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1 The Higgs Mechanism

We begin by applying the Higgs mechanism to an abelian, $U(1)$ gauge theory, to demonstrate how the mass of the corresponding gauge boson (the photon) comes about. The abelian example will then be generalized in a straightforward way to the non-abelian Glashow-Weinberg-Salam theory (the electroweak Standard Model). It succinctly can be specified as a gauge theory with the symmetry group $SU(2) \otimes U(1)$ and describes the weak and electromagnetic interactions due to the exchange of the corresponding spin 1 gauge fields: three massive bosons, $W^{\pm}$ and Z, for the weak interaction, and one massless photon for the electromagnetic interaction. Both the fermions and the electroweak gauge bosons will obtain their masses through the Higgs mechanism.

1.1 An Abelian Example

The $U(1)$ gauge invariant kinetic term of the photon is given by

$$\mathcal{L}_{kin} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ . \tag{2}$$

That is, $\mathcal{L}_{kin}$ is invariant under the transformation: $A_\mu(x) \to A_\mu(x) - \partial_\mu \eta(x)$ for any η and x. If we naively add a mass term for the photon to the Lagrangian,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu, \tag{3}$$

we will soon find out that the mass term violates the local gauge symmetry. The $U(1)$ gauge symmetry thus requires the photon to be massless.

Now extend the model by introducing a complex scalar field with charge $(-e)$ that couples both to itself and to the photon:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi), \tag{4}$$

where $D_\mu = \partial_\mu - ieA_\mu$ and $V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$. It is easily discerned that this Lagrangian is invariant under the gauge transformations:

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \eta(x), \tag{5}$$

$$\phi(x) \to e^{ie\eta(x)} \phi(x). \tag{6}$$

If $\mu^2 < 0$, the state of minimum energy will be that with $\phi = 0$ and the potential will preserve the symmetries of the Lagrangian. Then the theory is simply QED with a massless photon and a charged scalar field ϕ with mass μ.

However, if $\mu^2 > 0$, the field ϕ will acquire a vacuum expectation value (VEV),

$$\langle\phi\rangle = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} \ , \tag{7}$$

and the global $U(1)$ gauge symmetry will be spontaneously broken.

It is convenient to parameterize ϕ as

$$\phi = \frac{v+h}{\sqrt{2}} e^{i\frac{\chi}{v}}, \tag{8}$$

where h and χ, which are referred to as the Higgs boson and the Goldstone boson, respectively, are real scalar fields which have no VEVs. Substituting (8) back into the Lagrangian, we find

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - evA_\mu\partial^\mu\chi + \frac{e^2v^2}{2}A_\mu A^\mu \\ &+\frac{1}{2}\left(\partial_\mu h\partial^\mu h - 2\mu^2 h^2\right) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi \\ &+(h,\ \chi \text{ interactions}).\end{aligned} \tag{9}$$

This Lagrangian now describes a theory with a photon of mass $m_A = ev$, a Higgs boson h with $m_h = \sqrt{2}\mu = \sqrt{2\lambda}v$, and a massless Goldstone χ. The strange χ–A_μ mixing can be removed by making the following gauge transformation:

$$A_\mu \to A'_\mu = A_\mu - \frac{1}{ev}\partial_\mu\chi \ . \tag{10}$$

The gauge choice with the transformation above is called the unitary gauge. The Goldstone χ will then completely disappear from the theory and one says that the Goldstone has been eaten to give the photon mass.

It is instructive to count the degrees of freedom (dof) before and after SSB has occurred.[1] We started out with a massless photon (2 dof) and a complex scalar field (2 dof) for a total number of 4 dof. After the SSB we have one massive photon (3 dof) and a real scalar field h (1 dof), again for a total of 4 dof.

1.2 The Higgs mechanism in the Electroweak Standard Model

Now we discuss a gauge theory that contains the combined electromagnetic and weak interactions, which is generally referred to as the electroweak unification. The electroweak Standard Model is based on the $SU(2) \otimes U(1)$ Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_f + \mathcal{L}_{Higgs} + \mathcal{L}_{Yuk} \ . \tag{11}$$

The fermion term is

$$\mathcal{L}_f = i\,\bar{\Psi}_L \not{D}\, \Psi_L + i\,\bar{\psi}_R \not{D}\, \psi_R \ , \tag{12}$$

[1] Massless gauge fields have two transverse dof, while a massive gauge field has an additional longitudinal component.

where the covariant derivative acts on Ψ_L and ψ_R, respectively, as

$$D_\mu \Psi_L = \left(\partial_\mu + ig\, W_\mu + ig'\, Y_L\, B_\mu\right) \Psi_L\ , \qquad D_\mu \psi_R = \left(\partial_\mu + ig'\, Y_R\, B_\mu\right) \psi_R\ . \tag{13}$$

