Abelian F-theory constructions

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Outline

The talk is split into two major parts:

1. Engineering elliptic Calabi-Yau manifolds
2. Detailed discussion of toric $U(1)$ symmetries in different codimensions

Literature

Over the past few years, there has been much interest in constructing both local and global F-theory compactifications with Abelian gauge factors. For references, see for example papers by Anderson, Blumenhagen, Borchmann, A. Braun, V. Braun, Collinucci, Cvetič, Dolan, Dudas, García-Etxebarria, Grassi, Grimm, Klevers, Marsano, Mayrhofer, Palti, Piragua, Saulina, Schäfer-Nameki, Weigand.

For recent progress on landscape/classification questions in F-theory see for instance papers by Grimm, Heckman, Johnson, Martini, Morrison, Park, Seiberg, Taylor, Vafa.

In this talk, I wish to discuss the program initiated in [arXiv:1306.0577] with Volker Braun and Thomas W. Grimm and discuss its extension to complete intersection elliptic curves that has been work in progress for quite some time.

Part I: Motivation

Engineer Calabi-Yau manifolds

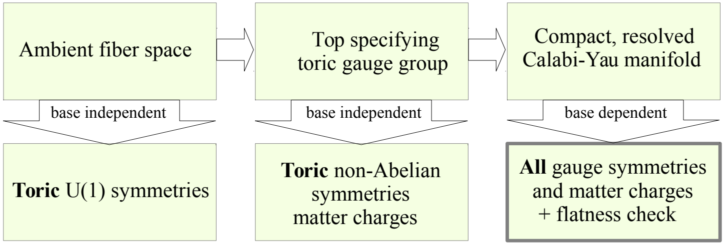
- Break up construction into different steps
- Define 'good' geometric quantities that can be considered independently
- Map geometric quantities to physical observables

Applications:

- Provide laboratory for F-theory models
- Landscape studies - classification of CYs and of their effective theories:
  $\Rightarrow$ possibly extend Kreuzer-Skarke classification to higher dimensions by restricting class of target geometries

Roadmap

In the following, I wish to summarize the approach to this problem suggested in [arXiv:1306.0577] with Volker Braun and Thomas W. Grimm.



In the second part of the talk, I will discuss step I in more detail.

Step I: Choosing a fiber

Choose a toric ambient space F to embed the elliptic fiber in. This choice fixes the *minimum* number of $U(1)$s of the total compactification, i.e. it determines a subgroup

$$\mathrm{MW}_T \subseteq \mathrm{MW}\,. \tag{1}$$

Important: In general

$$\mathsf{rk}\,\mathrm{MW}_T \neq \#\text{toric sections of fibration}\,. \tag{2}$$

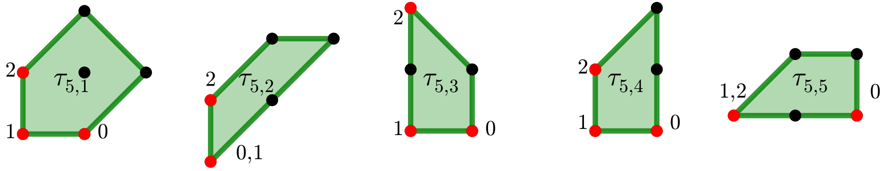
Only this subgroup is independent of the base manifold.

Step II: Choosing a Top

The algorithm in [Bouchard, Skarke '03] allows to construct all possible tops that induce a given non-Abelian gauge group *if the fiber ambient space is two-dimensional*.

Example

Modding out automorphisms, we find 5 different $SU(5)$ tops for the fiber dP_2 (see also [Borchmann, Mayrhofer, Palti, Weigand '13]):



$U(1)$ Charges

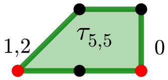
The choice of top already determines the charge of the **10** and fixes the charges of the **5** representations modulo 5 for $SU(5)$.

