Analog models of pre-heating and the back-reaction effect

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Quantum Gravity, Hydrodynamics and Emergent Cosmology - Munich, 8 December 2022

Outline

The back-reaction problem

Analog models of gravity

Analog of cosmological pre-heating

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Analog of cosmological pre-heating

The back-reaction problem

Quantum fields in curved spacetime

Cosmological particle creation

Dynamical Casimir

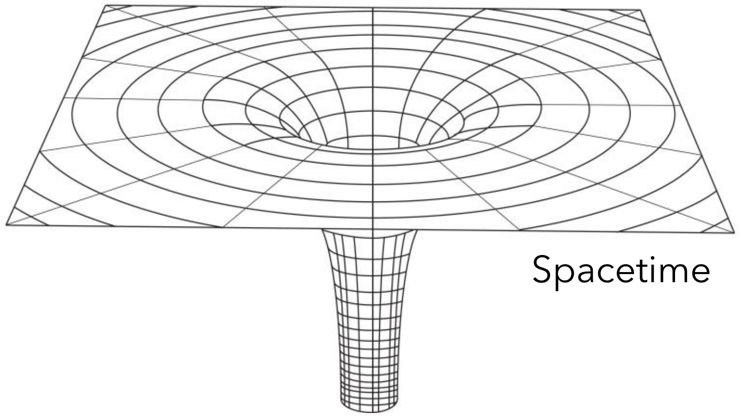
Action

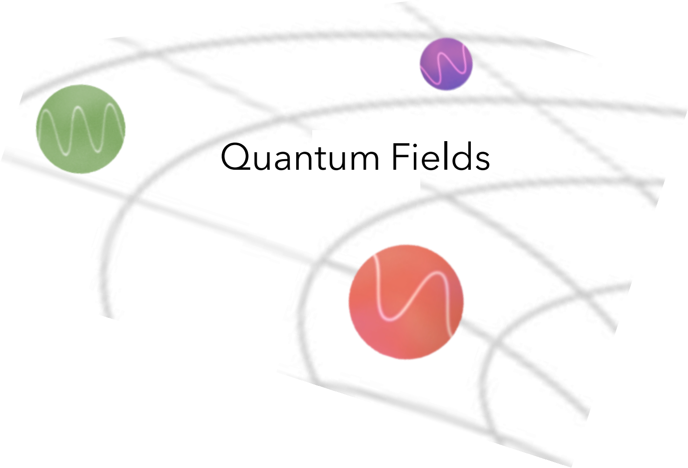
Hawking radiation

QFT in curved spacetime

Gravity

Quantum Mechanics





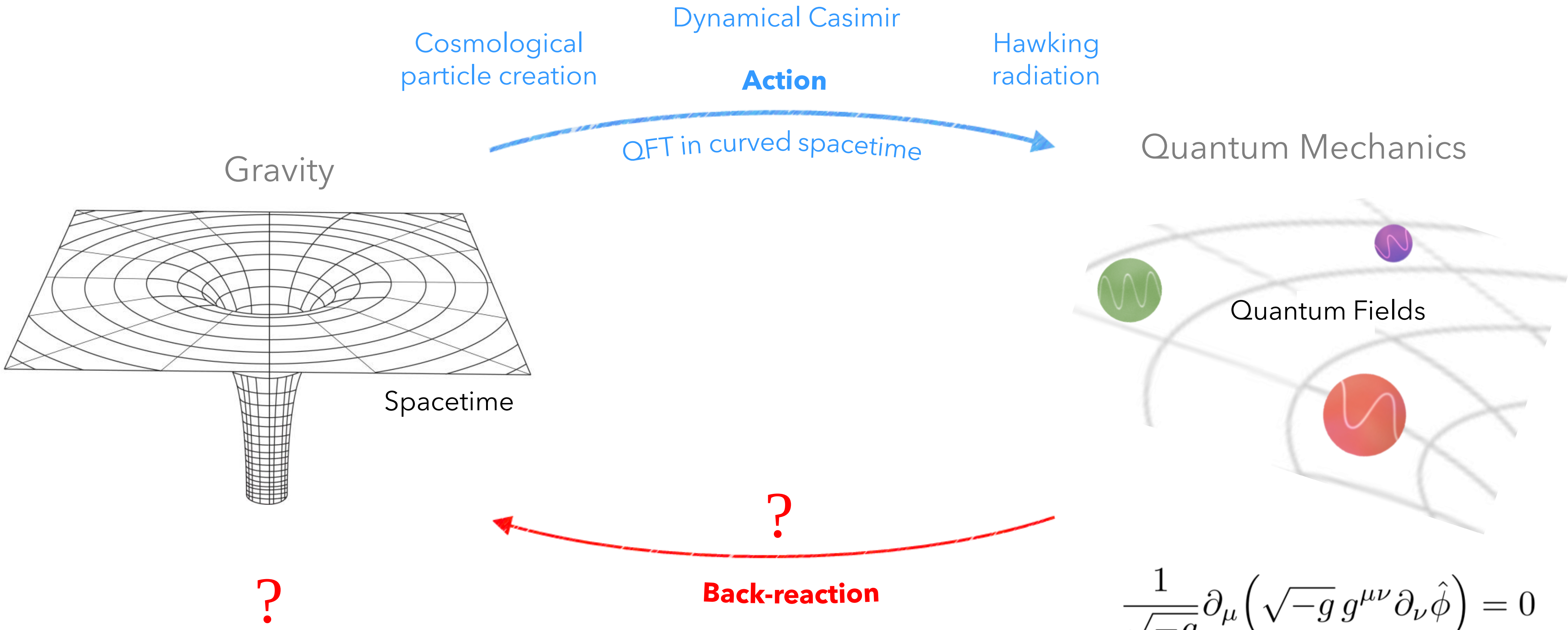
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T^{\mathrm{cl}}_{\mu\nu}$$

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \hat{\phi} \right) = 0$$

See books:
Birrell & Davies; Parker & Toms; Fulling
QFT in curved spacetime (CUP)

The back-reaction problem

Existing theories



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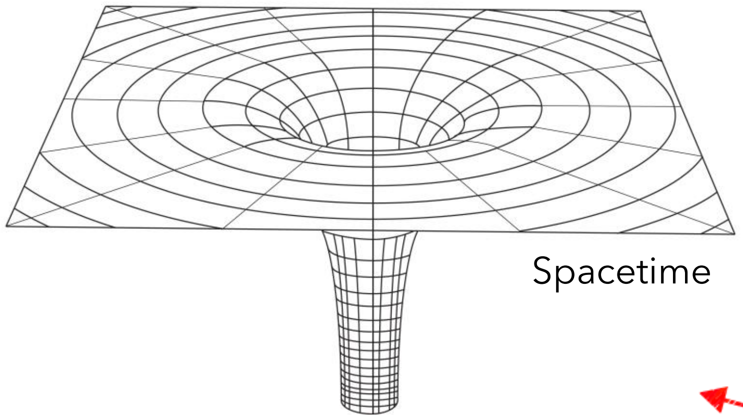
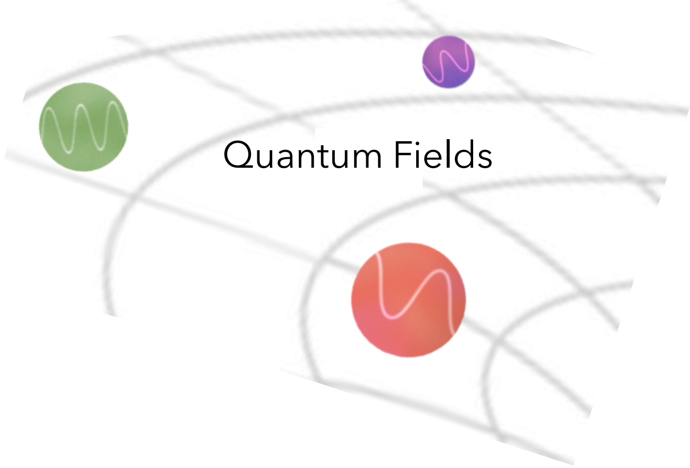




Semi-classical gravity

Back-reaction

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T^{\mathrm{cl}}_{\mu\nu} + \langle\Phi|\, \hat{T}_{\mu\nu}(\hat{\phi})\, |\Phi\rangle\right)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\hat{\phi}\right) = 0$$

Zeldovich & Starobinsky *72;* Parker & Fulling *73;* Hu & Parker *78;*
Fischetti Hartle & Hu *79;* Hartle & Hu *79, 80;* Hartle *80; …*

The back-reaction problem

Existing theories

Cosmological particle creation

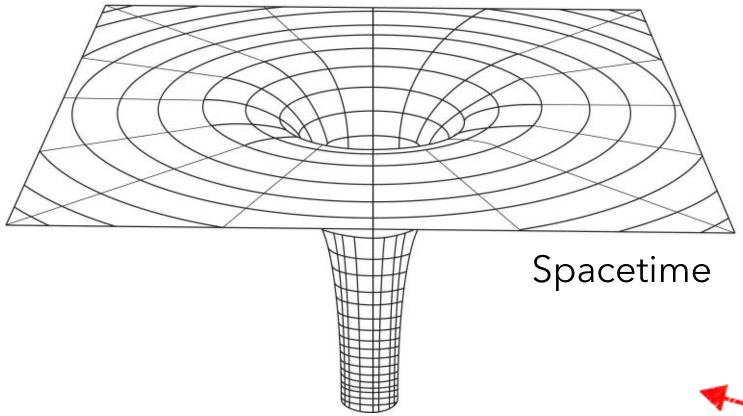
Dynamical Casimir

Hawking radiation

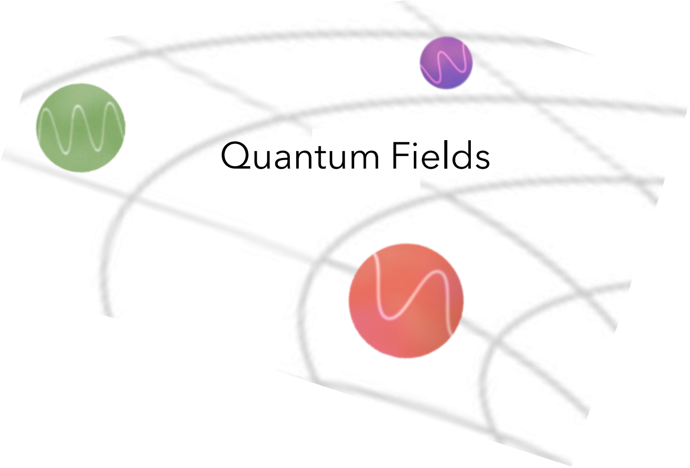
Action

QFT in curved spacetime

Gravity



Quantum Mechanics



Stochastic gravity

Back-reaction

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} \left(T^{\text{cl}}_{\mu\nu} + \langle \Phi | \hat{T}_{\mu\nu}(\hat{\phi}) | \Phi \rangle + \xi_{\mu\nu}(\hat{\phi}) \right)$$

$$\left\langle \xi_{\alpha\beta} \xi_{\gamma\delta} \right\rangle_s \sim \left\langle \hat{T}_{\alpha\beta} \hat{T}_{\gamma\delta} \right\rangle$$

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \hat{\phi} \right) = 0$$

Calzetta & Hu *94;* Hu & Sinha *95;* Hu & Matacz *95;* Hu Roura & Verdaguer *04;* ...
(See also Hu & Verdaguer, *Liv. Rev. Rel. 08* ; Hu & Verdaguer, *Class. Quant. Grav. 03*)

The back-reaction problem

Top-down approach

Macroscopic theory

	Matter sector		Spacetime
(0) Classic Gravity No quantum effects	$T^{\mathrm{cl}}_{\alpha\beta}$	$\longrightarrow$	$g_{\alpha\beta}$
(1) Semi-classical Gravity Meanfield approximation	$\langle \hat{T}_{\alpha\beta} \rangle$	$\longrightarrow$	$\langle \hat{g}_{\alpha\beta} \rangle$
(2) Stochastic Gravity Stress-tensor fluctuations	$\langle \hat{T}_{\alpha\beta} \hat{T}_{\gamma\delta} \rangle$	$\longrightarrow$	$\langle \hat{g}_{\alpha\beta} \hat{g}_{\gamma\delta} \rangle$
…	…		…
…	…		…
…	…		…
…	…		…
…	…		…
(∞) Quantum Gravity			