The L (R) refer to the left (right) chiral projections $\psi_{L\,(R)} = (1 \mp \gamma_5)\psi/2$. In the electroweak gauge theory the left-handed quarks and leptons (Ψ_L)

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L , \qquad l_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

are arranged in doublets, while the right handed fields (ψ_R)

$$u_R,\ d_R,\ \nu_{eR},\ e^-_R$$

are singlets. The gauge transformations are

$$\Psi_L \to \Psi'_L = e^{i\, Y_L\, \theta(x)}\, U_L\, \Psi_L\ , \tag{14}$$

$$\psi_R \to \psi'_R = e^{i\, Y_R\, \theta(x)}\, \psi_R\ . \tag{15}$$

The $SU(2)_L$ transformation that only acts on the doublet fields is

$$U_L = e^{iT^i \beta^i(x)}, \tag{16}$$

where $T^i = \frac{\tau^i}{2}$ (τ^i are the three Pauli matrices) denote the generators of the fundamental representation of the $SU(2)_L$ Lie algebra, i.e. they comply with

$$\left[T^i, T^j\right] = i\epsilon^{ijk} T^k. \tag{17}$$

The real and totally antisymmetric ϵ^{ijk} are the $SU(2)_L$ structure constants.

The transformation properties of B_μ and W_μ (W_μ can be written in terms of the generators: $W_\mu = W^i_\mu T^i$) are fixed by the gauge symmetry of the fermion Lagrangian. Thus,

$$B_\mu \to B'_\mu = B_\mu - \frac{1}{g'} \partial_\mu \theta\ , \tag{18}$$

$$W_\mu \to W'_\mu = U_L W_\mu U_L^\dagger + \frac{1}{g} \left(\partial_\mu U_L\right) U_L^\dagger\ . \tag{19}$$

Note that the four gauge parameters in the electroweak symmetry, $\beta^i(x)$ and $\theta(x)$ respectively involve three $SU(2)_L$ gauge bosons, W^i, $i = 1, 2, 3$, that couple to the weak-isospin T, and one $U(1)_Y$ gauge boson, B, that couples to hypercharge. The electroweak symmetry will turn out to be spontaneously broken, generating masses for the physical gauge bosons $W^\pm$ and Z. Also, it will be apparent that the photon and the Z boson are formed by the mixing between the B and W^3 fields, resulting from the Higgs mechanism.

The values of the hypercharge are fixed in such a way that the sum of the hypercharge and the third component of the weak-isospin generators of a specific fermion give its electric charge,

$$Q = T^3 + Y. \tag{20}$$

For left-handed doublets T^3 is $\frac{\tau^3}{2}$ (i.e. $T^3 = \pm\frac{1}{2}$), while for right-handed singlets $T^3 = 0$. Hence the hypercharge eigenvalues for the leptons are

$$Y(l_L) = -\frac{1}{2}\ , \qquad Y(l_R) = -1\ ,$$

and for the quarks we find

$$Y(q_L) = \frac{1}{6}\,, \qquad Y(u_R) = \frac{2}{3}\,, \qquad Y(d_R) = -\frac{1}{3}\,.$$

The gauge part is

$$\mathcal{L}_{gauge} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}G^{i\mu\nu}G^i_{\mu\nu}\,, \tag{21}$$

where

$$G^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon^{ijk}W^j_\mu W^k_\nu, \tag{22}$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,. \tag{23}$$

The quadratic G term above gives rise to cubic and quartic self-interactions among the gauge fields.

As in the case of gauge bosons, it is not possible to simply add a mass term for fermions to the Lagrangian. Such a Dirac mass term,

$$m\,\bar\psi\,\psi = m(\bar\psi_L + \bar\psi_R)(\psi_L + \psi_R) = m(\bar\psi_L\,\psi_R + \bar\psi_R\,\psi_L)\,, \tag{24}$$

contains couplings of left- and right-handed fields, which have different transformation properties, spoiling the gauge symmetry.

In the following we will dicuss the mass generation both for the electroweak gauge bosons and for the fermions through the elaborate mechanism of spontaneous symmetry breaking.

Spontaneous Symmetry Breaking

The abelian example can now be generalized in a straightforward way to a non-abelian gauge theory. The scalar Higgs part of the Lagrangian is given by

$$\mathcal{L}_{Higgs} = (D^\mu\phi)^\dagger\,(D_\mu\phi) - V(\phi)\,. \tag{25}$$

To spontaneously break the symmetry, consider a complex scalar field in the spinor representation of $SU(2)_L$,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}\,, \tag{26}$$

with $U(1)$ charge $Y(\phi) = +1/2$. An additional $U(1)_Y$ symmetry was needed in order for the theory to lead to a system with a massless gauge boson. The covariant derivative of ϕ is

$$D_\mu\phi = \left(\partial_\mu + ig\,T^iW^i_\mu + i\frac{1}{2}g'B_\mu\right)\phi\,, \tag{27}$$

where W^i_μ and B_μ are, respectively, the $SU(2)_L$ and $U(1)_Y$ gauge bosons. Note that the square of the covariant derivative involves three and four point interactions between the gauge and scalar fields.