Example

Pick f_2 as zero section, f_0 and f_1 as generators for $U(1)_0$ and $U(1)_1$, respectively. Then

$$Q_{U(1)_0}(\mathbf{5}) \equiv 2 \mod 5 \qquad Q_{U(1)_1}(\mathbf{5}) \equiv 0 \mod 5 \tag{3}$$

$$Q_{U(1)_0}(\mathbf{10}) = -1 \qquad Q_{U(1)_1}(\mathbf{10}) = 0 \tag{4}$$

for the top $\tau_{5,5}$: 

Step III: Choosing a Base

Last of all, choose the base manifold with $\dim_{\mathbb{C}} \mathcal{B} = n - 1$.

Question: How can one classify and construct all possible reflexive polyhedra with given top and base?

Polytope of Compactifications

Answer: There exists a simple geometric algorithm and the fibrations are encoded in the integral points of a $h^{1,1}(\mathcal{B}) \times \dim F$-dimensional lattice polytope.

Example

For $\tau_{5,5}$ with $\mathcal{B} = \mathbb{P}^3$, there are 30 inequivalent fourfolds.

Warning

The choice of top fixes the toric gauge group, not, however, the non-toric parts [Braun, Grimm, Keitel '13.02] for an example). A complete analysis of gauge groups, both Abelian and non-Abelian, is therefore *base dependent*.

Part II: $U(1)$ gauge factors

Part II: Motivation

Over the past two years, much activity has focused on studying $U(1)$ gauge symmetries in F-theory. Two main questions come to mind:

1. Why should one bother with $U(1)$s at all?
2. Why are $U(1)$s so tricky to handle in F-theory?

Why should one bother with $U(1)$s at all?

In fact, there are plenty of useful scenarios involving $U(1)$ gauge symmetries and many in the audience have worked on (some variation) of them.

A few include:

- $U(1)$s can be used to forbid proton decay operators in GUTs.
- $U(1)$s can be used to generate flavor hierarchies.
- $U(1)$s can be used to induce chirality by supporting gauge flux.

Why are $U(1)$s so tricky to handle in F-theory?

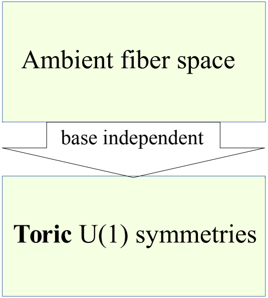
$U(1)$s are intrinsically global in character. For example, in 4d there are gravitational-Abelian anomaly conditions.

- Non-Abelian gauge groups are located on a stack of branes and can be described *locally*.
- Abelian gauge groups in F-theory are intrinsically global objects.
- This is reflected in their geometric realization: They correspond to *global* sections of the elliptic fibration and generate the Mordell-Weil group $\mathrm{MW}(\mathcal{E})$ of the elliptic fiber $\mathcal{E}$.

Embedding the elliptic fiber

Let us now discuss Step I, choosing an ambient space F for the elliptic fiber, in more detail.
The relevant quantities depending on the choice of F are:

- Intersection numbers
- Map of the elliptic curve $\mathcal{E}$ to Weierstrass form
- Mordell-Weil group law on $\mathcal{E}$

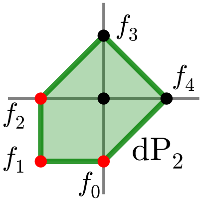


Note: Complementary approaches to systematically study $U(1)$s by [Morrison, Park], [Mayrhofer, Palti, Weigand] and [Cvetič, Klevers, Piragua]

Example dP_2, I

We begin with an example: $\mathcal{E}$ inside dP_2.

- 5 homogeneous coordinates f_i
- $V(f_i) \cap \mathcal{E} = 1$ for $i = 1, 2, 3$
  $V(f_i) \cap \mathcal{E} = 2$ for $i = 4, 5$
  $\Rightarrow$ 3 toric sections $V(f_1)$, $V(f_2)$, $V(f_3)$
- Defining equation for $\mathcal{E}$:



$$p = a_1 f_0^2 f_1^3 f_2^2 + a_2 f_0 f_1^2 f_2^2 f_3 + a_3 f_0^2 f_1^2 f_2 f_4 + a_4 f_1 f_2^2 f_3^2 + a_5 f_0 f_1 f_2 f_3 f_4 + a_6 f_0^2 f_1 f_4^2 + a_7 f_2 f_3^2 f_4 + a_8 f_0 f_3 f_4^2 = 0$$

After blowing down dP_2, p becomes a non-generic cubic inside $\mathbb{P}^2$

$$p\Big|_{f_0=f_2=1} = a_1 f_1^3 + a_2 f_1^2 f_3 + a_3 f_1^2 f_4 + a_4 f_1 f_3^2 + a_5 f_1 f_3 f_4 + a_6 f_1 f_4^2 + a_7 f_3^2 f_4 + a_8 f_3 f_4^2 = 0\,,$$

for which the map WF to Weierstrass form is known.