Microscopic theory

The back-reaction problem

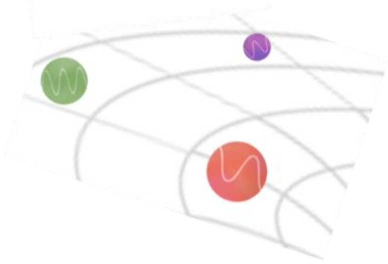
Bottom-up approach

Macroscopic theory

Classic Gravity

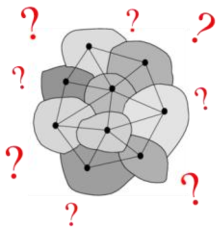
Mesoscopic level

Self-consistent theory of Spacetime + Quantum Fields



Coarse grain

Quantum Gravity



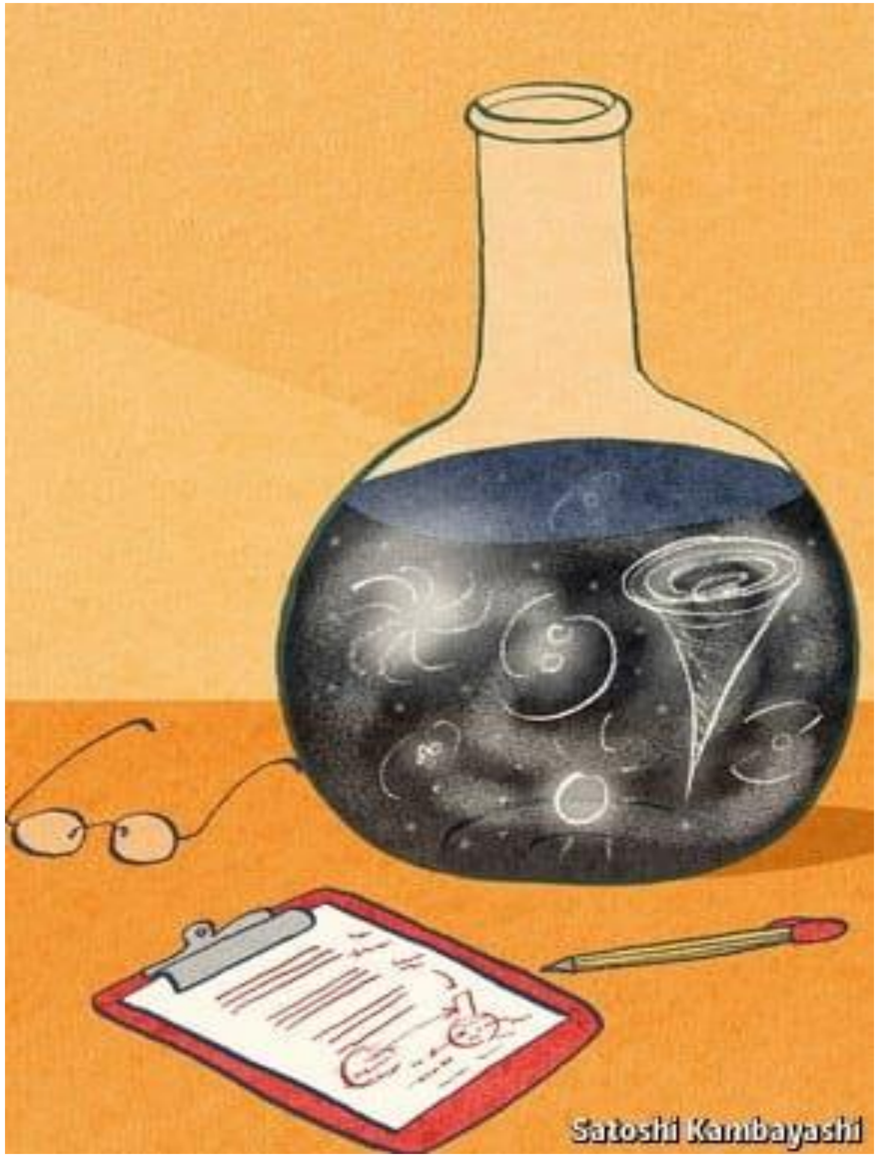
Microscopic theory

The back-reaction problem

Analog models of gravity

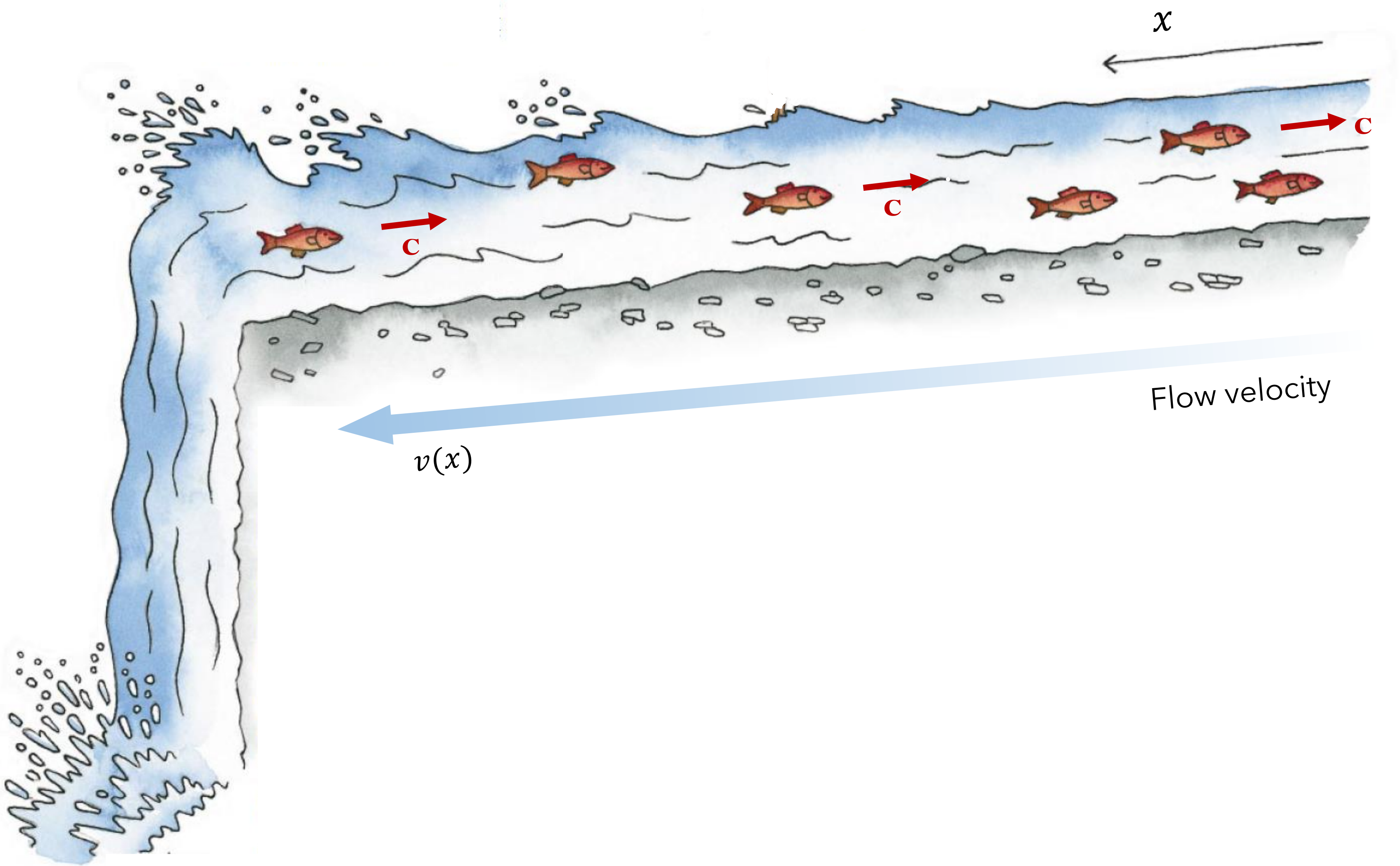
QFT in effective spacetime

Analog of cosmological pre-heating



Analogue models of gravity

A fish black hole



Analogue models of gravity

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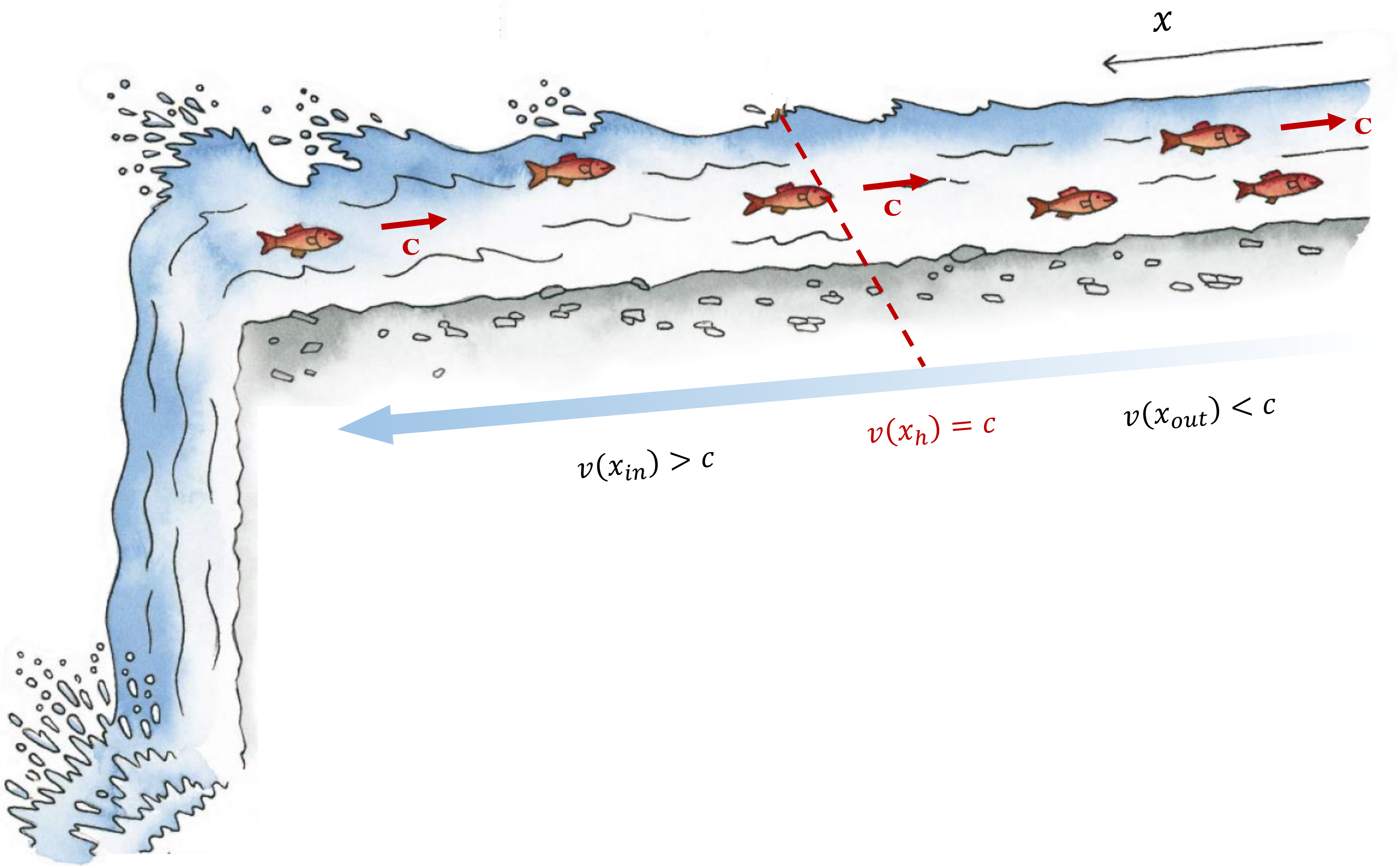
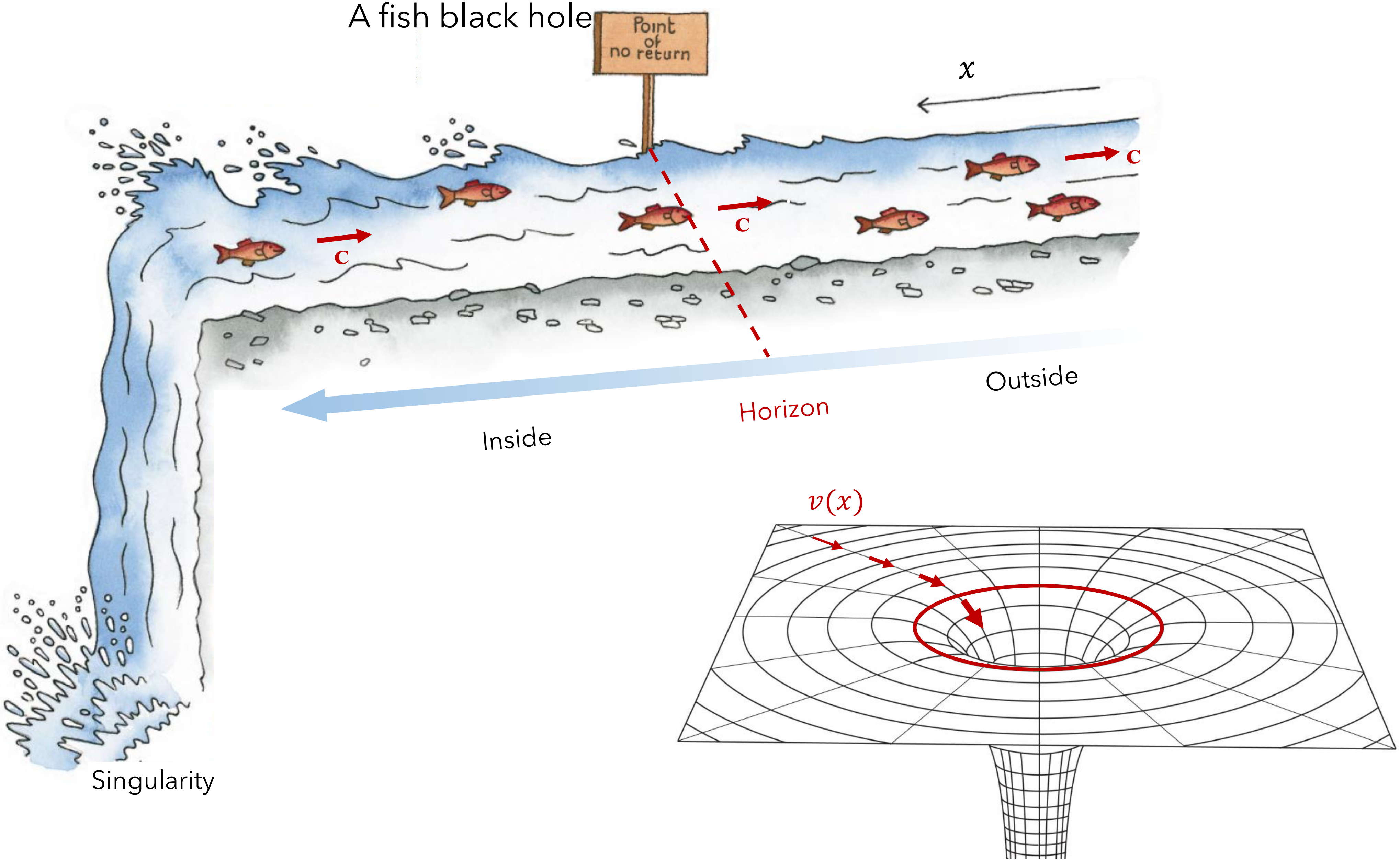


Analogue models of gravity

A fish black hole



Analogue models of gravity

Effective spacetime

White hole in the sea

W. G. Unruh. , *PRL* **46**, 1351 (1981)
C. Barcelo' et al. , *Liv. Rev. Rel.* **14**, 3 (2011)

In the case of (i) irrotational flow (no viscosity)

Velocity → $v \equiv -\boldsymbol{\nabla} S$ ← Velocity potential

and (ii) barotropic medium

Density ⟶ $\rho = \rho(p)$

Linear excitations evolve as a field in curved spacetime:

$$S = S_0 + \epsilon S_1 \text{ , etc.}$$

Eq. of motion: $$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial S_1}{\partial x^\nu}\right) = 0$$

Background density, Background velocity, Speed of sound

Metric: $$g_{\mu\nu} = \frac{\rho_0}{c_0}\begin{pmatrix} -(c_0^2 - v_0^2) & -v_{j0} \\ -v_{i0} & \delta_{ij} \end{pmatrix}$$

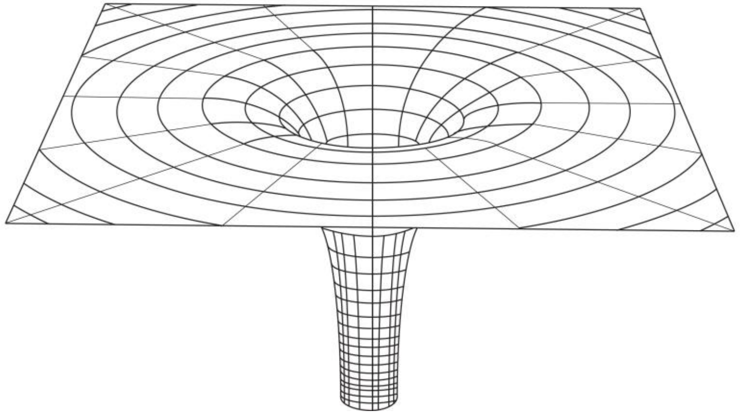
Analogue models of gravity

QFT in effective spacetime

Action

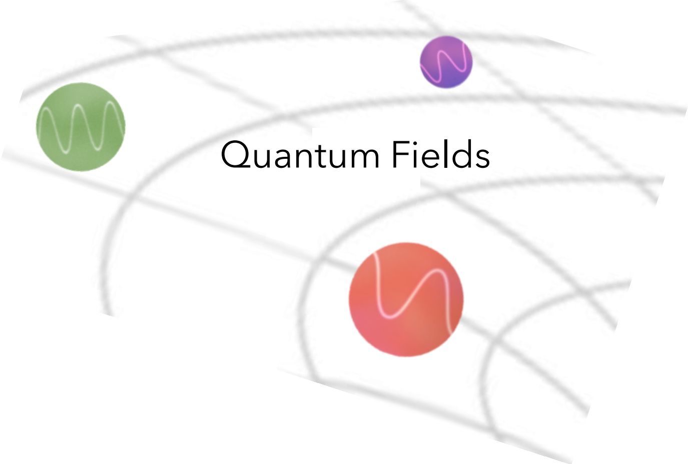
QFT in curved spacetime

Effective spacetime



Hydrodynamic equations
of the analog system

Excitation fields



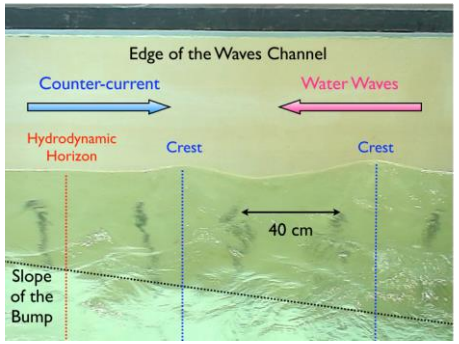
Effective field equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial S_1}{\partial x^{\nu}}\right)=0$$

Analogue models of gravity

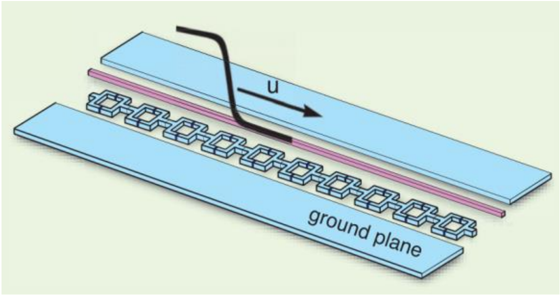
In the lab

Surface waves in water



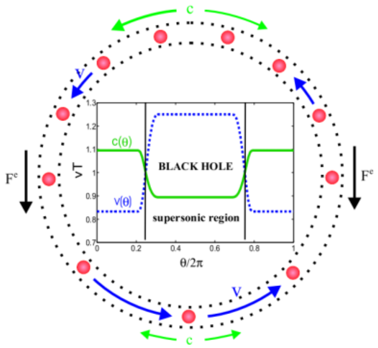
G. Rousseaux et al., *NJP* (2010)

Superconducting circuits



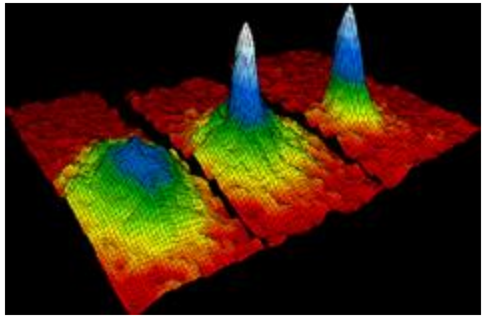
P. D. Nation et al., *RMP* (2012)

Rings of trapped ions



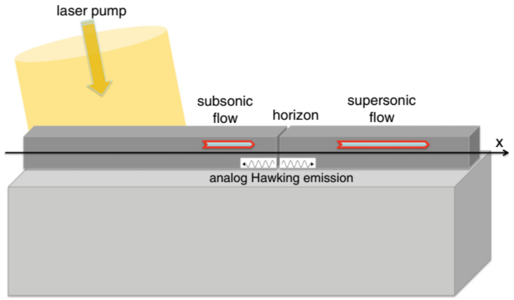
B. Horstmann et al., *PRL* (2010)

BECs of ultra-cold atoms



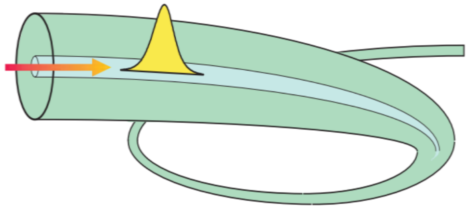
M. H. Anderson et al., *Science* (1995)

Quantum fluids of light



D. Gerace et al., *PRB* (2012)

Nonlinear optical systems



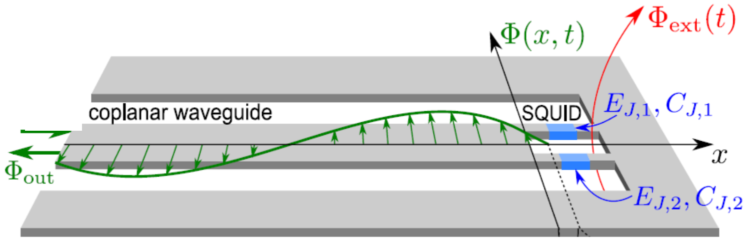
T. G. Philbin et al., *Science* (2008)

Analogue models of gravity

In the lab

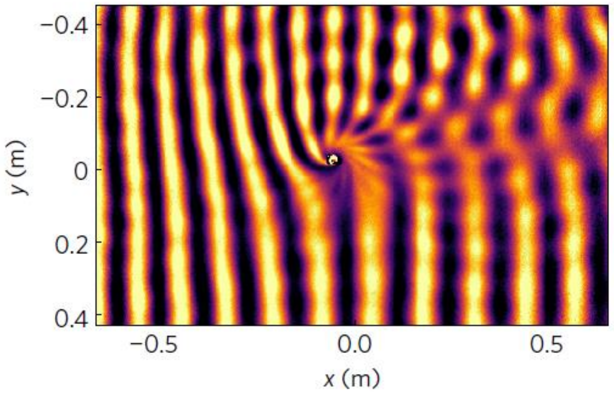
Observation of the dynamical Casimir effect in a superconducting circuit

