Arnold Sommerfeld Center for Theoretical Physics
Ludwig-Maximilians-Universität München
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Self-organization and pattern formation
Sheet 9

Exercise 24 – Conceptual Questions

a) Think about the three codimension-one bifurcations in one dimension (fold, transcritical, pitchfork). Which symmetries does each obey? Why is the fold (saddle-node) considered the most generic bifurcation? Why do we come across the others (pitchfork, transcritical) so often in physics?

b) When handling gradient terms, why do we (often) go into Fourier space?

c) Consider the free energy density $f = u\phi^2 + v\phi^4$. What signs do we allow for u, v and what behavior do we expect the system to exhibit?

d) What are the conceptional differences between the Allen-Cahn and the Cahn-Hilliard Dynamics?

e) What does it mean when we say a system is active?

f) Consider a mass-conserving system with an symmetric free energy, which expresses a domain wall between a high and a low-density phase. Can we move this domain freely?

g) What does the Linear stability analysis tell us about the final state of a system?

h) What is the difference between a convective and an absolute instability?

i) You have seen in the lecture that each conserved quantity in a reaction-diffusion system allows to reduce a stationary state equation to a Laplace equation. What happens in the case where there are as many conserved quantities as there are components? What about the case with one conservation law less?

j) In some recipes, butter is added to hot chocolate, which, upon cooling can create a phase-separating effect. Which thermodynamic effect is responsible for mixing and which for phase separation? In the regime of phase separation, recall the coarsening discussed in the lecture. What is the difference between a system where butter is lighter than the

chocolate (flows to the surface), and the system where the butter has around the same density as the chocolate (is evenly distributed in the bulk)?

This sheet is not graded, but is meant as a food-for-thought during the winter break.