Renormalizability and $SU(2)_L \otimes U(1)_Y$ invariance require the Higgs potential $V(\phi)$ to be of the form

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2. \tag{28}$$

The λ term describes quartic self-interactions among the scalar fields. Vacuum stability demands λ to be greater than zero.

Just as in the abelian example, the scalar field develops a nonzero VEV for $\mu^2 > 0$, which spontaneously breaks the symmetry. Due to the symmetry of $V(\phi)$ there is an infinite number of degenerate states with minimum energy satisfying $\phi^\dagger \phi = v^2/2$. Since the potential depends only on the combination $\phi^\dagger \phi$, we arbitrarily choose

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} . \tag{29}$$

Owing to the conservation of electric charge only a neutral scalar field can acquire a VEV. Thus, with the choice above, ϕ^0 is to be interpreted as the neutral component of the doublet, and above all $Q(\phi) = 0$. That is, electromagnetism is unbroken by the scalar VEV. The scalar VEV of Eq. (29) hence yields the breaking scheme,

$$SU(2)_L \otimes U(1)_Y \; \rightarrow \; U(1)_Q \, , \tag{30}$$

which is by construction still a true vacuum symmetry.

The W and Z gauge masses can now be generated in the same manner as the Higgs mechanism generated the photon mass in the abelian example. Recall from earlier considerations that in the unitary gauge the spectrum is obvious and there are no Goldstone bosons, but only the physical Higgs after SSB. For convenience, we thus write the scalar doublet in the unitary gauge as follows:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} . \tag{31}$$

Because we are interested only in the contribution for the gauge boson masses, we omit any h - mixed terms in what follows. The piece generating the gauge boson masses is

$$\begin{aligned} (D^\mu \phi)^\dagger (D_\mu \phi) &= \left| \left(\partial_\mu + \frac{i}{2} g \tau^k W^k_\mu + \frac{i}{2} g' B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\ &= \frac{v^2}{8} \left| \left(g \tau^k W^k_\mu + g' B_\mu \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2 \\ &= \frac{v^2}{8} \left| \begin{pmatrix} g W^1_\mu - i g W^2_\mu \\ -g W^3_\mu + g' B_\mu \end{pmatrix} \right|^2 \\ &= \frac{v^2}{8} \left[g^2 \left(\left(W^1_\mu\right)^2 + \left(W^2_\mu\right)^2 \right) + \left(g W^3_\mu - g' B_\mu \right)^2 \right] . \end{aligned} \tag{32}$$

The charged vector boson, W^-_μ, and its complex conjugate are defined as

$$W^\pm_\mu \equiv \frac{1}{\sqrt{2}} \left(W^1_\mu \mp i W^2_\mu \right) . \tag{33}$$

Thereby the g^2 term in Eq. (32) becomes

$$\frac{1}{2} \left(\frac{g \, v}{2} \right)^2 W^\dagger_\mu W^\mu , \tag{34}$$

yielding the W mass:

$$m_W = \frac{g \, v}{2} \, . \tag{35}$$

The two remaining neutral gauge bosons, Z and A, are defined as follows:

$$\begin{aligned} Z_\mu &\equiv \frac{1}{\sqrt{g^2+g'^2}}\left(gW^3_\mu - g'B_\mu\right) \qquad \text{with mass} \qquad m_Z = \frac{v}{2}\sqrt{g^2+g'^2}\ , \\ A_\mu &\equiv \frac{1}{\sqrt{g^2+g'^2}}\left(g'W^3_\mu + gB_\mu\right) \qquad \text{with mass} \qquad m_A = 0\ . \end{aligned} \tag{36}$$

It is again instructive to count the degrees of freedom before and after the Higgs mechanism. At the outset we had a complex doublet ϕ with four degrees of freedom, one massless B with two degrees of freedom and three massless W^i gauge fields with six for a total number of 12 degrees of freedom. At the end of the day, after SSB we have a real scalar Higgs field h with one degree of freedom, three massive weak bosons, $W^\pm$ and Z , with nine, and one massless photon with two degrees of freedom, yielding again a total of 12. One says that the scalar degrees of freedom have been eaten to give the $W^\pm$ and Z bosons their longitudinal components.