C. M. Wilson et al. , *Nature* **479**, 376-379 (2011)



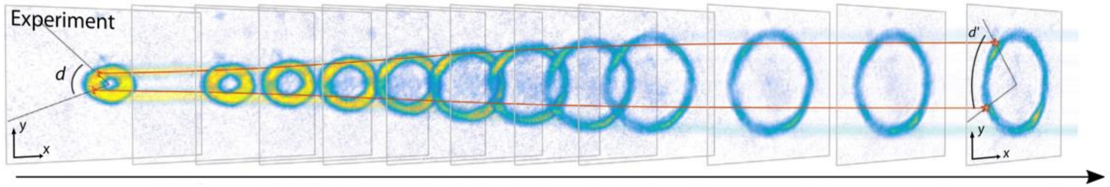
Rotational superradiant scattering in a vortex flow

S. Weinfurtner et al.,
Nat. Phys. **13**, 833 (2017)



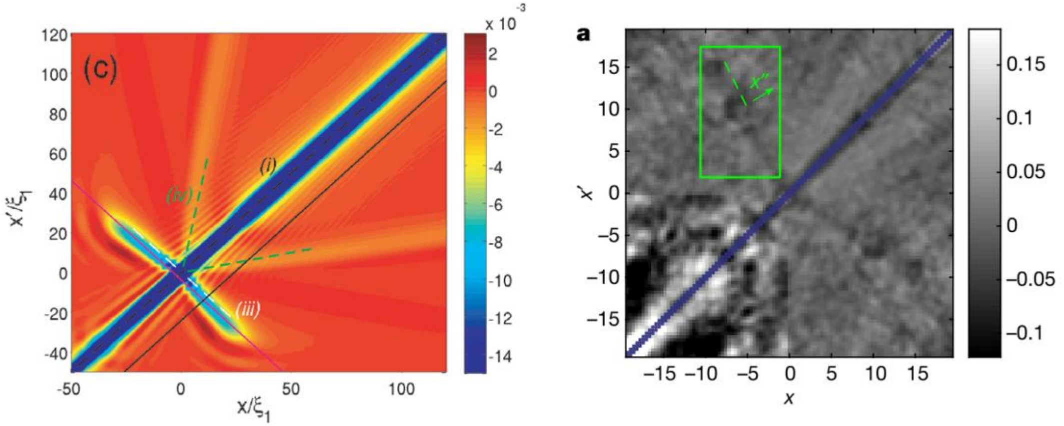
A Rapidly Expanding Bose-Einstein Condensate: An Expanding Universe in the Lab

S. Eckel et al. , *PRX* **8**, 021021 (2018)



Observation of thermal Hawking radiation and its temperature in an analogue black hole

J. Steinhauer et al. , *Nature* **12**, 688 (2019)



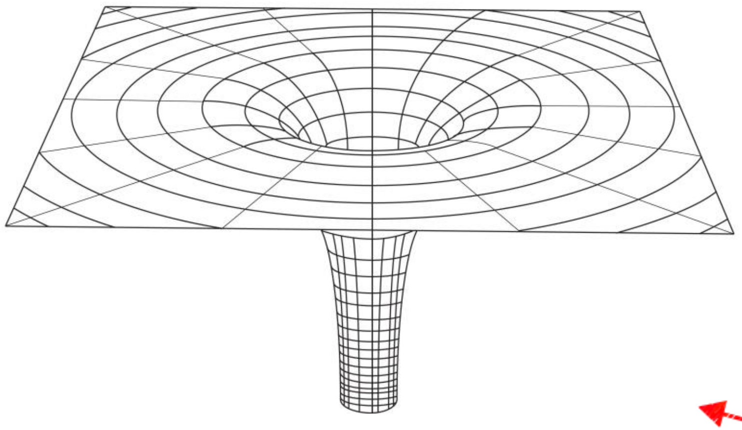
Analogue models of gravity

Back-reaction

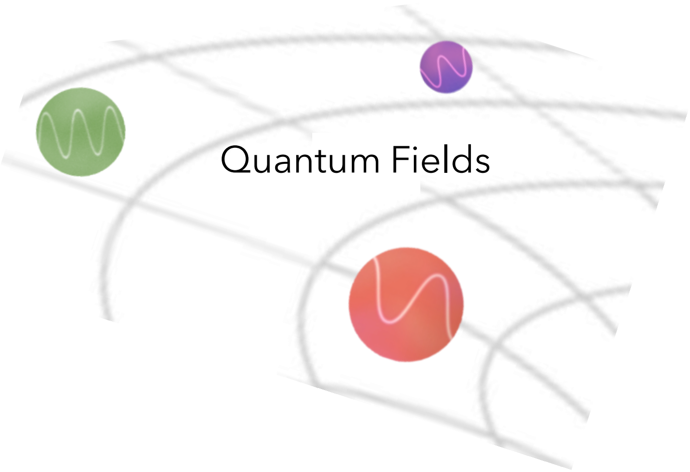
Action

QFT in curved spacetime

Effective spacetime



Excitation fields



Back-reaction

?

Hydrodynamic equations
of the analog system

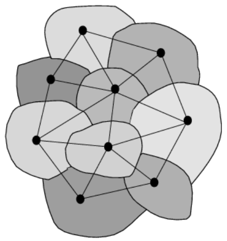
Effective field equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial S_1}{\partial x^\nu}\right) = 0$$

See:
R. Balbinot et al., PRL **94**, 161302 (2005)
Schutzhold et al. PRD **72** 105005 (2005)
S. Weinfurtner et al. PRL **126** 041105 (2021)

WARNING!

The microscopic
physics is different



Analogue models of gravity

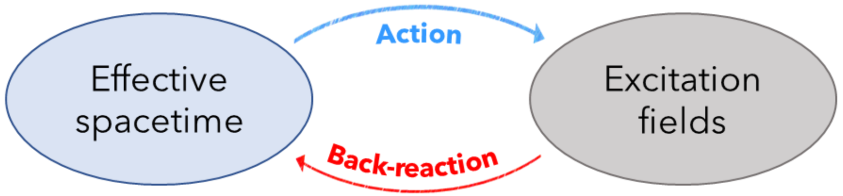
Back-reaction

Mesoscopic level

Gravity

Self-consistent theory of
Spacetime + Quantum Fields

Self-consistent quantum theory of:



Fluctuations
Dissipation
De-coherence

Microscopic details are blurred. Mesoscopic back-reaction effects are Universal!

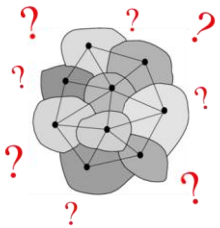
Analogue systems

Self-consistent theory of
Effective Spacetime +
Quantum excitation Fields

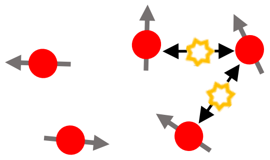
Coarse grain

Coarse grain

Quantum Gravity



Microscopic theory



Atoms, electrons, light, etc.

The back-reaction problem

Analog models of gravity

Analog of cosmological pre-heating

Based on:
S. Butera and I. Carusotto , *arXiv: 2207.00311* (2022)
S. Butera and I. Carusotto , *PRD* ***104*** *083503* (2021)

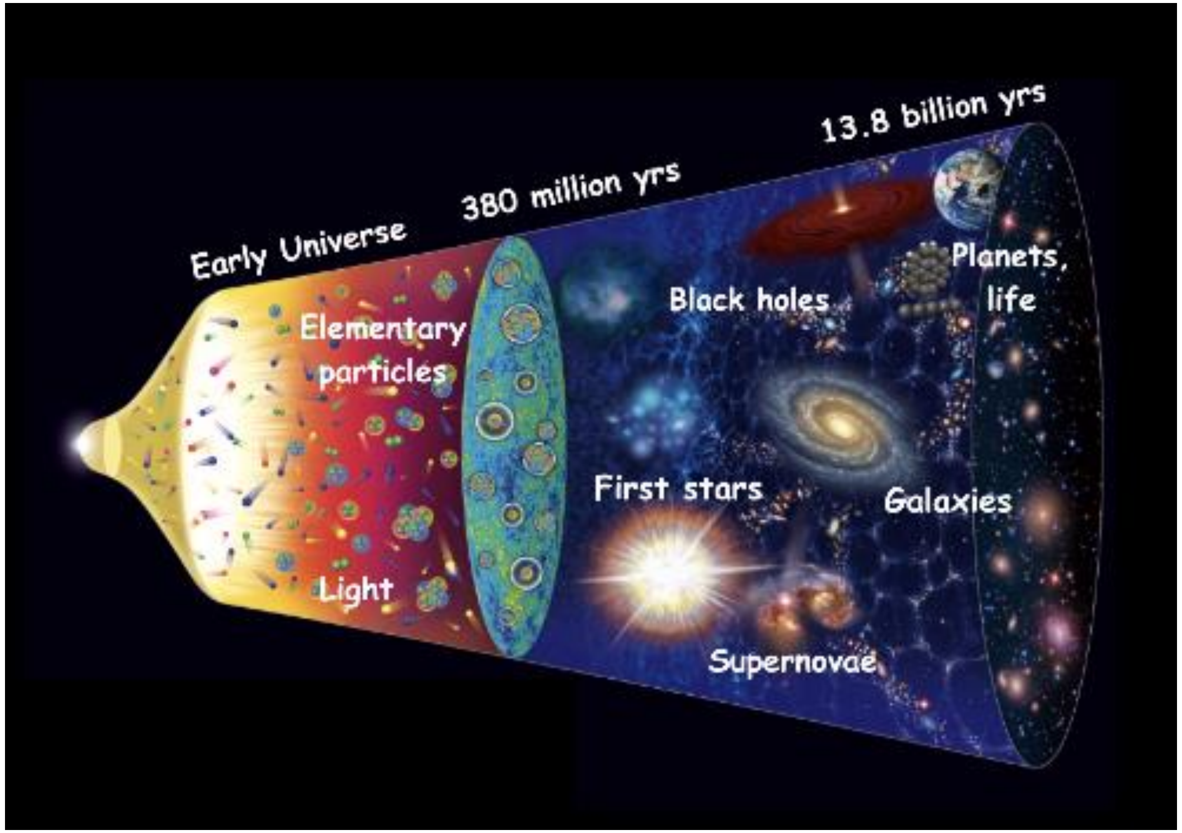


Figure: N. Yoshida, The First Generation of Stars and Blackholes in the Universe (2021)
`https://media.ed.ac.uk/media/1_fe4wdyse`

Pre-heating in the early Universe

The back-reaction problem

Explosive creation of matter at the end of inflation → Creates the hot soup that feeds the standard Big Bang evolution

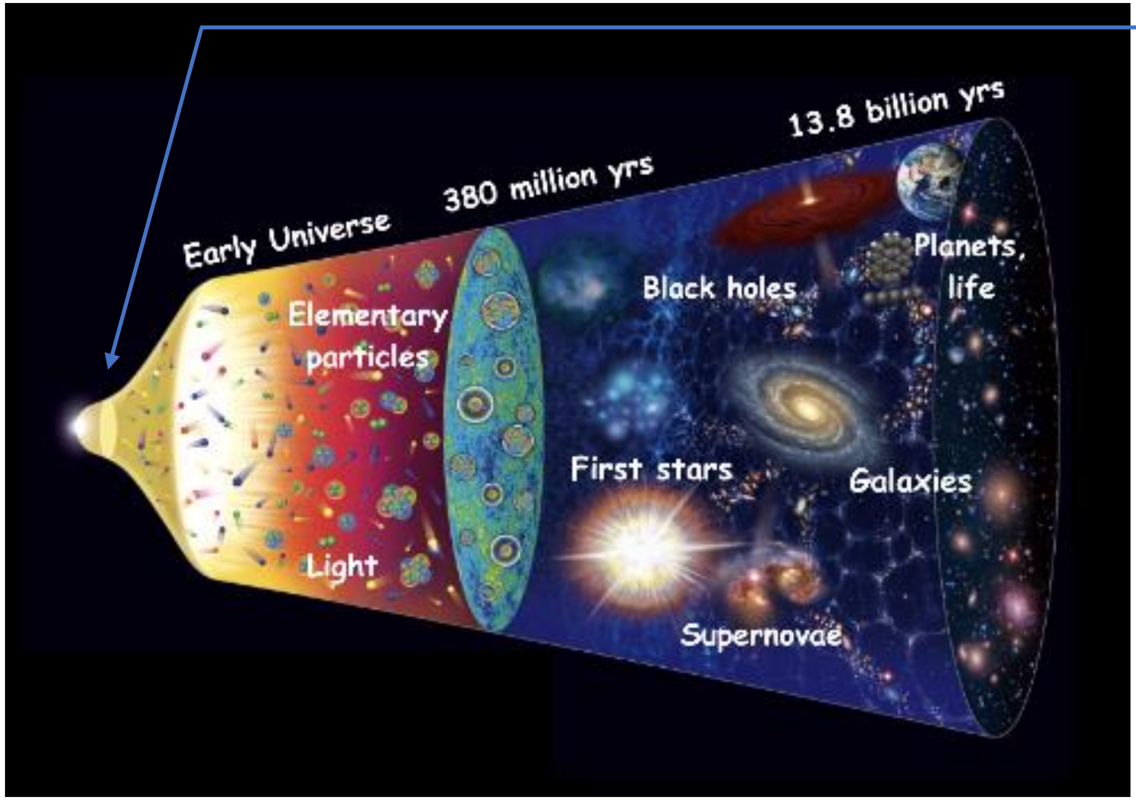


Figure: N. Yoshida, The First Generation of Stars and Blackholes in the Universe (2021)
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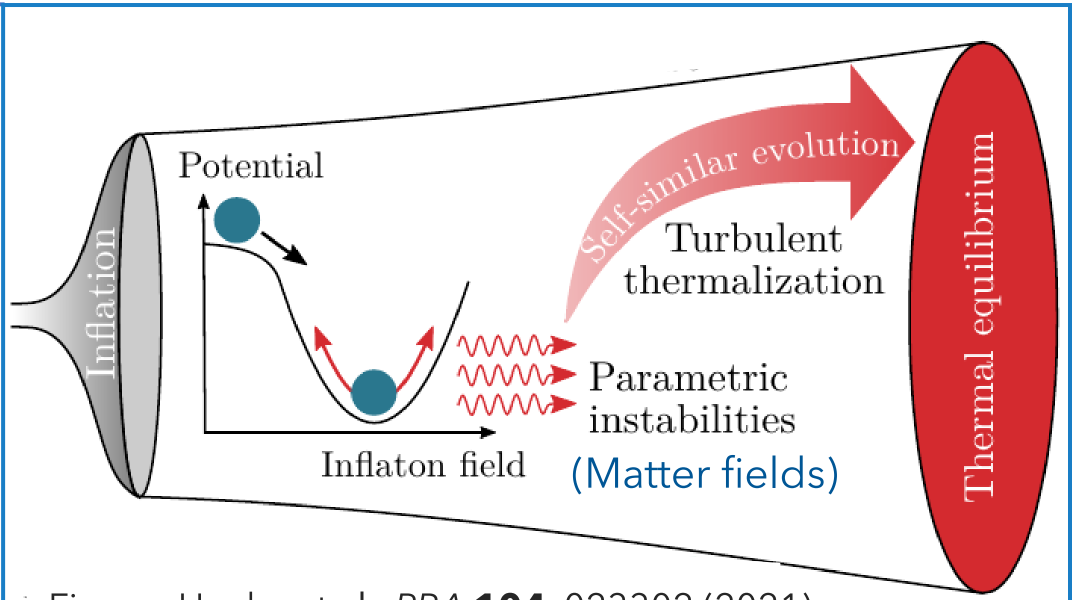


Figure: Hauke et al., *PRA* **104**, 023302 (2021)

Schematic of physical processes

Pre-heating in the early Universe

The back-reaction problem

Why we need an analogue simulator?

- The objective is study the dynamics of the inflaton field, induced by the creation of matter: **back-reaction** problem.
- It is a non-perturbative effect. Interaction are strong and **quantum effects are important**.
- We have a clear understanding of the microscopic dynamics of analogue systems, and we can perform table-top experiments.

  (See experiments at BEC-Center in Trento, Italy)

- The characteristic effects of the back-reaction are **universal** and the basic mechanism are qualitatively the same.

