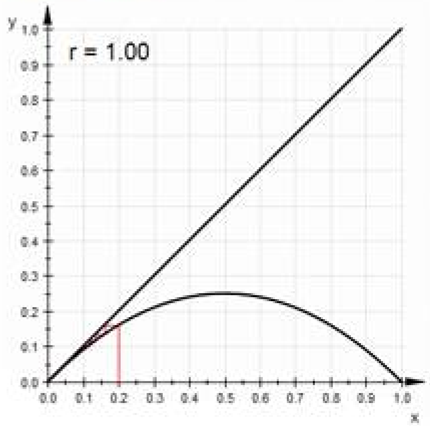
Dewetting & Logistic Equation

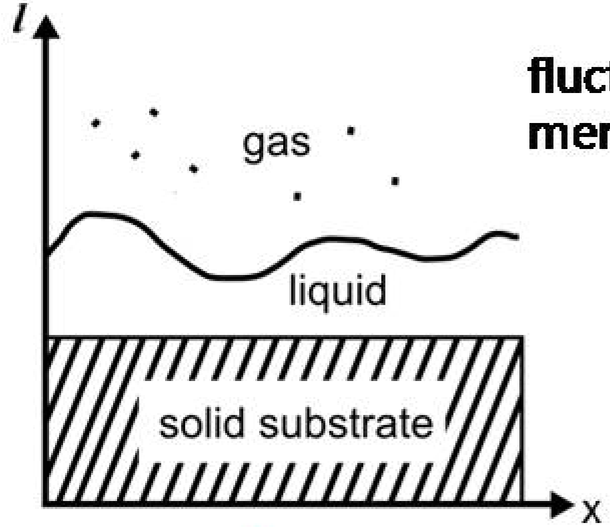
two examples for an exact functional renormalization

- dewetting transition

- logistic mapping, Feigenbaum constant



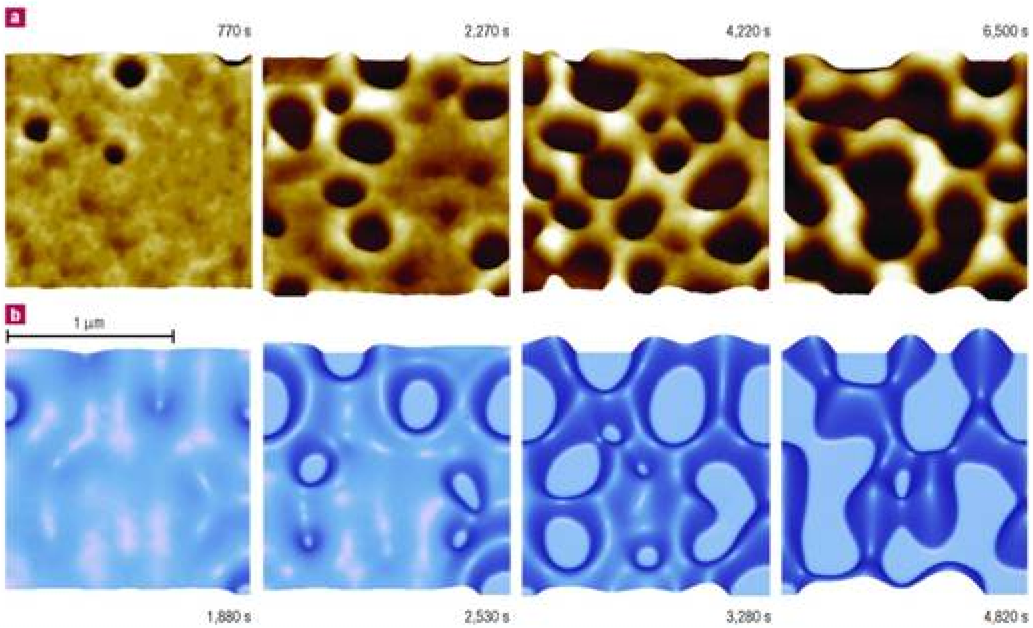
dewetting transition



fluctuating liquid-gas interface is described by a membrane in an external potential V(*l*), *l*=*l*(x)