To see how the fermion mass comes about, consider the last missing piece of the final Lagrangian of the electroweak Standard Model, the Yukawa Lagrangian:

$$\begin{aligned} \mathcal{L}_{Yuk} = {} & \Gamma^u_{mn}\bar{q}_{m,L}\,\tilde{\phi}\,u_{n,R} + \Gamma^d_{mn}\bar{q}_{m,L}\,\phi\,d_{n,R} \\ & +\Gamma^e_{mn}\bar{l}_{m,L}\,\phi\,e_{n,R} + \Gamma^\nu_{mn}\bar{l}_{m,L}\,\tilde{\phi}\,\nu_{n,R} + h.c.\ , \end{aligned} \tag{37}$$

with an implicit sum over the family indices m and n. The matrices Γ_{mn} describe the so called Yukawa couplings between the single Higgs doublet ϕ and the fermions. The Yukawa Lagrangian is, of course, gauge invariant, as the combinations $\bar{L}\phi R$ are $SU(2)_L$ singlets. Taking into account that the mass terms should be hyperchargeless, two representations of Higgs fields with $Y=+\frac{1}{2}$ and $-\frac{1}{2}$ are needed to give masses to the down quarks and electrons, and to the up quarks and neutrinos. As the neutrino has no right-handed partner in the SM, it can not acquire a mass term through Yukawa coupling.

The representations are

$$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \qquad \text{with} \qquad Y(\phi) = +\frac{1}{2}\ , \tag{38}$$

and $\tilde{\phi}_i = \epsilon_{ij}\ \phi^*_j$:

$$\tilde{\phi} = \begin{pmatrix}\phi^{0*}\\ -\phi^-\end{pmatrix} \qquad \text{with} \qquad Y(\tilde{\phi}) = -\frac{1}{2}\ . \tag{39}$$

Under $SU(2)$ both representations transform as

$$\phi_i\ \rightarrow\ \phi'_i = U_{ij}\ \phi_j\ , \qquad \tilde{\phi}_i\ \rightarrow\ \tilde{\phi}'_i = U_{ij}\ \tilde{\phi}_j\ . \tag{40}$$

The transformation properties of $\tilde{\phi}$ is in fact true, since[2]

$$\tilde{\phi}'_i = \epsilon_{ij}\ \phi'^*_j = \epsilon_{ij}\ U^*_{jk}\ \phi^*_k = (U^\dagger)_{kj}\epsilon_{ij}\ \phi^*_k = U_{il}\ \epsilon_{lk}\ \phi^*_k\ .$$

[2]Note that

$$(U^\dagger)_{ln}(U^\dagger)_{kj}\ \epsilon_{nj} = \det(U^\dagger)\ \epsilon_{lk} \quad \Rightarrow \quad (U^\dagger)_{kj}\ \epsilon_{ij} = U_{il}\ \epsilon_{lk}\ .$$

In principle, all of the fermion masses can now be generated with a single Higgs-doublet by making use of both ϕ and $\tilde{\phi}$. We do this for the first family as an example:

$$\mathcal{L}_{Yuk} = f_e \, \bar{l}_L \, \phi \, e_R + f_u \, \bar{q}_L \, \tilde{\phi} \, u_R + f_d \, \bar{q}_L \, \phi \, d_R + h.c. \tag{41}$$

Choose

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \Rightarrow \quad \tilde{\phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}, \tag{42}$$

then the Langrangian takes the form

$$\mathcal{L}_{Yuk} = \frac{f_e \, v}{\sqrt{2}} \underbrace{(\bar{e}_L e_R + \bar{e}_R e_L)}_{\bar{e}\, e} + \frac{f_u \, v}{\sqrt{2}} (\bar{u}_L u_R + \bar{u}_R u_L) + \frac{f_d \, v}{\sqrt{2}} (\bar{d}_L d_R + \bar{d}_R d_L) \,, \tag{43}$$

from which the masses for the fermions in question can be read off:

$$m_i = -\frac{f_i \, v}{\sqrt{2}} \,, \qquad i = e, \ u, \ d \,. \tag{44}$$

If we were to compute radiative corrections (e.g. to the Weinberg angle), we would discover that heavy quarks give rise to large corrections. This result is a direct consequence of a gauge theory with SSB. In a renormalizable gauge theory without SBB, heavy quarks would decouple at energy scales much smaller than their masses. However, in the GSW model, the longitudinal components of the W and Z bosons are generated by the Higgs mechanism, and their coupling increase with the masses. Thus heavy quarks do not decouple in the SM.

The top quark, due to its large mass, plays an essential role both in and beyond the SM. Precision measurements of m_t set contraints on the masses of particles to which the top quark makes radiative corrections, including the unobserved Higgs boson and new particles that might contribute additional radiative corrections. The precision electroweak measurements indicate that $m_h \leq 163$ GeV (one-sided 95% confidence level, see [**?**]), and according to the LEP Higgs Working Group m_h must be heavier than 114.4 GeV (95% confidence level).

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