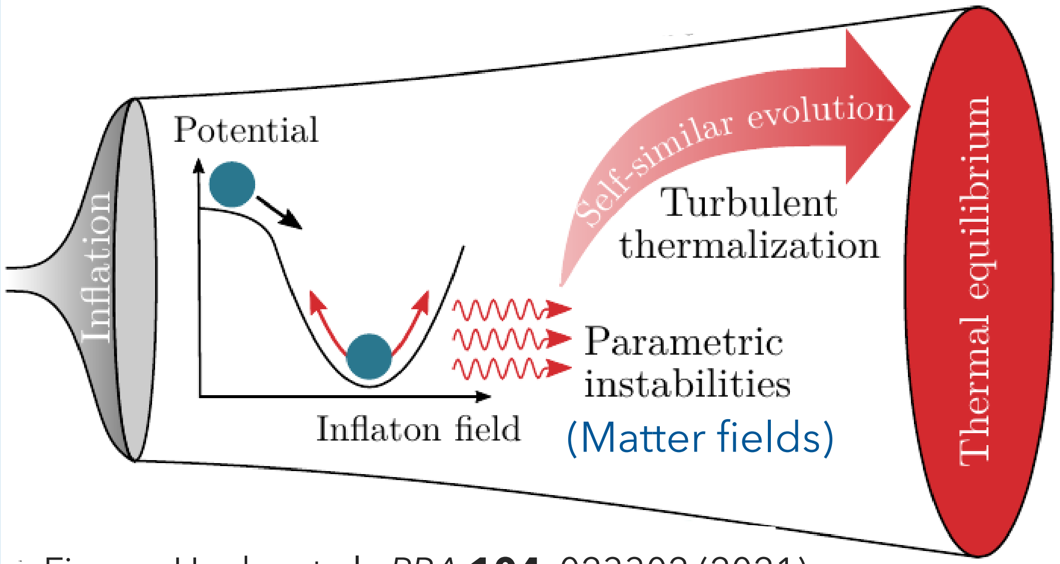


Figure: Hauke et al., *PRA* **104**, 023302 (2021)

Schematic of physical processes

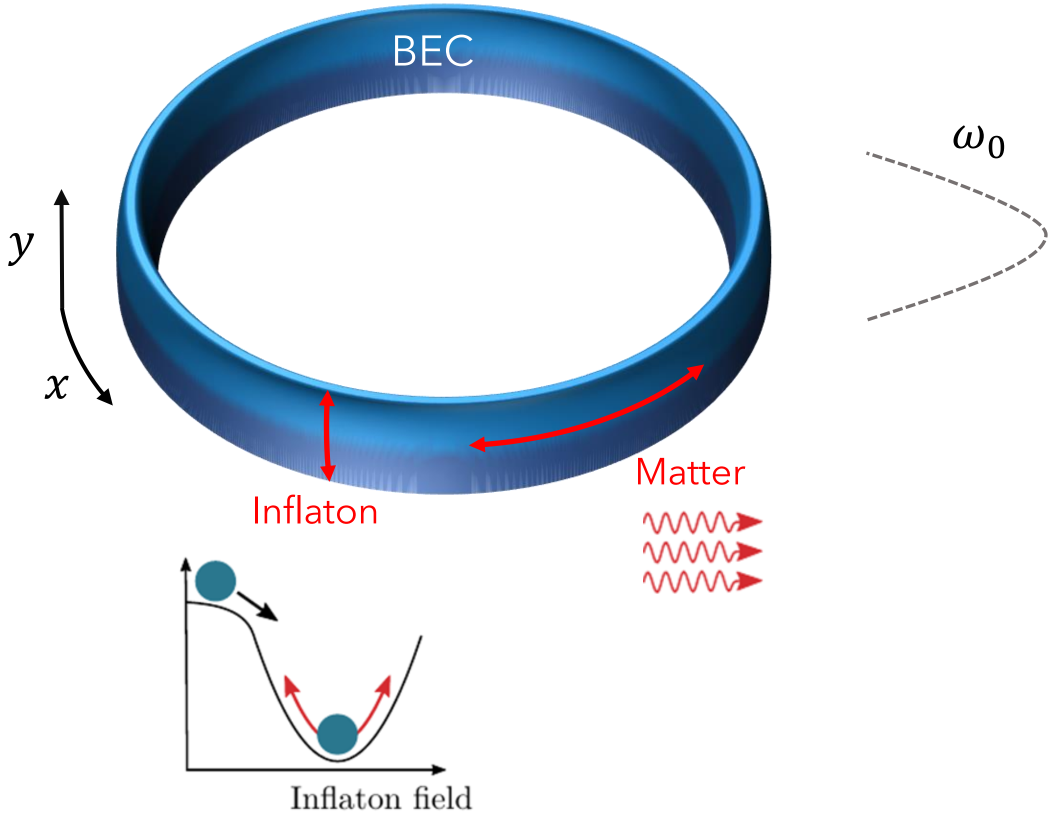
The analogue system

2D BEC

- Degrees-of-freedom

Inflaton field ⟶ (High energy) Transverse modes

Matter field ⟶ (Low energy) Longitudinal modes



S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

The analogue system

2D BEC

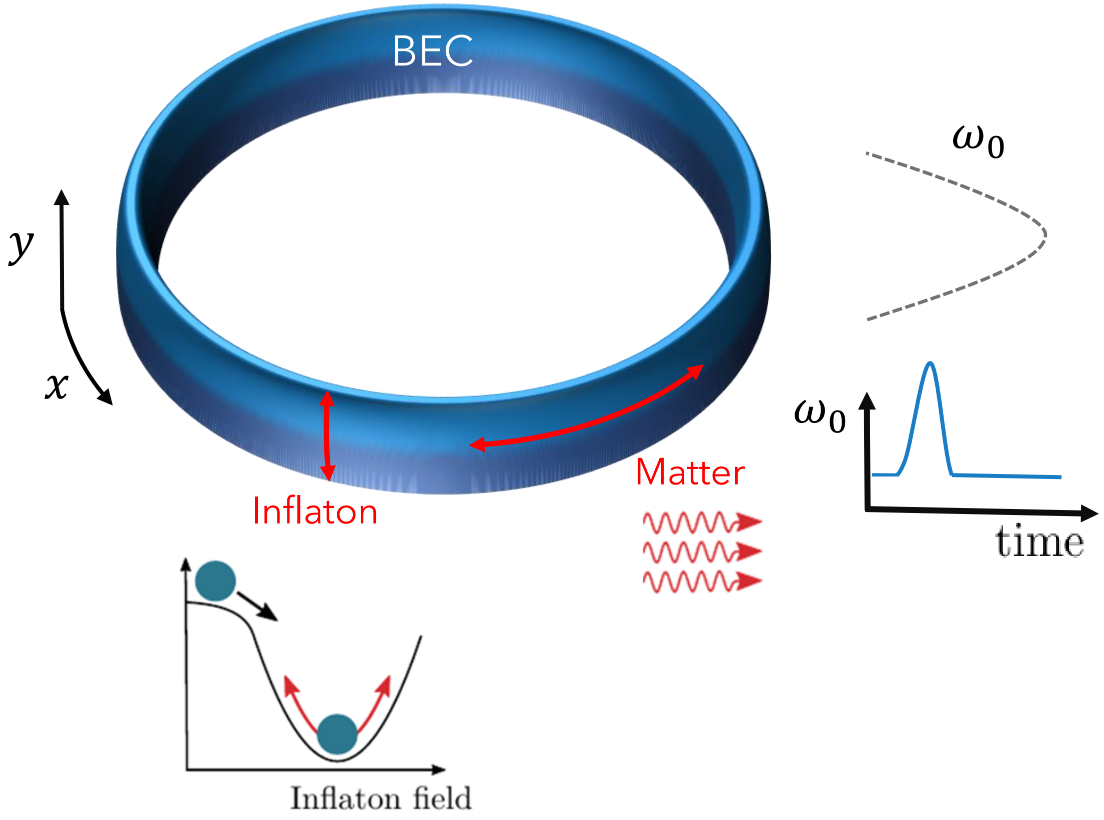
- Degrees-of-freedom

Inflaton field ⟶ (High energy) Transverse modes

Matter field ⟶ (Low energy) Longitudinal modes

- Inflaton oscillations

Excite the transverse modes



S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

The analogue system

2D BEC

- Degrees-of-freedom

Inflaton field ⟶ (High energy) Transverse modes

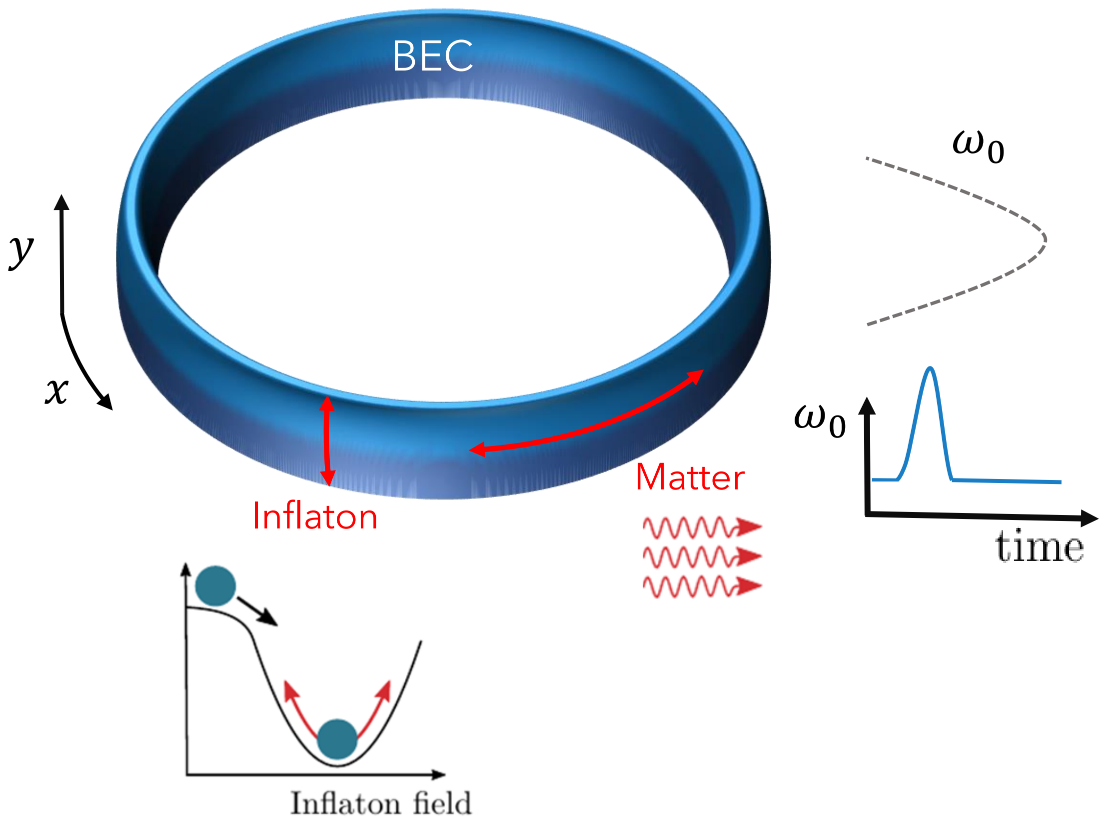
Matter field ⟶ (Low energy) Longitudinal modes

- Inflaton oscillations

Excite the transverse modes

- Inflaton ⟷ matter interaction

Collisional atomic interactions



S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

The method

Semiclassical phase-space representation

Characteristic function

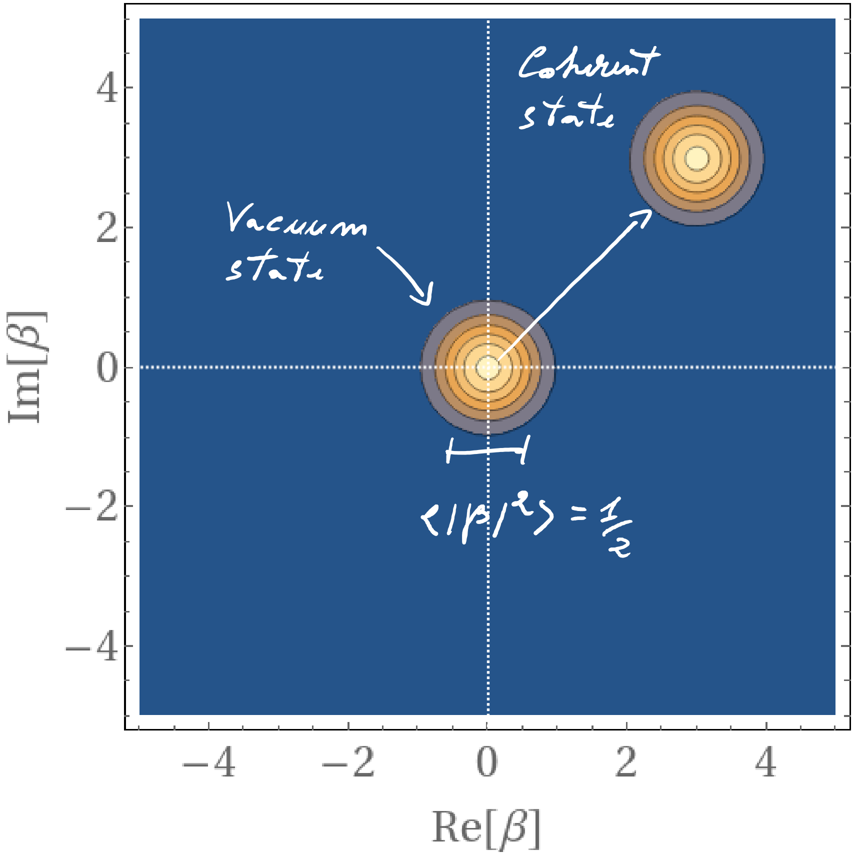
$$\chi_S(z, z^*) \equiv \mathrm{Tr}\left(\hat{\rho} e^{iz^*\hat{b}^\dagger + iz\hat{b}}\right)$$

Wigner function (quasi-probability distribution)

$$W(\beta, \beta^*) \equiv \frac{1}{\pi^2}\int d^2z\, \chi_S(z, z^*) e^{-iz^*\beta^*} e^{-iz\beta}$$

Symmetric averages

$$\left\langle \left(\hat{b}^{\dagger p}\hat{b}^q\right)_S \right\rangle = \int d^2\beta\, W(\beta, \beta^*) \beta^{*p}\beta^q$$



The method

Semiclassical phase-space representation

Dynamics of the BEC in the phase space

$$i\hbar \frac{d\hat{\rho}}{dt} = [\hat{H}, \hat{\rho}] \quad \Longrightarrow \quad \frac{\partial W(\psi,\psi^*)}{\partial t} = i \int dx \left\{ \underbrace{\frac{\delta}{\delta\psi}\left[h\psi + g(|\psi|^2 - 1)\psi\right]}_{\text{Meanfield}} - \underbrace{\frac{1}{4}\frac{\delta^3}{\delta^2\psi\delta\psi^*}\psi}_{\text{Quantum fluctuations}} \right\} W(\psi,\psi^*) + \text{c.c.}$$

- BEC Hamiltonian

$$\hat{H} = \int dx\, \hat{\psi}^\dagger h \psi + \frac{g}{2}\int dx\, \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi}\hat{\psi}$$

Steel et al., *PRA* **58** 4824 (1998)

The method

Semiclassical phase-space representation

Dynamics of the BEC in the phase space

Truncated Wigner approximation

$$\frac{\partial W(\psi,\psi^*)}{\partial t} = i\int dx \left\{ \underbrace{\frac{\delta}{\delta\psi}\left[h\psi + g(|\psi|^2 - 1)\psi\right]}_{\text{Meanfield}} - \underbrace{\frac{1}{4}\frac{\delta^3}{\delta^2\psi\,\delta\psi^*}\psi}_{\text{Quantum fluctuations}} \right\} W(\psi,\psi^*) + \text{c.c.}$$

- The system evolves classically, according to the Gross-Pitaevskii equation

$$i\hbar\frac{\partial\psi}{\partial t} = h\psi + g|\psi|^2\psi$$

- Quantum fluctuations are accounted for **only** in the initial condition.

Steel et al., *PRA* **58** 4824 (1998)

The initial state

Bogoliubov vacuum

$$\psi\left(\mathbf{r},t=0\right)=\underbrace{\phi(\mathbf{r})}_{\text{Condensate}}+\underbrace{\sum_{n,r}\left(\beta_n^r u_n^r(\mathbf{r})+\beta_n^{r\,*}v_n^{r\,*}(\mathbf{r})\right)}_{\text{Vacuum fluctuations}}$$

Bogoliubov modes

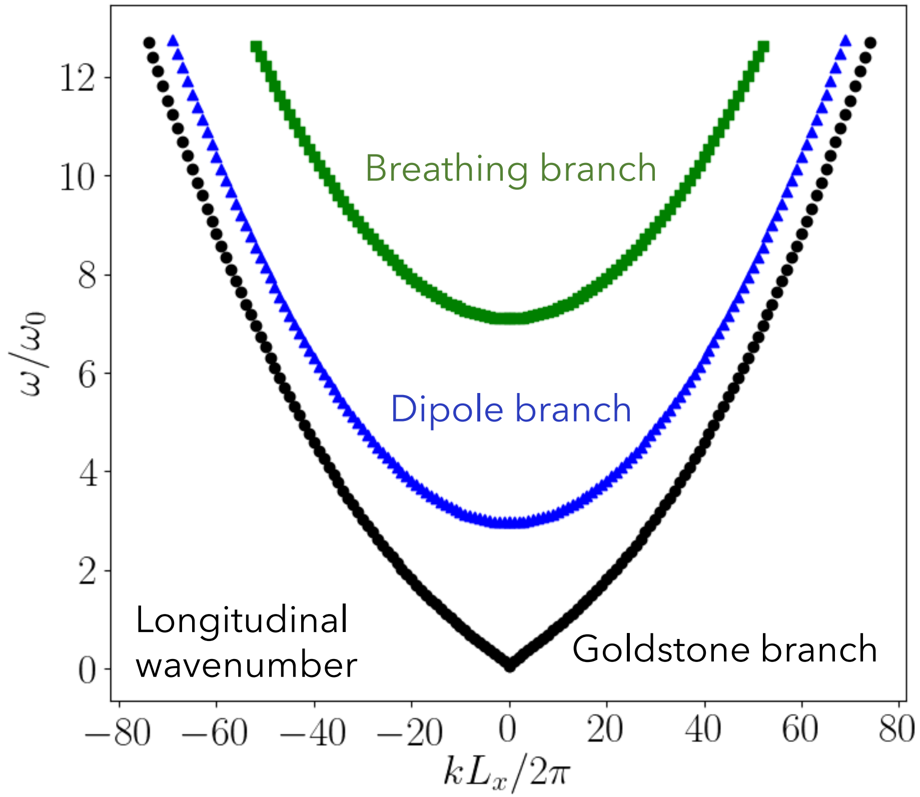
$$\{u_n^r, v_n^r\}\in\mathbb{N}$$

Gaussian random variables

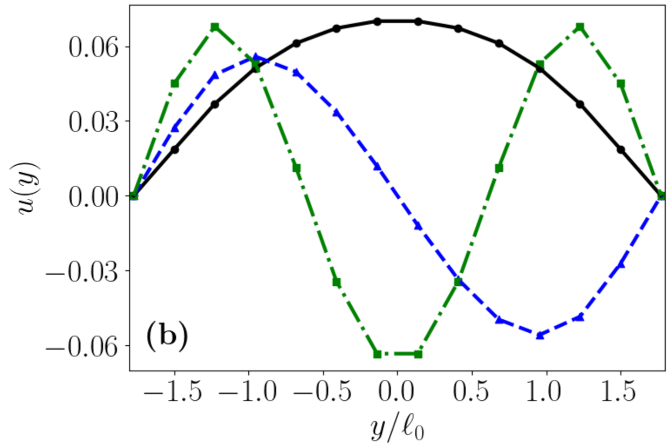
$$\overline{\beta_n^r\beta_m^s}=0\,,\qquad \overline{\beta_n^{r\,*}\beta_m^s}=\frac{1}{2}\delta_{nm}\delta_{rs}$$