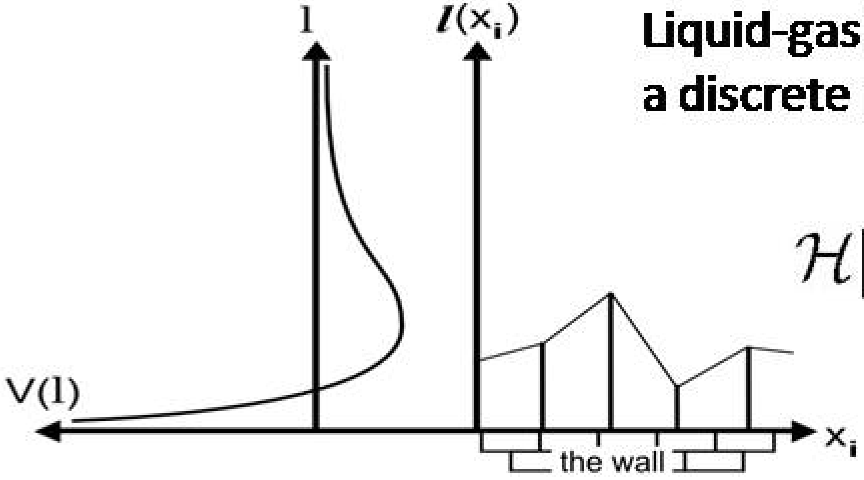
$$\mathcal{H}[l] = \int dx \left[\frac{\sigma}{2} \left(\nabla l \right)^2 + V\left(l \right) \right]$$

σ surface tension



a) AFM recording of an experiment with a 3.9 nm polysterene film on an oxidized Si wafer. b) Simulation. Nature materials Vol 2 JAN 2003 Jürgen Becker et al.

dewetting transition



Liquid-gas interface will be described by a discrete model $l(x_i) = l_i$ in 1+1 dim

$$\mathcal{H}[l_i] = \sum_{i=1}^{N} \frac{J}{2} \left(l_i - l_{i+1}\right)^2 + V(l_i)$$

Transfermatrix: $T(l, l')$

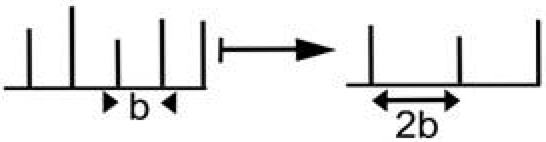
Proper rescaling of vaiables: $z := \sqrt{J\beta}, \qquad U(z) = \beta \cdot V(\sqrt{J\beta} \cdot z)$

$$T(z, z') = exp\left[-\frac{1}{2}(z - z')^2 - \frac{1}{2}\left(U(z) + U(z')\right)\right]$$

Symmetrized bare potential: $U^0(z, z') = \frac{1}{2}\left(U(z) + U(z')\right)$

dewetting transition

- we integrate over every second degree of freedom $T \mapsto T' = T \circ T$

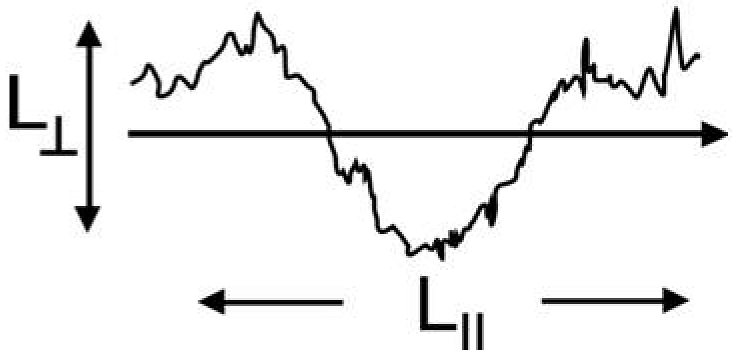


- we have to rescale the system in the l-direction:

  -> scaling behaviour of the fluctuations of the free interface:

$$L_\perp \sim L_{\parallel}^{\zeta}$$

ζ : roughness exponent



x-direction rescaled by: 2 $\quad T \mapsto T \circ T$

y-direction rescaled by: $\sqrt{2}$ $\quad z \mapsto z\sqrt{2}$

dewetting transition

Abb:Funktionale Renormierung an einem Modell zum Benetzungsübergang – Frank Jülicher

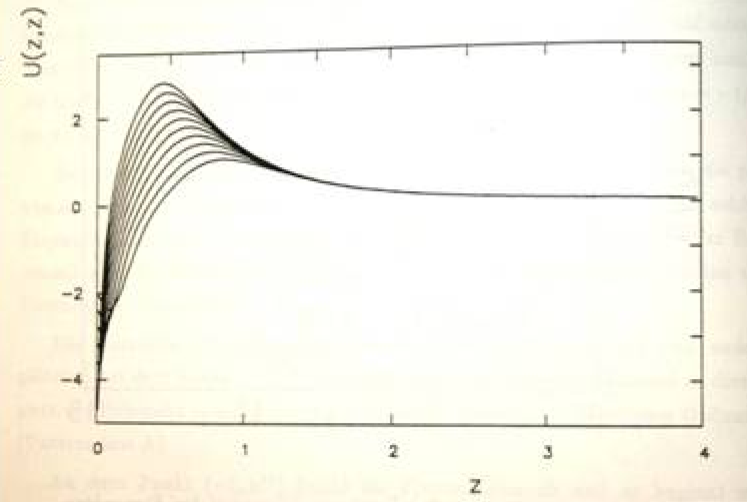


Abb. 10: Einige Iterationen der RG am kritischen Punkt im Unterregime C. Dargestellt sind die Diagonalelemente $U^{(N)}(z,z)$ des Potentials. Der RG-Fluß konvergiert nicht gegen einen Fixpunkt.

bare potential:

$$U^0(z,z') = \frac{1}{2}\left(U(z) + U(z')\right)$$

$$exp[-U'(z,z'')] = \int_0^\infty \frac{dz'}{\sqrt{\pi}} exp\left[-\left(z' + \frac{z+z''}{\sqrt{2}}\right)^2 - U(z',\sqrt{2}z) - U(z',\sqrt{2}z'')\right]$$

- **microscopic fluctuations lead to a renormalization of the interaction potential U(z,z‘) on larger length-scales**
- **we do not consider the flow of couplings but the flow of U(z,z‘) in a space of all possible functions**
- **two non trivial fixpoints are:**

$$U^*_\pm(z,z') = -ln\left(1 \pm e^{-2zz'}\right) = U(z\cdot z')$$

dewetting transition

•All important thermodynamic properties can be extracted from the eigenvalues of T

eigenvalue equation: $$T\psi(x) = \int_0^\infty dy T(x,y)\psi(y) = \lambda_q \psi_q$$

•spectral resolution of the transfermatrix/potential:

$$exp[-\bar{U}(z^2)] = \sqrt{2\pi} z \cdot exp[-z^2] I_\nu(z^2)$$

modified Bessel function of the first kind

$$\upsilon > -1$$

$$2\rho = \upsilon^2 - \frac{1}{4}$$

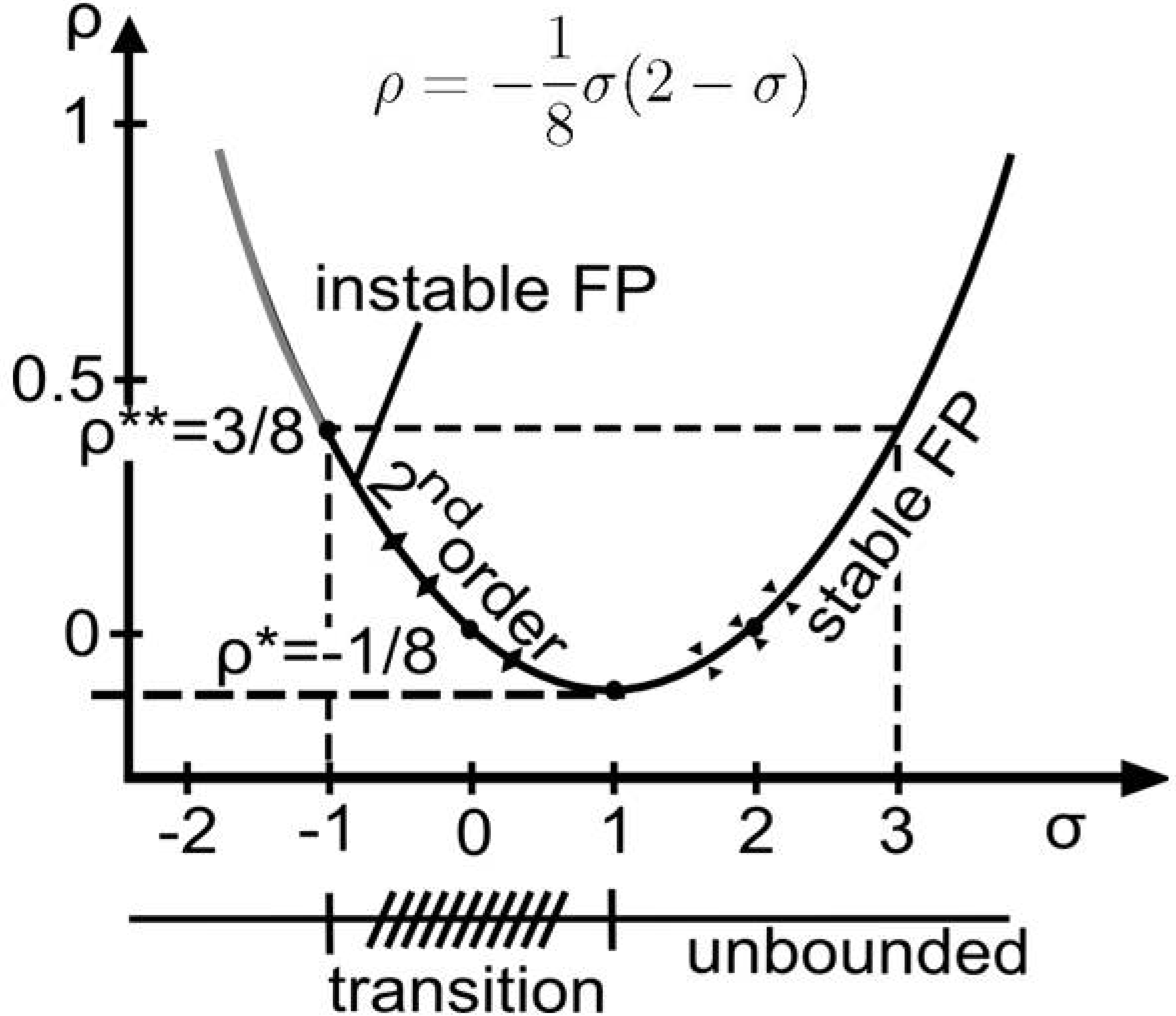
dewetting transition

- **Asymptotic behaviour of the fix points:**

$$U(z^2) \approx \begin{cases} \dfrac{\rho}{z^2} & z \to \infty \\ -\dfrac{\sigma}{2}\ln z & z \to 0 \end{cases}$$

the line of exact fix points:

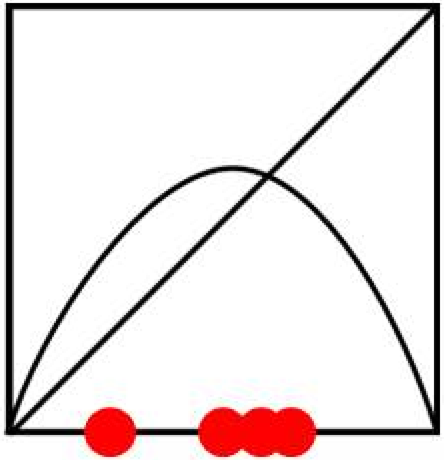
$$\rho = -\frac{1}{8}\sigma(2-\sigma)$$



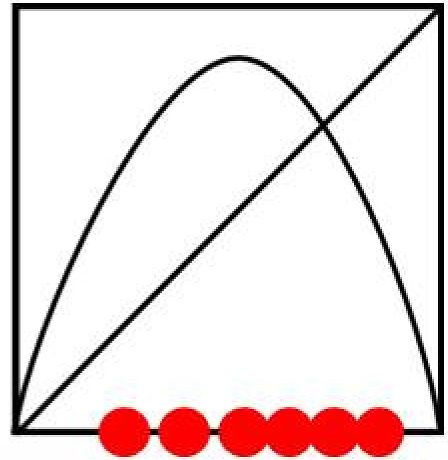
Logistic equation

$$x_{n+1} = r \cdot x_n (1 - x_n)$$

r = 2.5



r = 3.5

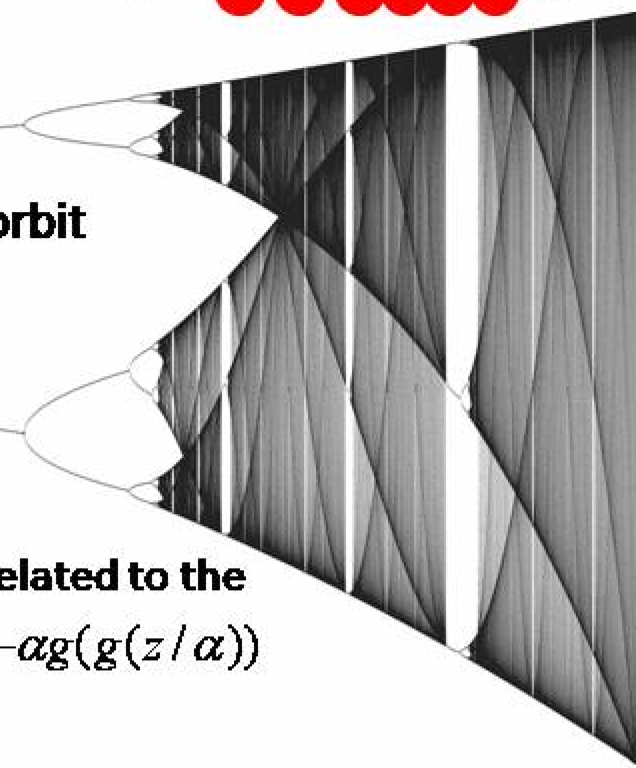


- **Feigenbaum constant:**

$$\frac{\Delta\lambda_{n-1}}{\Delta\lambda_n} = \delta = 4.6692\ldots$$

- **difference from starting point 1/2 to value at half the orbit**

$$\frac{\frac{1}{2} - f^{2^{n-1}}_{\lambda_n}(x = \frac{1}{2})}{\frac{1}{2} - f^{2^n}_{\lambda_{n+1}}(x = \frac{1}{2})} \approx -\alpha = -2.502907875\ldots$$



Feigenbaum's renormalization equation:

The fact that δ, α are universal for a large class of systems is related to the existence of a universal function: (fixpoint equation) $g(z) = -\alpha g(g(z/\alpha))$

