_o .
- ▶ $\mu_{v_o}(\pi_\phi) \simeq c_{v_o}\pi_\phi$, with $4c_{v_o}^2 = 12\pi G$.

$$(\bar{V}'/\bar{V})^2 = 12\pi G\tilde{\pi}_\phi \quad (\bar{V}'/\bar{V})' = 0$$

Perturbations

Classical

- ▶ First order harmonic gauge.
- ▶ Negligible contribution of reference matter.
- ▶ Define $V(x) = \sqrt{\det q_{ij}} \equiv \bar{V} + \delta V$.

$$\delta V'' - 6\mathcal{H}\delta V' + 9\mathcal{H}^2\delta V - \bar{V}^{4/3}\nabla^2\delta V = 0.$$

Quantum

- ▶ Wavefunction peaked on $\pi_\phi = \tilde{\pi}_\phi$.
- ▶ Domination of single v_o : $\delta V \equiv 2\bar{\rho}_{v_o}\delta\rho_{v_o}$.
- ▶ $\mu_{v_o}(\pi_\phi) \simeq c_{v_o}\pi_\phi$, with $4c_{v_o}^2 = 12\pi G$.

$$\delta V'' - 3\mathcal{H}\delta V' + \text{Re}(\alpha^2)\nabla^2\delta V = 0.$$

Super-horizon

- ▶ Matches the classical solution $\delta V \propto \bar{V}$.

Sub-horizon

- ▶ Same diff. structure but different powers of \bar{V} .

No matching with GR for arbitrary modes.

Matter at late times

Background

Classical

- ▶ Harmonic gauge: $N = a^3$.
- ▶ Negligible contribution of ref. matter.

$$\bar{\phi}'' = 0,$$

$$\pi_{\phi}^{(c)} = \text{const.}$$

Quantum

- ▶ Wavefunction peaked on $\pi_{\phi} = \tilde{\pi}_{\phi}$.
- ▶ Domination of single v_o .

$$\langle \hat{\Pi}_{\phi} \rangle_{\bar{\sigma}} = \tilde{\pi}_{\phi} \bar{N},$$

$$\langle \hat{\Phi} \rangle_{\bar{\sigma}} = \left[-\partial_{\pi_{\phi}} \left[\frac{Q_{v_o}}{\mu_{v_o}} \right] + Q_{v_o} \frac{\partial_{\pi_{\phi}} \mu_{v_o}}{\mu_{v_o}} x^0 \right]_{\pi_{\phi} = \tilde{\pi}_{\phi}}$$

Matching conditions

- ▶ $\pi_{\phi}^{(c)} \equiv \langle \hat{\Pi}_{\phi} \rangle_{\bar{\sigma}} / \bar{N} = \tilde{\pi}_{\phi}$.
- ▶ $\phi \equiv \langle \hat{\Phi} \rangle_{\bar{\sigma}} = -c_{v_o}^{-1} + \tilde{\pi}_{\phi} x^0$, $Q_{v_o} \simeq \pi_{\phi}^2$!
- ▶ Peaking in $\pi_{\phi} \rightarrow$ peaking in matter field momenta.
- ▶ Emergent G related to matter content!

Perturbations

Classical

- ▶ First order harmonic gauge.
- ▶ Negligible contribution of ref. matter.

$$\delta\phi'' - \bar{V}^{4/3} \nabla^2 \delta\phi = 0.$$

Quantum

- ▶ Wavefunction peaked on $\pi_{\phi} = \tilde{\pi}_{\phi}$.
- ▶ Domination of single spin v_o : $\delta V \equiv 2\bar{\rho}_{v_o} \delta\rho_{v_o}$.

$$\delta\phi = \delta V / \bar{V} + \bar{N} [\partial_{\pi_{\phi}} \theta_{v_o}]_{\pi_{\phi} = \tilde{\pi}_{\phi}}.$$

- ▶ Matching at super-horizon scales
- ▶ No matching for intermediate scales.