S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

Bogoliubov spectrum

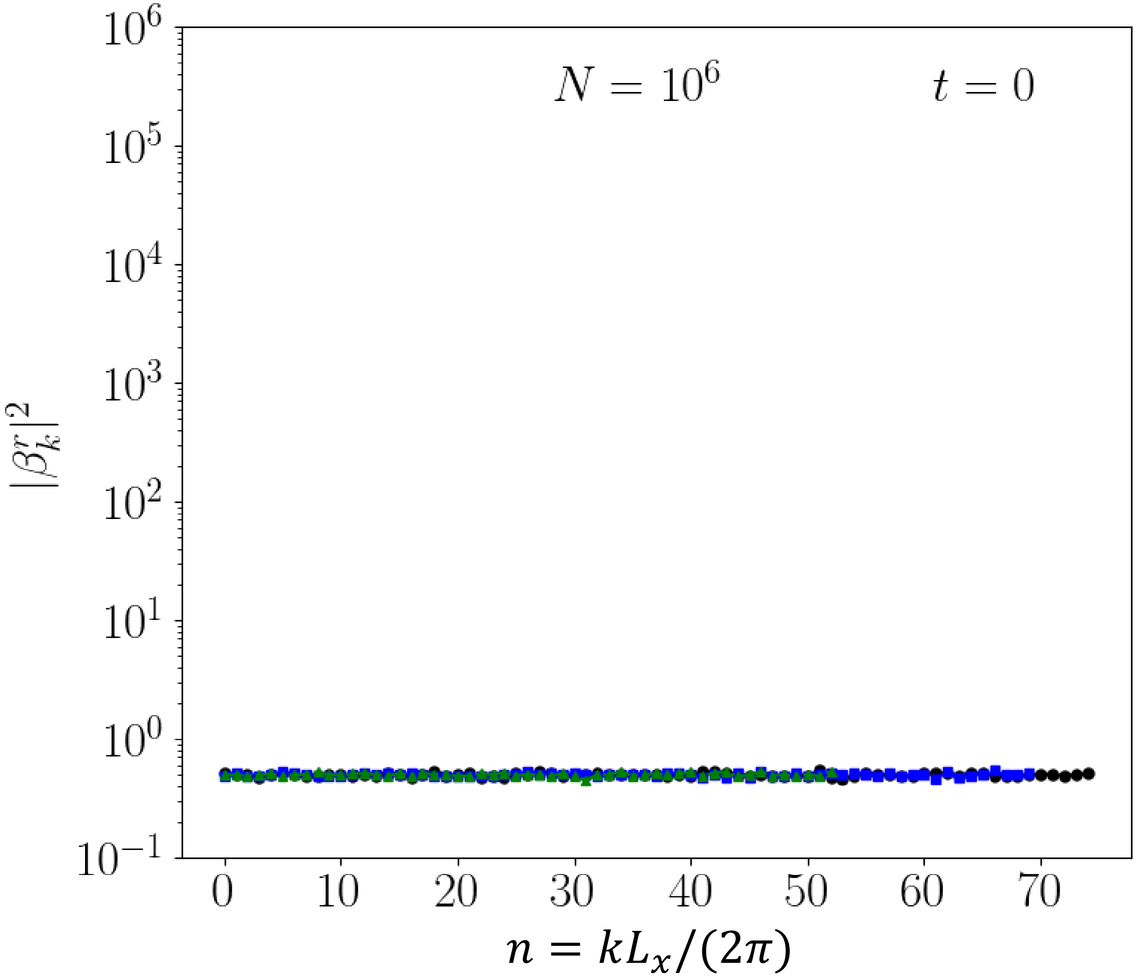


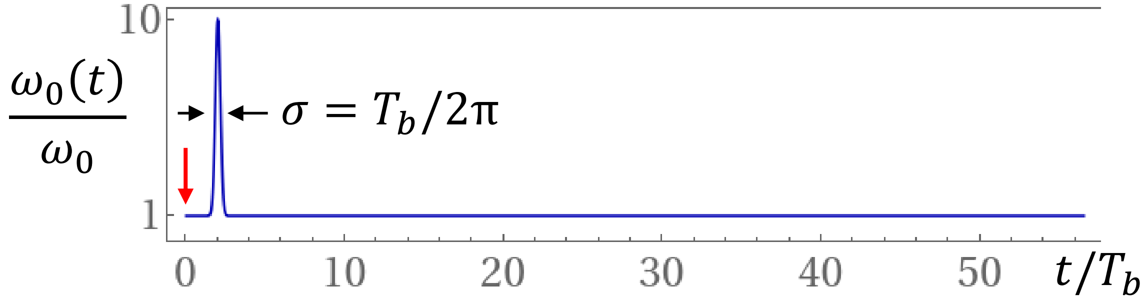
Transverse profiles

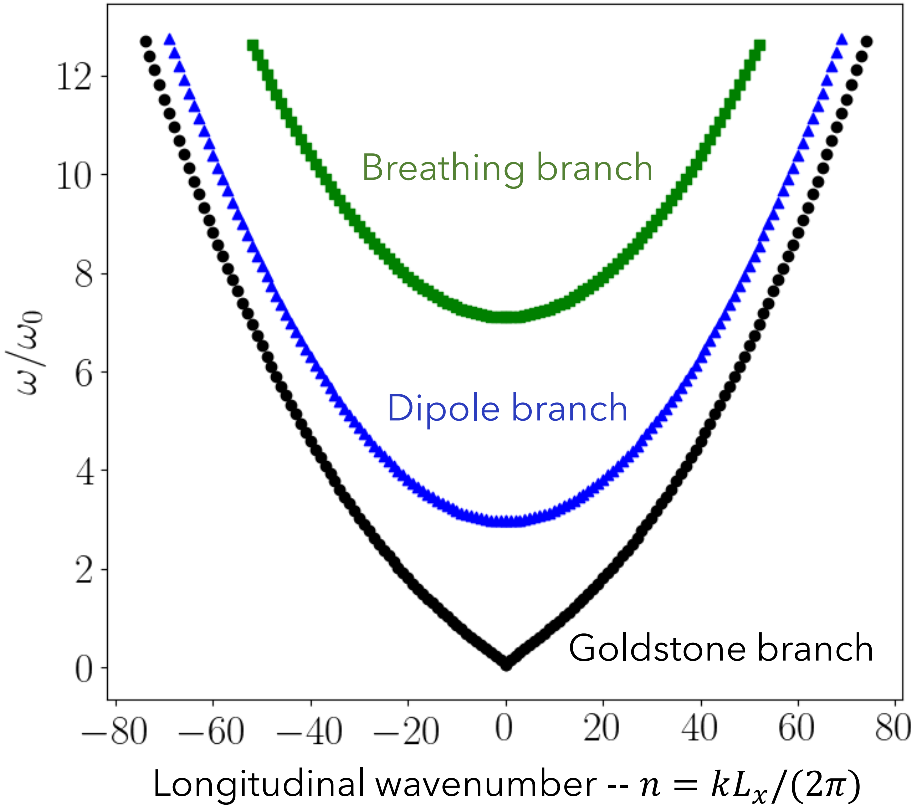


Results

Bogoliubov modes evolution





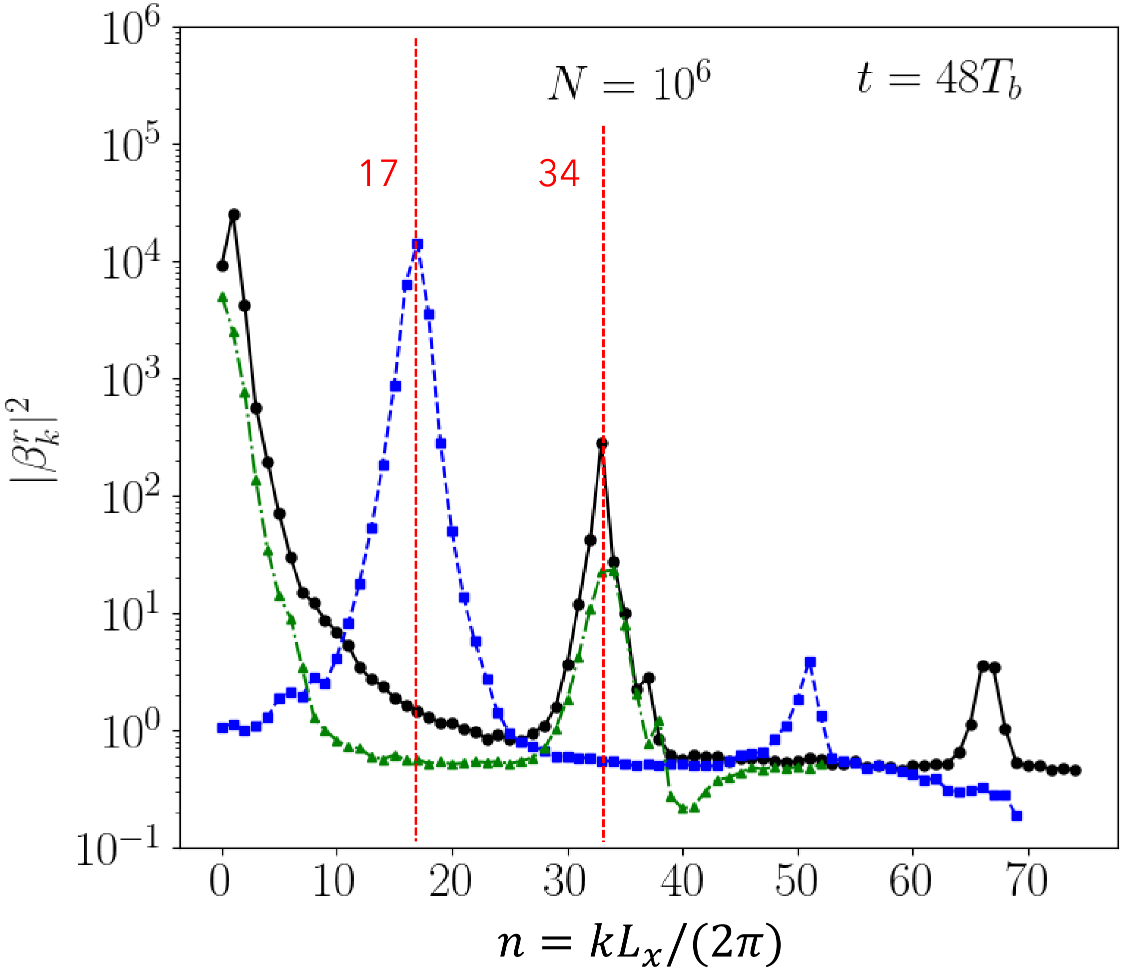


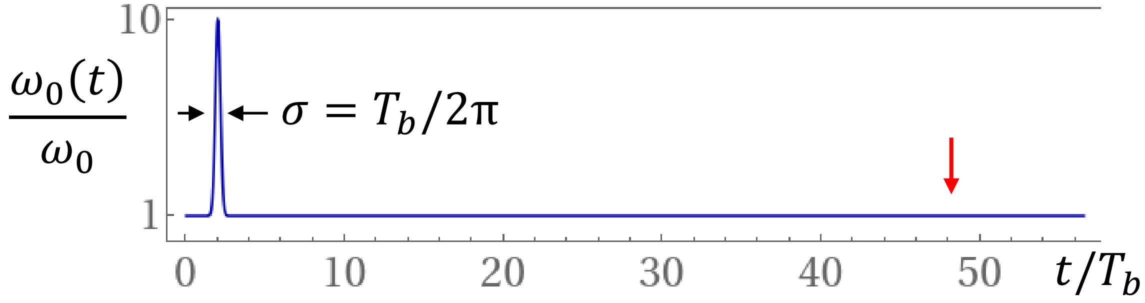
S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

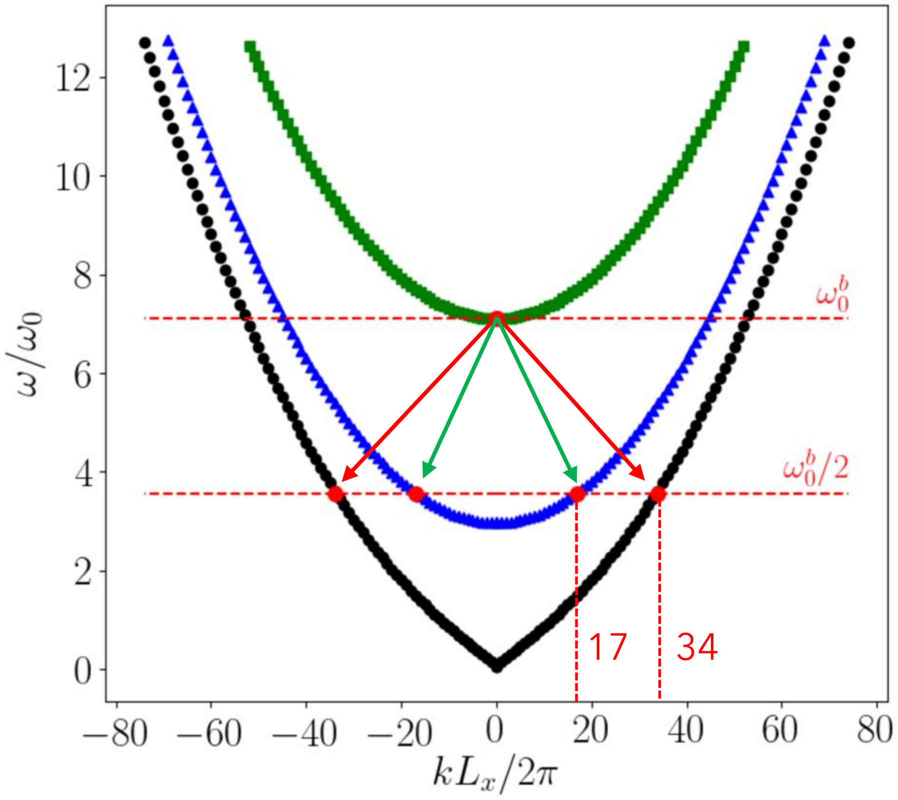
Results

Bogoliubov modes evolution

Parametric amplification of vacuum fluctuations in the resonant modes





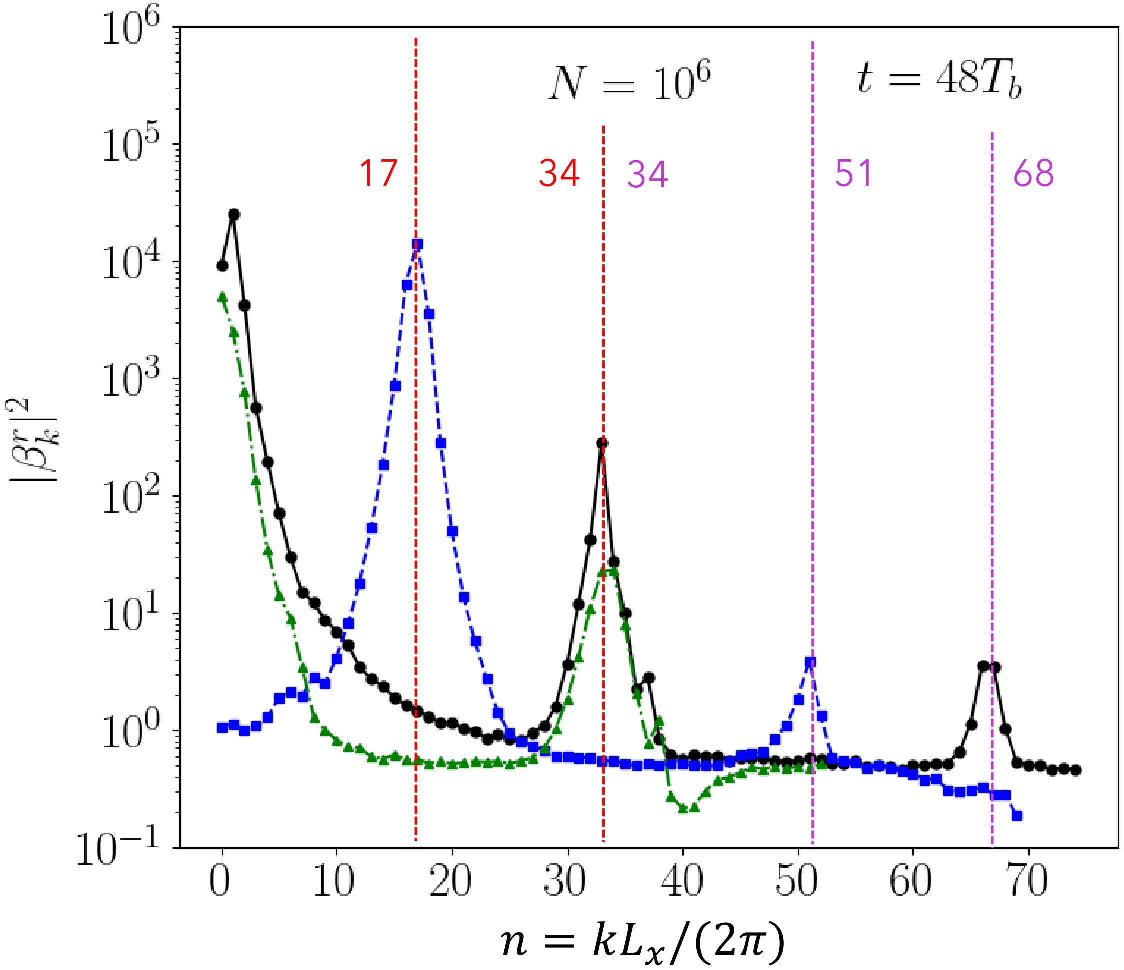


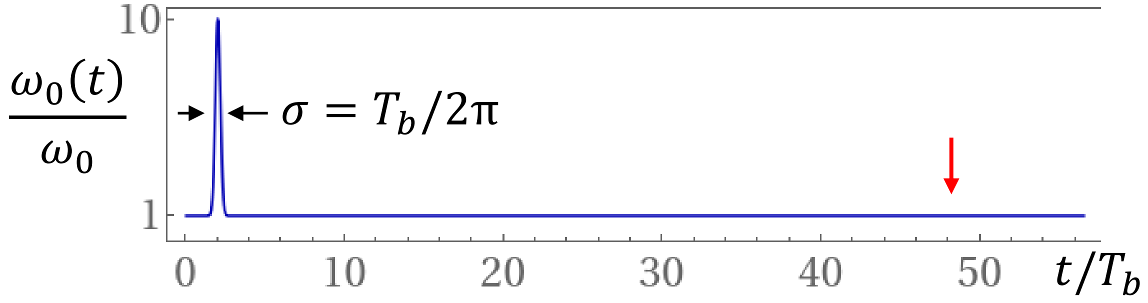
S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