Logistic equation

$$x_{n+1} = r \cdot x_n (1 - x_n)$$

Feigenbaum's renormalization equation:

$$g(z) = -\alpha g(g(z/\alpha))$$

A simpler derivation of Feigenbaum's renormalization group equation for the period-doubling bifurcation sequence –
S.N. Coppersmith, Am. J.Phys. 67 Jan 1999

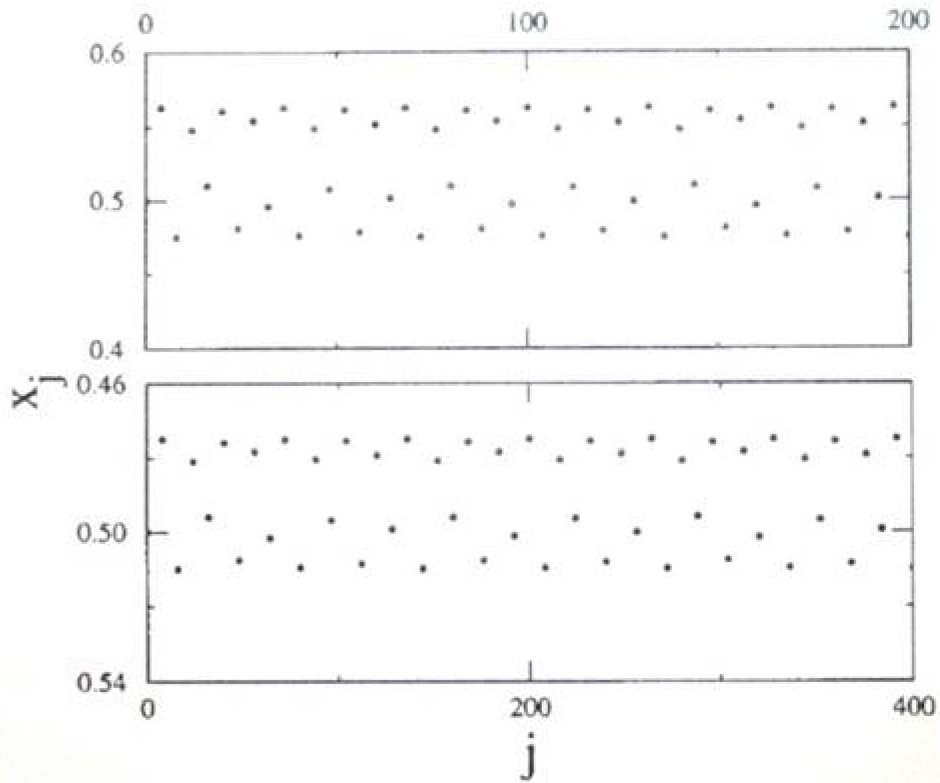


Fig. 1. Two plots of the time series of x values for the logistic map ($x_{j+1} = \lambda x_j(1-x_j)$) with the parameter value $\lambda = \lambda_\infty = 3.569\,945\,669...$, starting with $x_{j=0} = \frac{1}{2}$. Note that the axes have different scales in the lower panel than in the upper panel: the horizontal axis is compressed by a factor of 2, and the vertical axis is inverted and expanded by a factor of 2.5.

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Summary

- Wetting, dewetting transition: bounded/unbounded interfaces
    - Real-space decimation RG, fixpoint equation for transfermatrices
    - Exact line of fix point potentials in 1+1 dim
- Logistic equation

References:

[1] Dissertation: Frank Jülicher – Funktionale Renormierung an einem Modell zum Benetzungsübergang
[2] H. Spohn – Europhys. Lett., 14 (7), pp. 689-692 (1991) Fixed Points of a Functional Renormalization Group for Critical Wetting
[3] S. N. Coppersmith Am. J. Phys. 67 (1), Jan 1999 Feigenbaum's RG equation