Scalar perturbations from causal (T)GFT condensates

Model

- ▶ Spacelike and timelike tetrahedra, generated respectively by ($g_l \in \text{SL}(2, \mathbb{C})$):

$$\hat{\varphi}_+(g_l, X_+, \chi^\mu, \cdot), \quad X_+ \in \mathbb{H}^3,$$

$$\hat{\varphi}_-(g_l, X_-, \chi^\mu, \cdot), \quad X_- \in \mathbb{H}^{1,2}.$$

- ▶ Causal properties of the frame encoded in $K = K_+ + K_-$, with $K_\pm = (\varphi_\pm^*, \mathcal{K}_\pm \varphi_\pm)$:

$$\mathcal{K}_+ = \mathcal{K}_+(g_l, g'_l; (\chi^0 - \chi'^0)^2, \cdot),$$

$$\mathcal{K}_- = \mathcal{K}_-(g_l, g'_l; |\chi^i - \chi'^i|^2, \cdot).$$

States

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I} + \mathbb{I} \otimes \hat{\tau} + \delta\hat{\Phi} \otimes \mathbb{I} + \delta\hat{\Psi} + \mathbb{I} \otimes \delta\hat{\Xi}) |0\rangle$$

Background:

- ▶ Two-sectors condensation: $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$, $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$;
- ▶ σ peaked on time, τ on space and time;
- ▶ $\tilde{\sigma}$ and $\tilde{\tau}$ only time dependent (homogeneity).

Perturbations: nearest neighbour two-body correlations

$$\delta\hat{\Phi} = (\delta\Phi, \hat{\varphi}_+^{\dagger 2}) \quad \delta\hat{\Psi} = (\delta\Psi, \hat{\varphi}_+^\dagger \otimes \hat{\varphi}_-^\dagger) \quad \delta\hat{\Xi} = (\delta\Xi, \hat{\varphi}_-^{\dagger 2}).$$

$$\langle \hat{V} \rangle_\psi = \bar{V}(x^0) + \delta\bar{V}(x^0) + \delta V(x^0, \mathbf{x}).$$

- ▶ $\delta\bar{V} \sim (\delta\Phi, (\sigma^*)^2)$ (bkg quantum correction).
- ▶ $\delta\bar{V} \sim (\delta\Psi, \sigma^* \tau^*)$ (inhomogeneities).
- ▶ Only two mean-field equations.
- ⚠ Move beyond mean-field?
- ⚠ Physical implications of $\delta\bar{V}$?

$$\delta V'' + c_1 \delta V' + c_2 \delta V'' + f(x^0) \nabla^2 \delta V = 0.$$

- ▶ Isotropy and single spin approximation.
- ✓ Structure of equation essentially fixed.
- ✓ Matching for sub-horizon modes possible with appropriate choice of τ .
- ⚠ Values of c_1 and c_2 to be determined.

Interactions: inflation and running couplings

Interactions

Perturbative analysis

$$U = (\mathcal{U}, \phi_I[\varphi^*, \varphi]), \quad \phi_I[\varphi^*, \varphi] \sim \begin{cases} \phi^{l+1} & \text{(phase dependent interactions)} \\ (\phi^*)^{(l+1)/2} \phi^{(l+1)/2} & \text{(modulus interactions)} \end{cases}$$

- ▶ $l + 1$ order of interaction ($l = 4$ simplicial).
- ▶ Effective dynamics from mean-field approx.
- ▶ $\mathcal{U} = \mathcal{U}(\cdot, \psi)$ only if $V_\psi^{(c)} \neq 0$.
- ▶ **Perturbative** analysis: small interactions.

Interacting scalar

Including more realistic matter: running couplings

$$\mathcal{H}^2 = \frac{8\pi G}{3} \left(\pi_\psi^2 / \pi_\chi^2 + (V^2 / \pi_\chi^2) V_\psi^{(c)} \right),$$

$$0 = \psi'' + (V^2 / \pi_\chi^2) V_{\psi, \psi}^{(c)}.$$

$$\mathcal{H}^2 = c_1 + c_2 V^2 V_\psi^{(q)},$$

$$0 = \psi'' + c_3 \chi^2 V^2 V_{\psi, \psi}^{(q)}.$$

Classical
Quantum

- ▶ $V_\psi^{(c)}$ classical scalar potential: $V_\psi^{(c)} \sim a_n \psi^n$.
- ▶ $V_\psi^{(q)}$ quantum scalar effective potential:
$$V_\psi^{(q)} = \begin{cases} V_\psi^{(c)} & \text{(modulus int.)} \\ V_\psi^{(c)} F & \text{(phase int., } F \text{ trig. function)} \end{cases}$$
- ▶ GR matching: $l = 5$ and **running couplings**:
 $G \rightarrow G/\chi^2, \pi_\psi \rightarrow \pi_\psi \chi, a_n \rightarrow \chi^2 a_n$.

Acceleration

Geometric inflation

- ▶ $l = 5$ generate cosmic acceleration.
- ▶ Phase int. produce **quantum gravity induced** trig. potential: $V = c_1 \sin(\omega\chi) + c_2 \cos(\omega\chi)$.
- ▶ Modulus interactions produce a de Sitter phase (no graceful exit).
- ▶  Detailed slow-roll analysis?