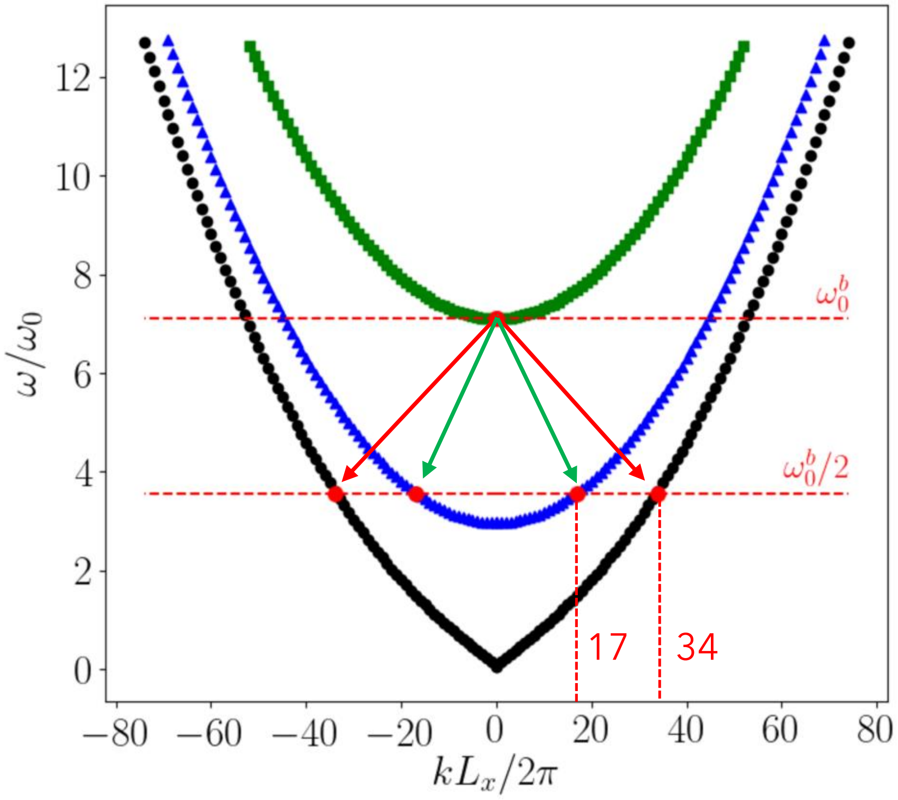
Results

Bogoliubov modes evolution

Non-linear scattering between quasi-particles generate higher harmonics







S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

See also: S.Weinfurtner et al. , *arXiv:2207.02199 (2022)*

Results

Trento experiments

PHYSICAL REVIEW LETTERS **128,** 210401 (2022)

Observation of Massless and Massive Collective Excitations with Faraday Patterns in a Two-Component Superfluid

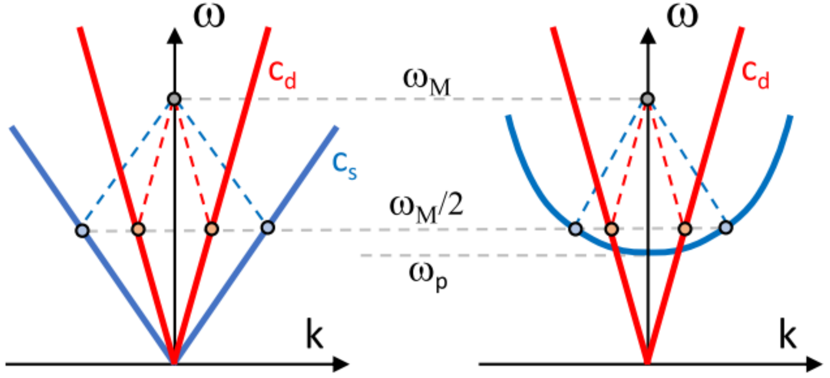
R. Cominotti, A. Berti, A. Farolfi, A. Zenesini,[*] G. Lamporesi,[†] I. Carusotto, A. Recati,[‡] and G. Ferrari
INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, and Trento Institute for Fundamental Physics and Applications, INFN, 38123 Povo, Italy

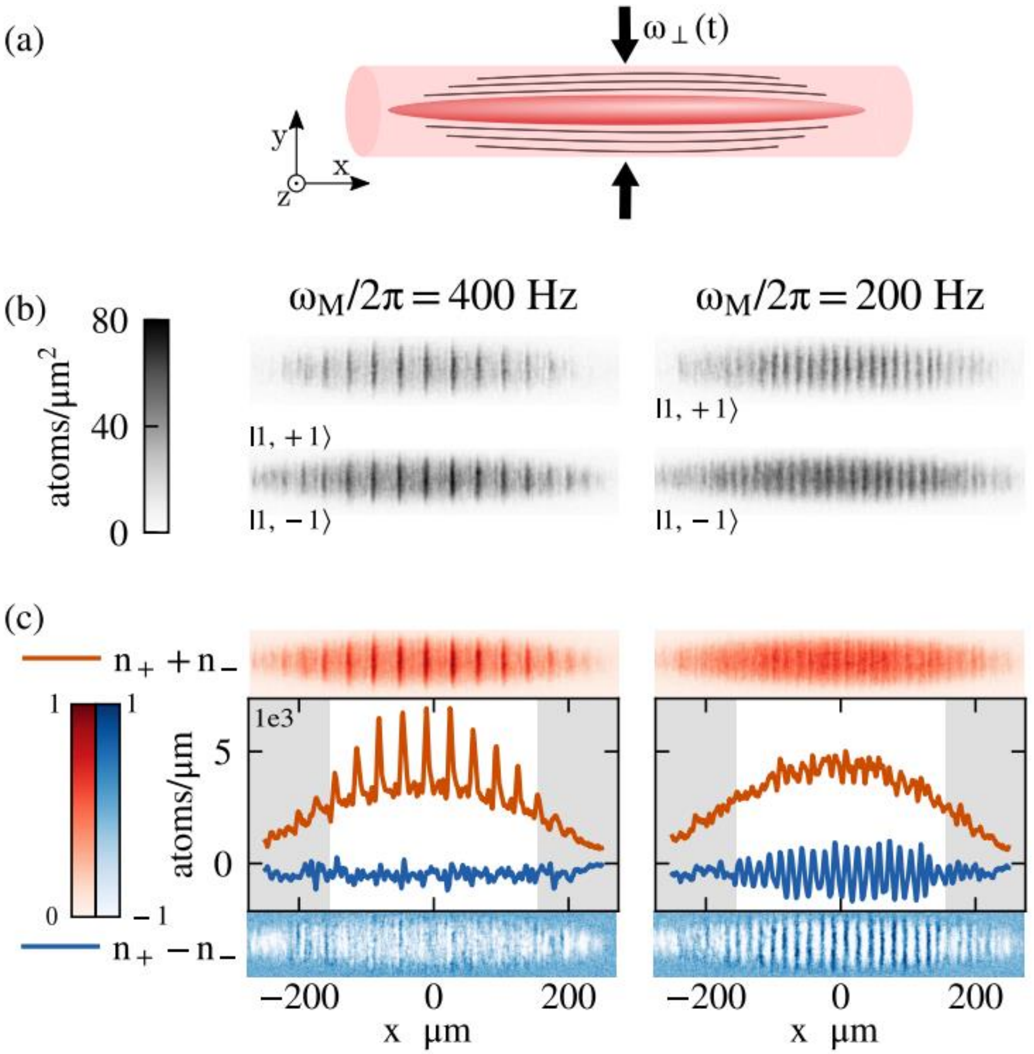
PHYSICAL REVIEW D **104,** 083503 (2021)

Particle creation in the spin modes of a dynamically oscillating two-component Bose-Einstein condensate

Salvatore Butera[1,*] and Iacopo Carusotto[2]
[1]*School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom*
[2]*INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, I-38123 Povo, Italy*

$\Omega_R=0$	$\Omega_R\neq0$
Massless density excitation	Massless density excitation
Massless spin excitation	**Massive** spin excitation

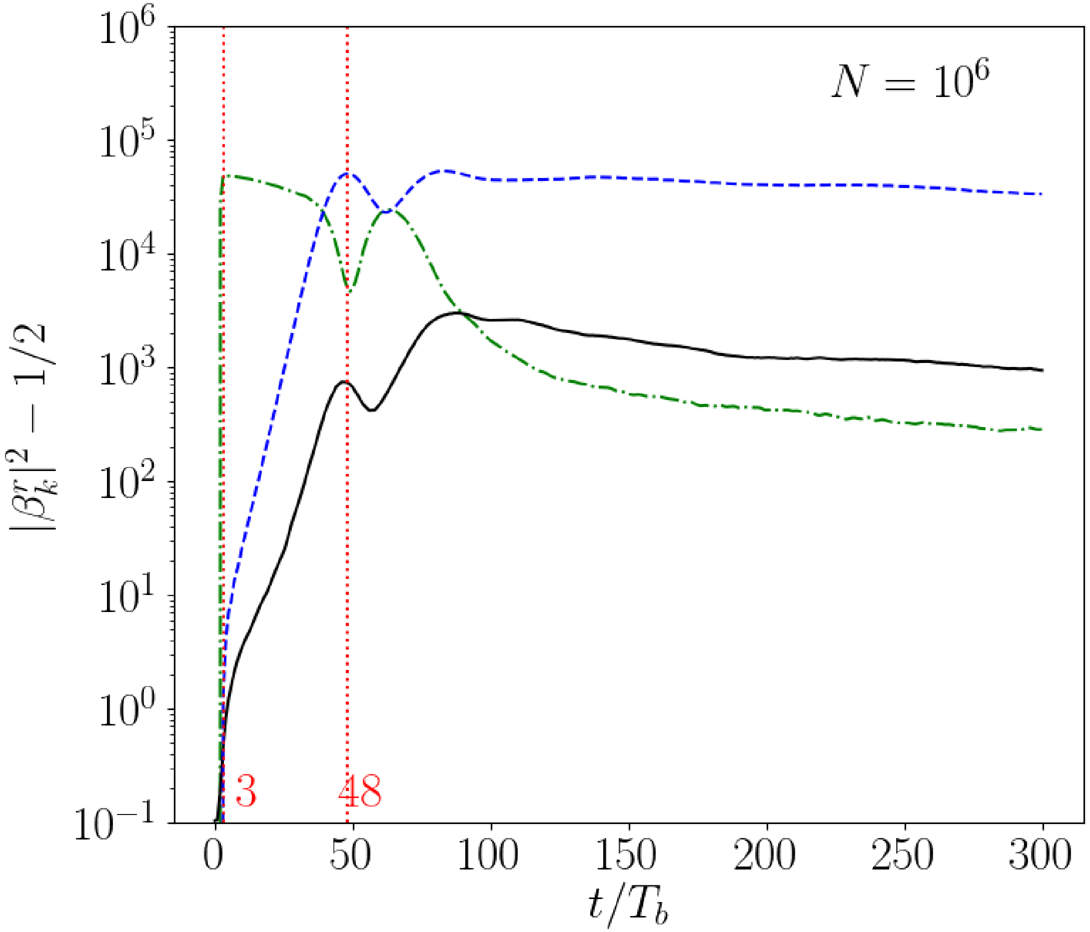


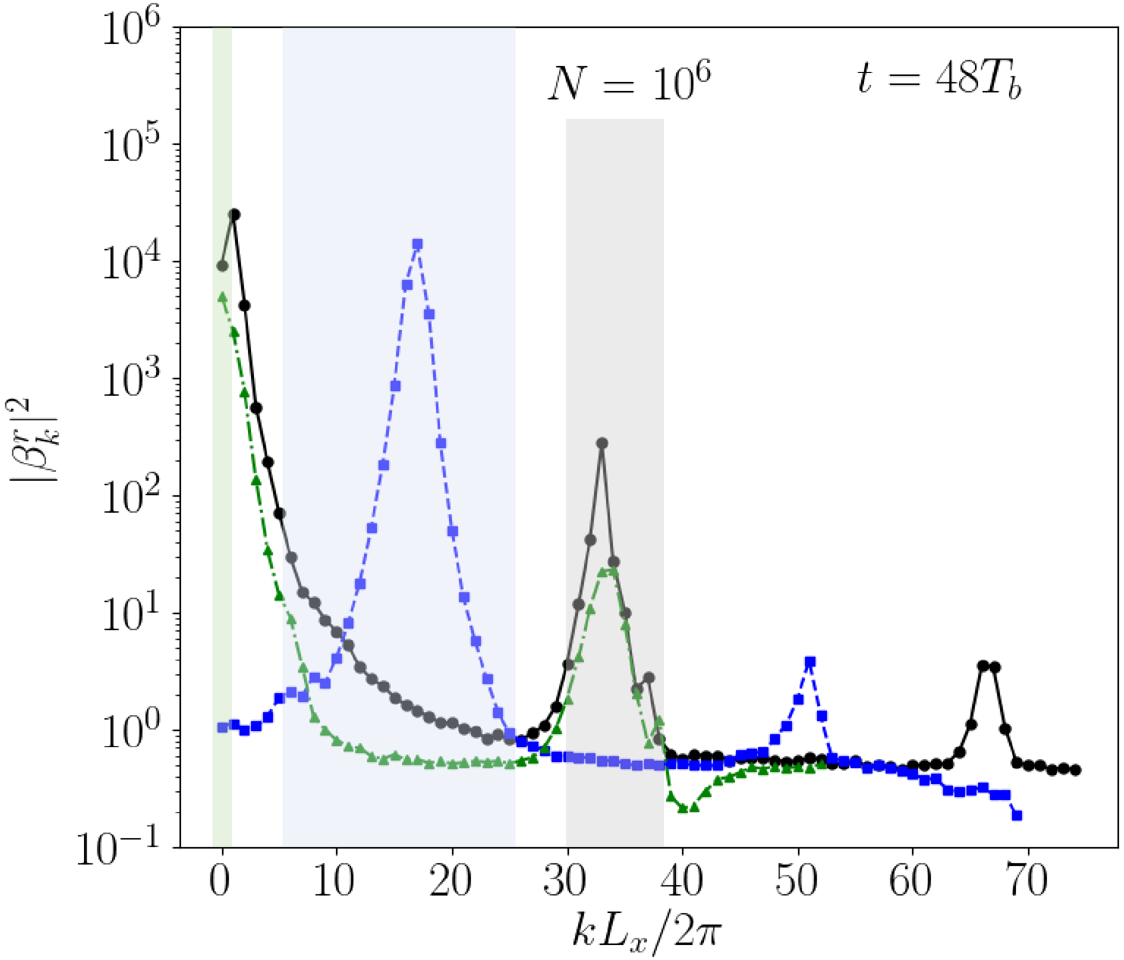


Results

Dissipation

Effect of the back-reaction that can be captured by a semiclassical theory



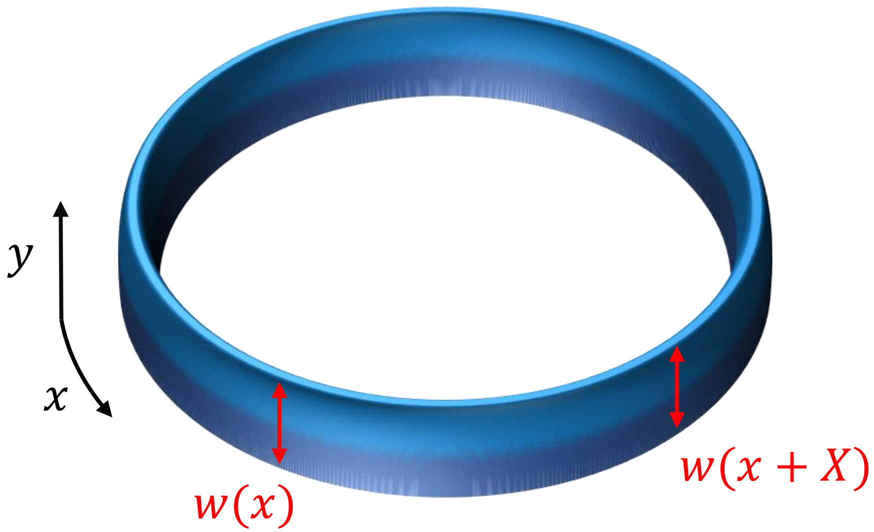


S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

Results

Decoherence

Effect of the back-reaction beyond the semiclassical level (that is due to quantum fluctuations)



Transverse BEC size

$$w(x,t) \equiv \frac{\int_0^{L_y} dy |\psi(\mathbf{r},t)|^2 y^2}{\int_0^{L_y} dy |\psi(\mathbf{r},t)|^2}$$

Spatial correlation of the transverse oscillations

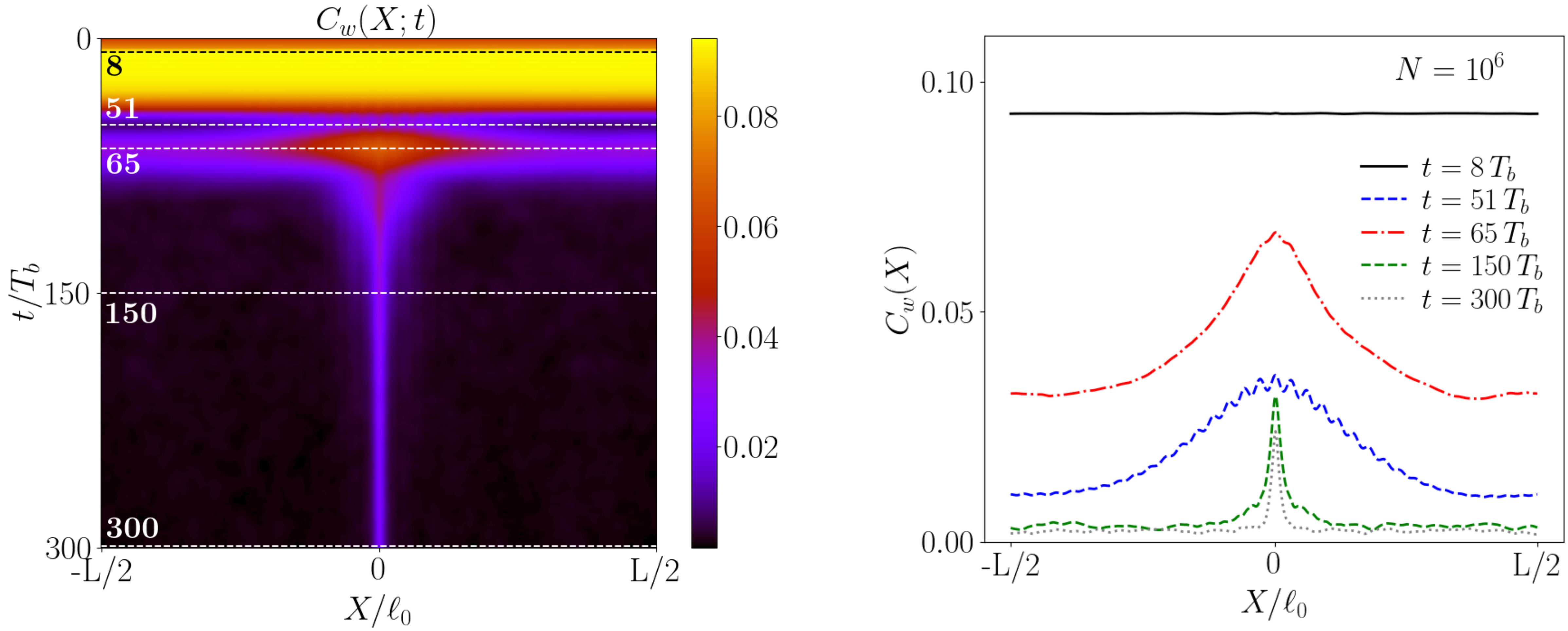
$$C_w(X;t) \equiv \left\langle \frac{\delta w(x,t)\delta w(x+X,t)}{\bar{w}^2(0)} \right\rangle_W$$

- $\delta w(x,t) = w(x,t) - \bar{w}(0)$

S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

Results

Decoherence



S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

Results

Tripartite entanglement

Linear entanglement entropy of the breathing mode:

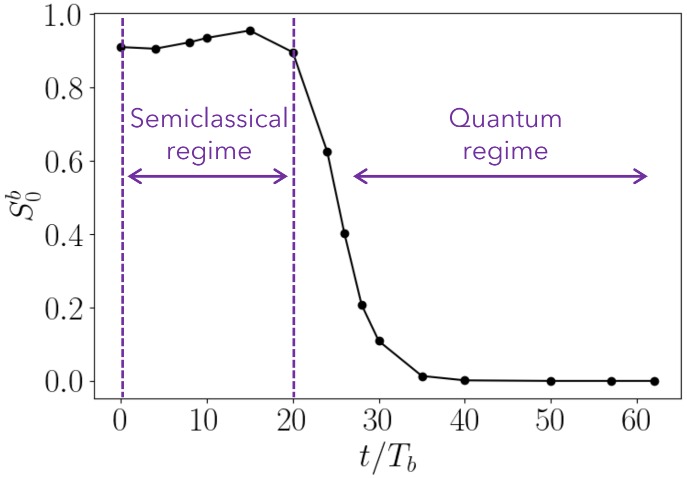
$$S_0^b \equiv Tr(\hat{\rho}_{b,0}^2) = \pi \int d^2\beta \, W_{b,0}(\beta, \beta^*)$$

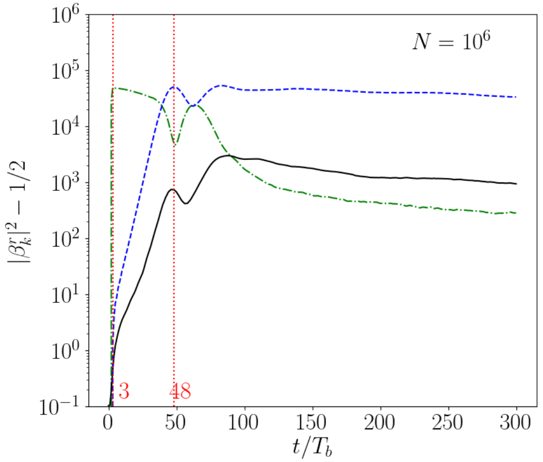
- Pure state:

$$S_{b,0} = Tr(\hat{\rho}_{b,0}^2) = Tr(\hat{\rho}_{b,0}) = 1$$

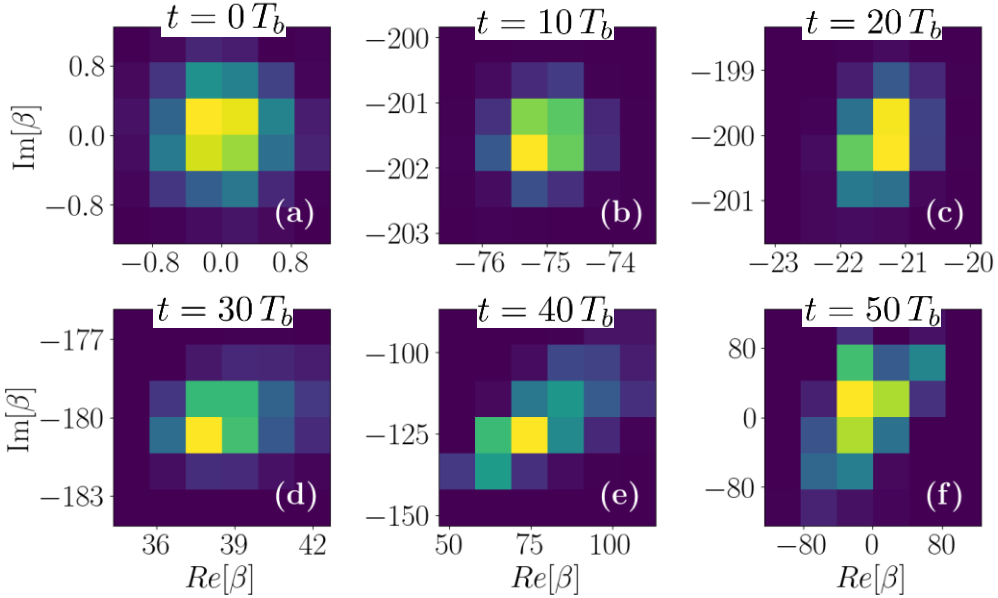
- Mixed state:

$$S_{b,0} < 1$$





Wigner distribution of the breathing mode



<u>S. Butera</u> and I. Carusotto , *in preparation* (2022)

Conclusions

- Analogue models are a powerful quantum simulation platform that can be used to investigate the physics of the back-reactions.
- Despite the microscopic physics of analogue systems is expected to be different from the (unknown) physics of gravity at the Planck scale, the qualitative, mesoscopic effects of the back-reaction are universal.
- We used a 2D BEC as analogue simulator of the pre-heating of the early universe, identifying the inflaton dofs in the transverse breathing mode, while the matter dofs in the longitudinal modes.
- We observed the test field effect of the parametric excitations of the vacuum fluctuations of the longitudinal modes, induced by the transverse modes oscillations of the system.
- We observed back-reaction effects in

    1. Damping (semiclassical)
    2. Decoherence (beyond semiclassical – quantum fluctuations)
    3. Entanglement (beyond semiclassical – quantum fluctuations)

  of the transverse modes oscillations.

Acknowledgments

In collaboration with:

Iacopo Carusotto
(INO-CNR BEC Centre – Trento, Italy)

LEVERHULME
TRUST

Acknowledgments

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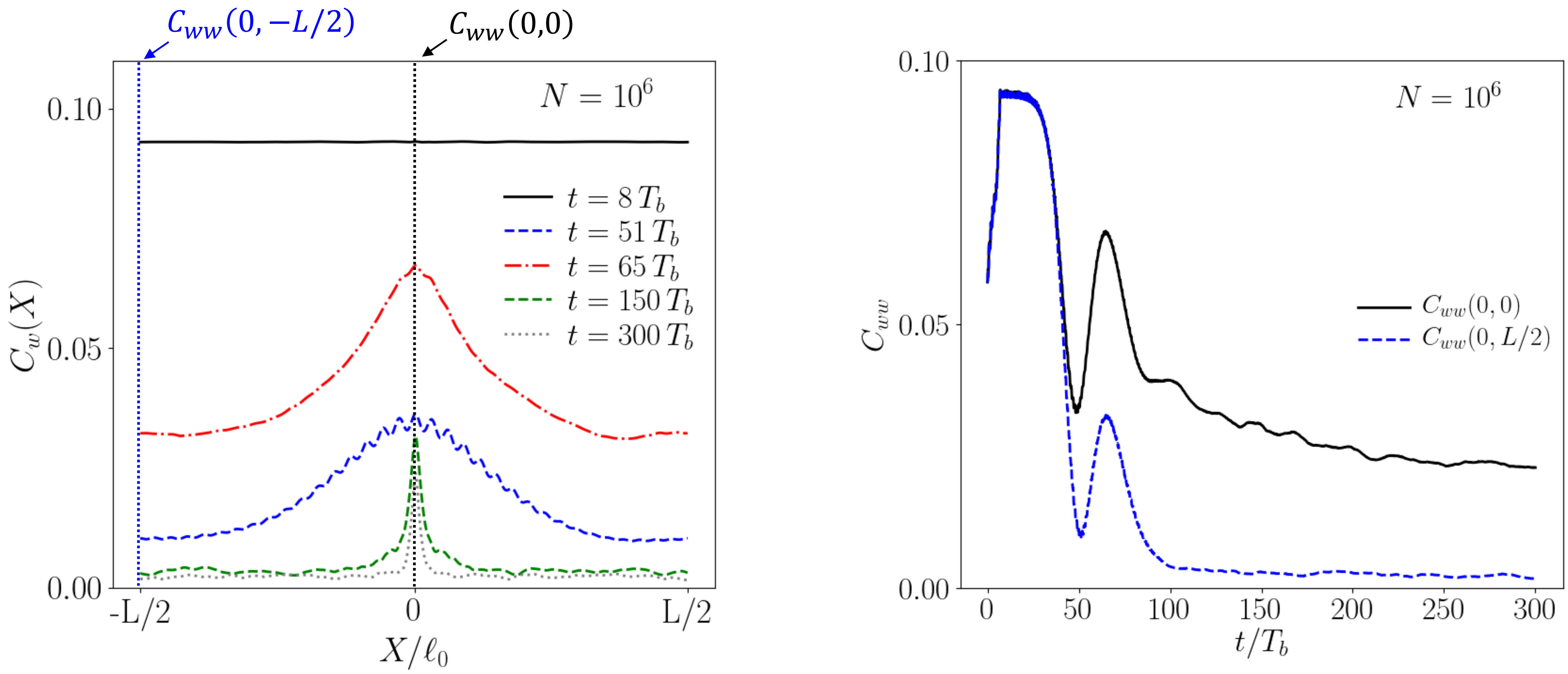
Iacopo Carusotto
(INO-CNR BEC Centre – Trento, Italy)

Thank you! ☺

LEVERHULME TRUST

Results

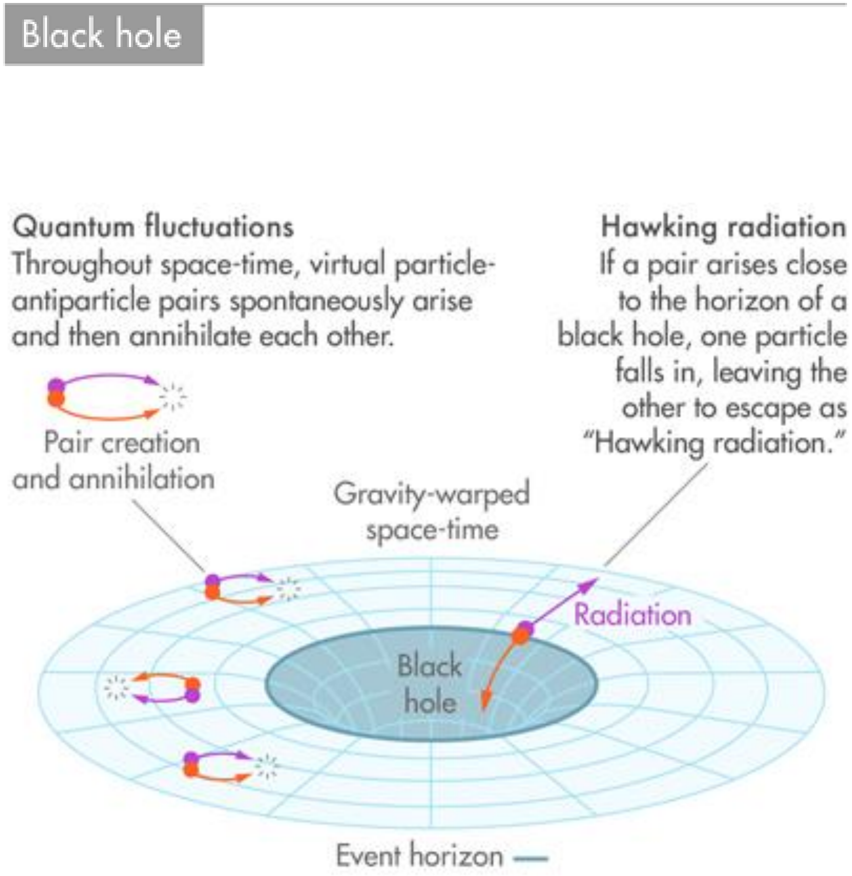
Decoherence



S. Butera and I. Carusotto , *arXiv:2207.00311* (2022)

Analogue models of gravity

Hawking radiation



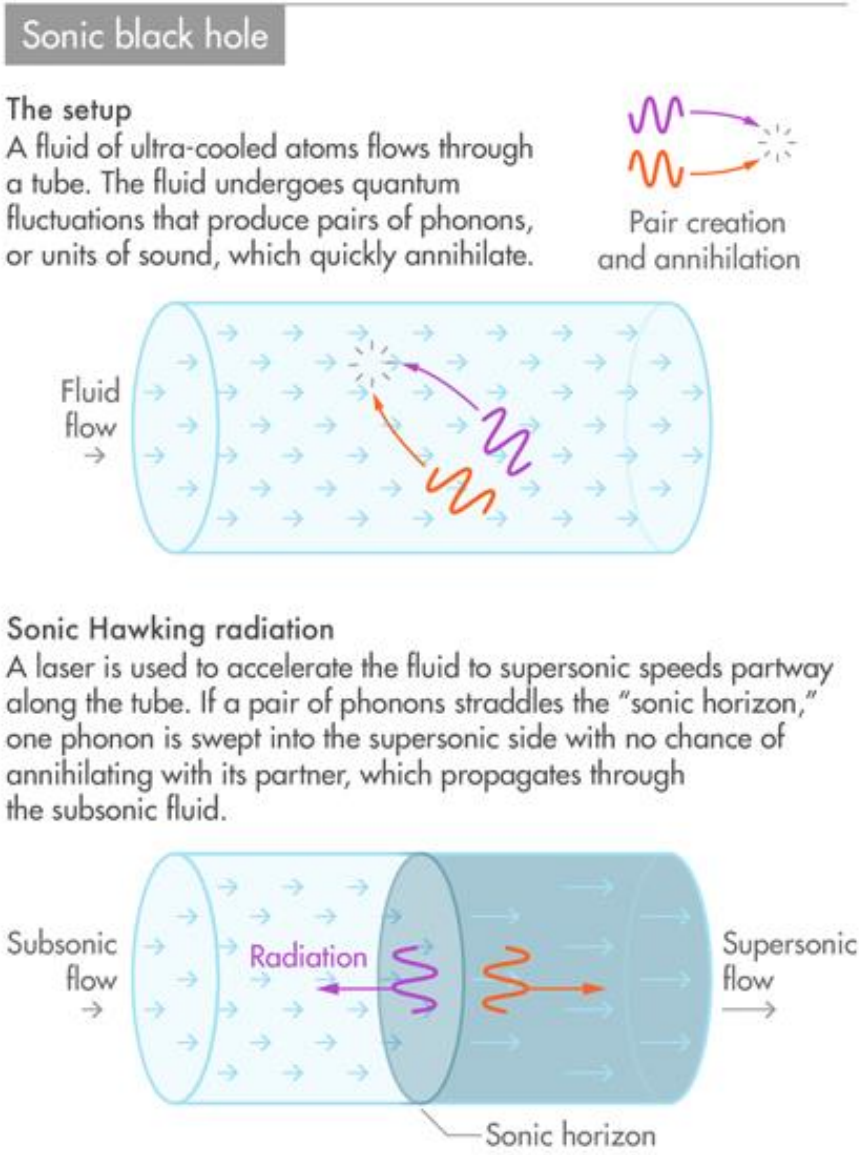
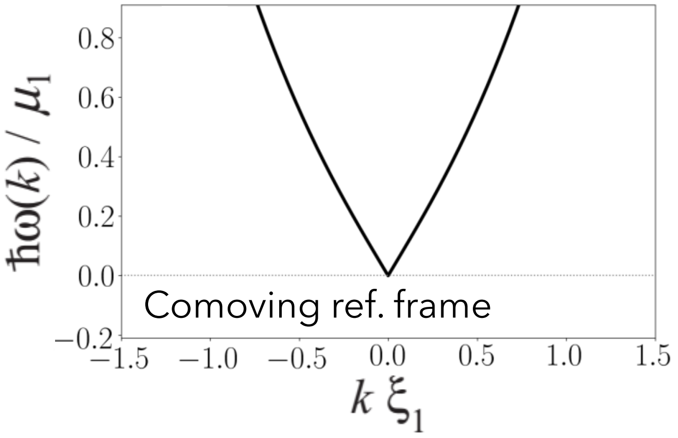


Figure: *Quanta Magazine, 25 June 2019*

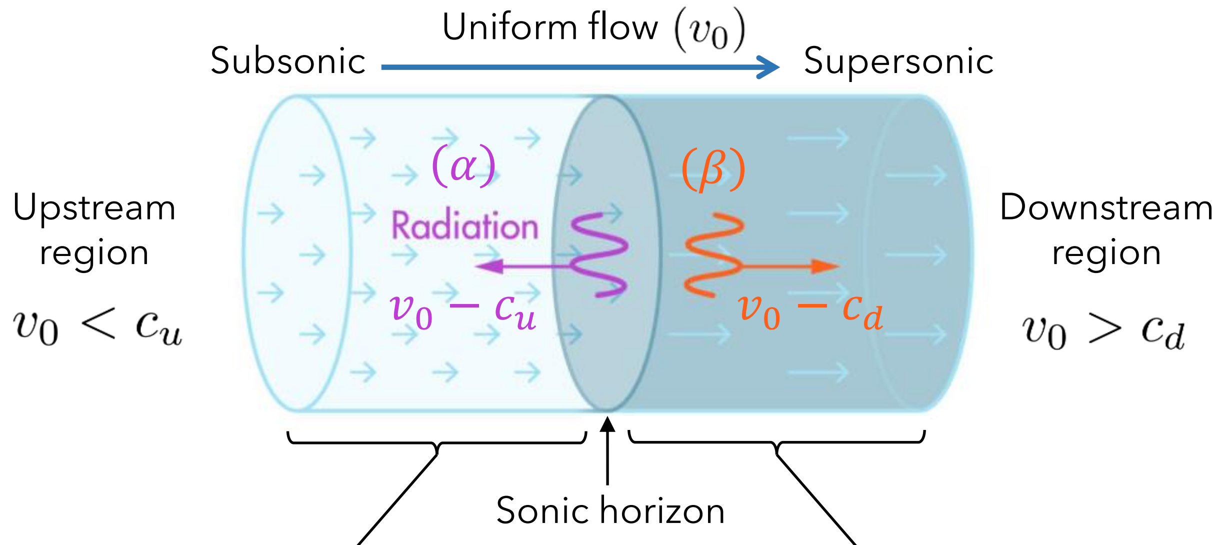
Analogue models of gravity

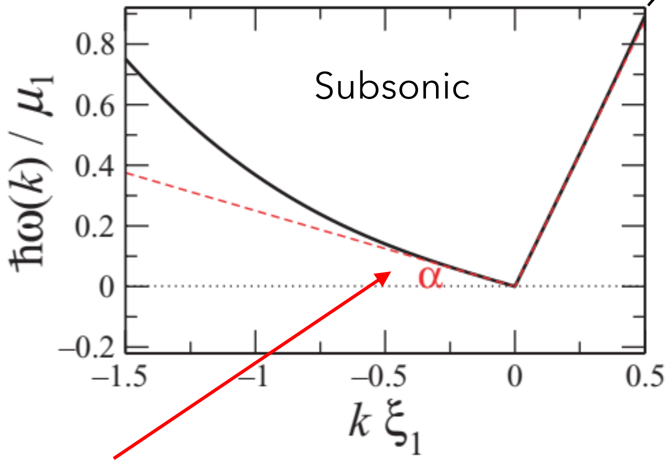
Bogoliubov spectrum

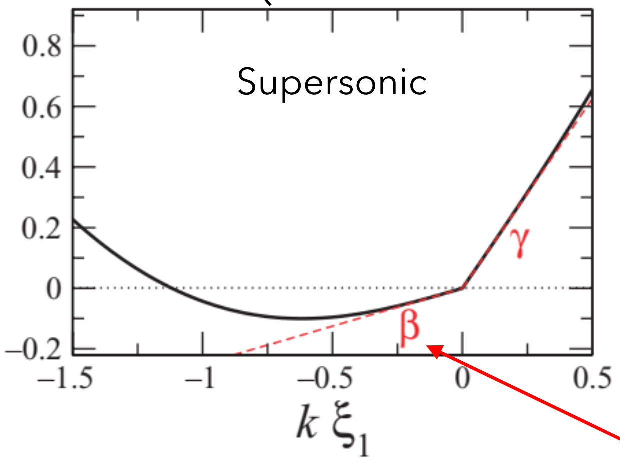


Doopler shift: $\omega_{\text{lab}} = \omega_{\text{com}} + kv$

Hawking radiation







$v_g = v_0 - c_u < 0$

$v_g = v_0 - c_d > 0$

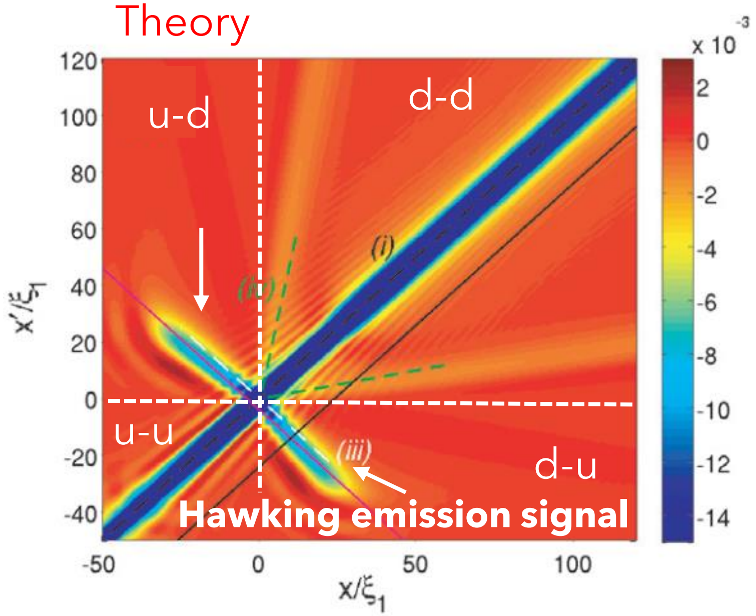
I. Carusotto et al. , *New J. Phys* **10** (2008)

Detecting Hawking radiation in a BEC

Density correlations

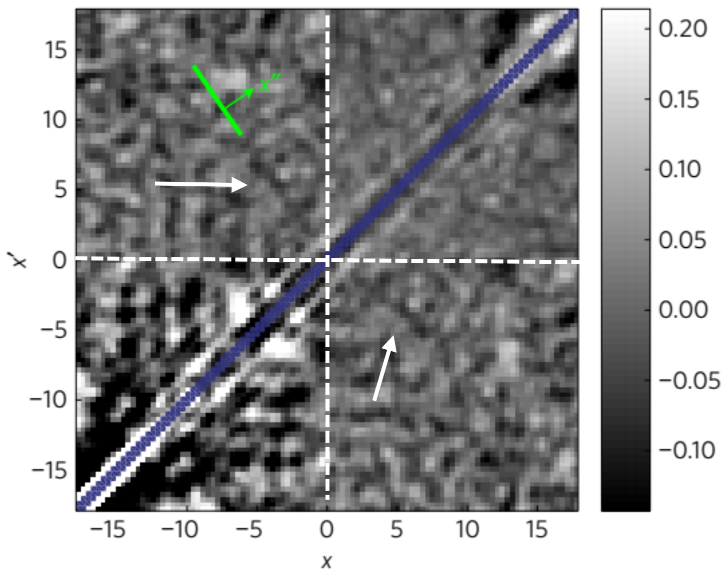
$$g^{(2)} = \frac{\langle : \hat{n}(x)\hat{n}(x') \rangle}{\langle \hat{n}(x) \rangle \langle \hat{n}(x') \rangle} - 1$$

Theory



I. Carusotto et al, *New J. Phys.* (2008)

Experiment



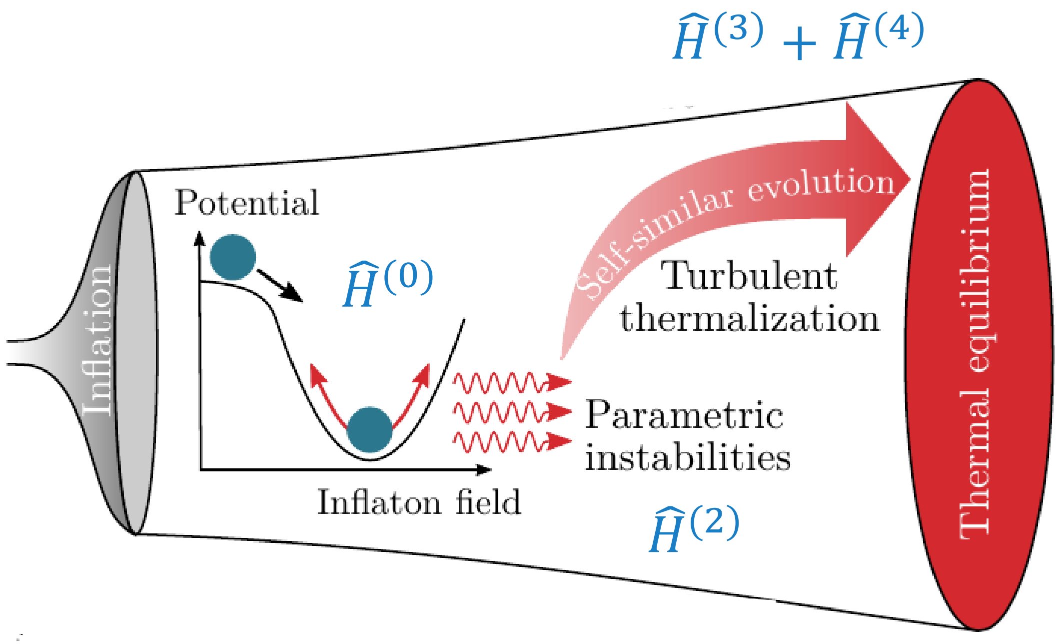
J. Steinhauer, *Nat. Phys.* (2016)

Pre-heating in the early Universe

Analogue BEC system

BEC Hamiltonian

$$\hat{H} = \int dx\, \hat{\psi}^\dagger h \psi + \frac{g}{2} \int dx\, \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} \xrightarrow{\hat{\psi} = \phi + \delta\hat{\psi}} \hat{H} = \overbrace{N\hat{H}^{(0)}}^{\text{GPE}} + \underbrace{\hat{H}^{(2)}}_{\text{Bogoliubov-de Gennes}} + \overbrace{\frac{1}{\sqrt{N}}\hat{H}^{(3)} + \frac{1}{N}\hat{H}^{(4)}}^{\text{Quasi-particles interaction}}$$



Back-reaction from analogue of Pre-heating

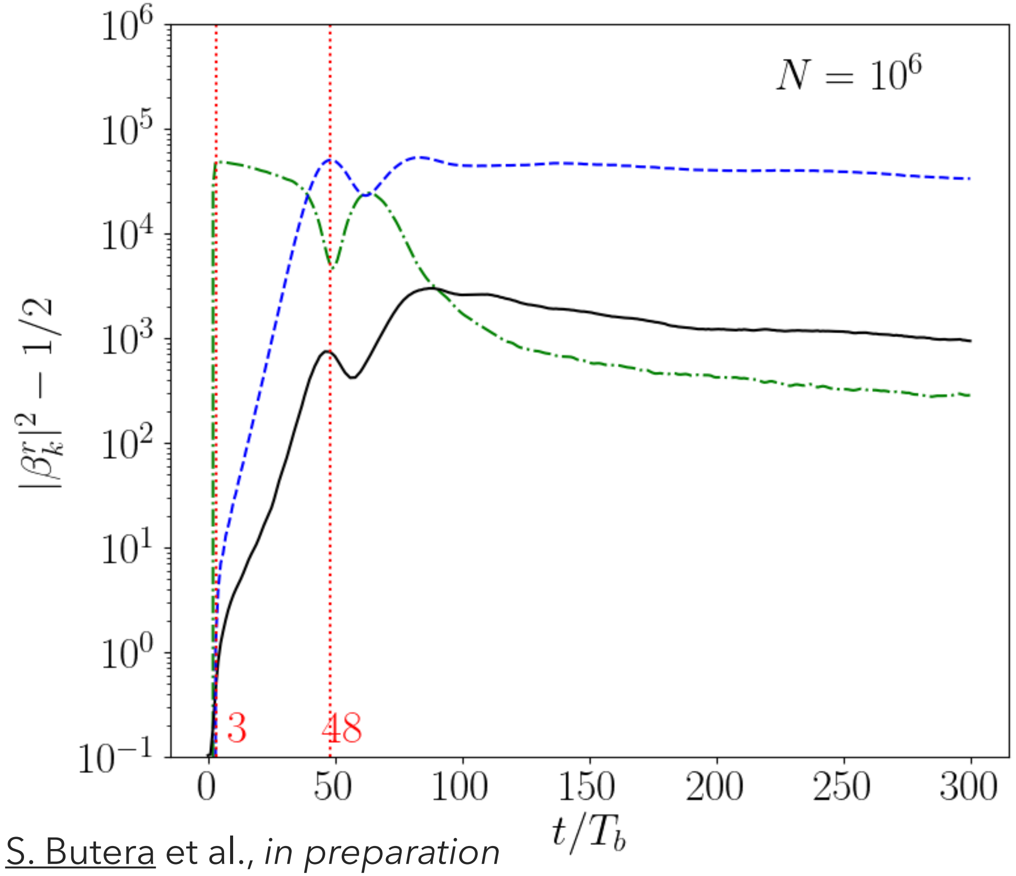
Dissipation

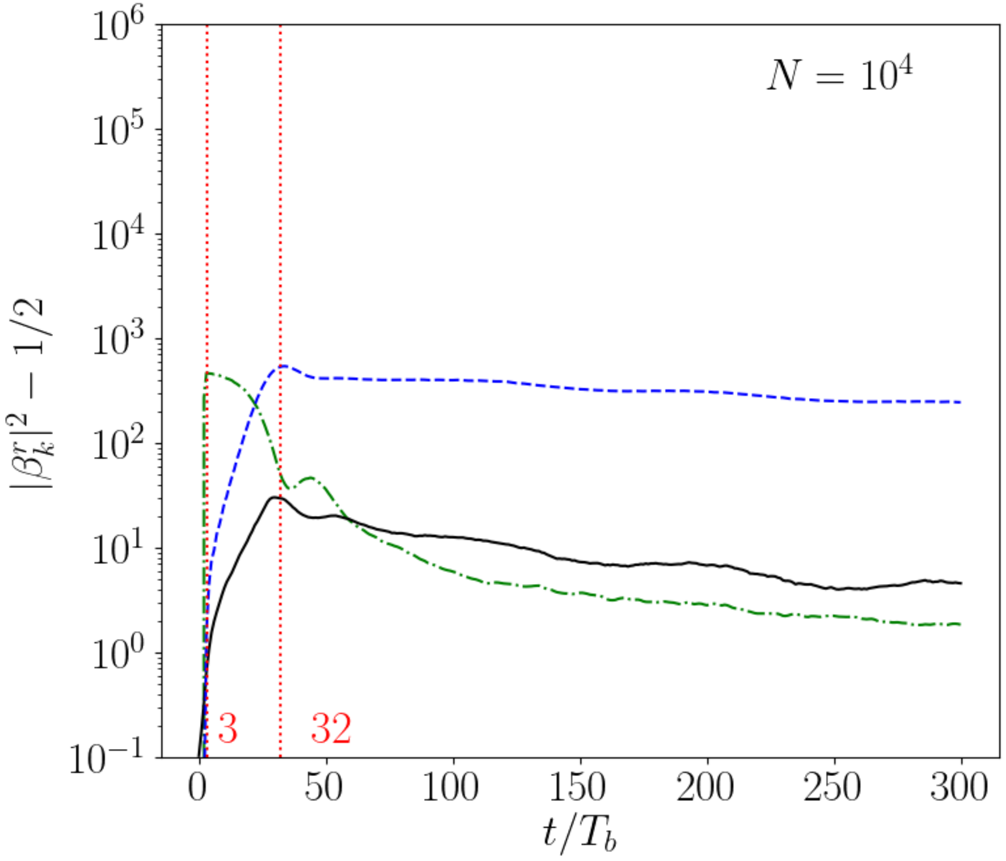
GPE

Quasi-particles interaction

$$\hat{H} = N\hat{H}^{(0)} + \hat{H}^{(2)} + \frac{1}{\sqrt{N}}\hat{H}^{(3)} + \frac{1}{N}\hat{H}^{(4)}$$

Bogoliubov-de Gennes





<u>S. Butera</u> et al., *in preparation*