Example dP_2, II

Given the map $WF : (f_1, f_2, f_3, f_4, f_5) \mapsto (x, y, z) \in \mathbb{P}_{231}$, one can determine the toric Mordell-Weil group inside dP_2.

- Map $V(f_i) \xrightarrow{WF} \mathbf{q}_i$ to obtain the points $\mathbf{q}_i$ on $WF(\mathcal{E}) \subset \mathbb{P}_{231}$.
- Check their relations under the usual group law to find MW_T.

In this case take $V(f_0)$ as neutral element. Then $V(f_1) - V(f_0)$ and $V(f_2) - V(f_0)$ are independent with respect to the group law.

$$\Rightarrow \mathrm{MW}_T(\mathcal{E} \subset dP_2) = \mathbb{Z} \oplus \mathbb{Z}$$

Results for two-dimensional ambient spaces

The same procedure can be repeated for all other 15 toric varieties corresponding to reflexive polygons. One finds:

- All elliptic curve equations can be mapped into non-generic equations inside $\mathbb{P}^1 \times \mathbb{P}^1$, $\mathbb{P}_{112}$ or $\mathbb{P}^2$. [V. Braun '11]
  For all of these one knows the map to Weierstrass form.
- If non-trivial, MW_T is one of $\{\mathbb{Z}, \mathbb{Z}\oplus\mathbb{Z}, \mathbb{Z}\oplus\mathbb{Z}\oplus\mathbb{Z}, \mathbb{Z}_2, \mathbb{Z}\oplus\mathbb{Z}_2, \mathbb{Z}_3\}$.
  Non-trivial torsion in MW_T has recently been studied by [Mayrhofer, Morrison, Till, Weigand '14.05].
- Three elliptic fibers do not have toric sections (see recent papers [Braun, Morrison '14.01], [Morrison, Taylor '14.04], [Anderson, García-Etxebarria, Grimm, JK '14.06]).

Results for two-dimensional ambient spaces, II

One can now construct all tops for a given gauge group and compute the matter charges of the fundamental and antisymmetric representations with respect to the toric $U(1)$ gauge fields.

More importantly, one can generally show that in compactifications where the elliptic fiber is a hypersurface in some toric space, all antisymmetric representations have the same $U(1)$ charges in a given compactification.

$\Rightarrow$ Let's look at complete intersection elliptic fibers!

To my knowledge, there only a single example in the literature by [Mayrhofer, Palti, Weigand] so far.

Complete intersection fibers

Instead of reflexive polygons, consider nef partitions of reflexive polytopes that define a torus.

Three dimensions

There are $4,319$ reflexive three-dimensional polytopes. After modding out automorphisms, these have $3,134$ nef partitions.

Four dimensions

There are $473,800,776$ reflexive four-dimensional polytopes with an unknown number of nef partitions. [Kreuzer, Skarke]

Complete intersection fibers

Let us now try and repeat the same procedure as before. Naively, one observes:

1. Find toric sections and multisections. ✓
2. Find map to Weierstrass equation. ?
3. Find MW_T. ?

⇒ The difficult part is to find a map to Weierstrass form. For an arbitrary elliptic curve such a map is guaranteed to exist, but finding it is in general an open problem.

Exceptions: biquadric in $\mathbb{P}^3$, see for example [Esole, Fullwood, Yau '11] or biquadric inside $\mathbb{P}^3$ blown-up at three points in Cvetič, Klevers, Piragua, Song '13.10].

Complete intersection fibers - Weierstrass form

1. In 2d, `V. Braun` simplified the problem by finding a minimal set of equations into which all other could be mapped and found only 3 equations.
2. In 3d, finding the equations is computationally involved. After a few weeks, computer cluster finds $\mathcal{O}(40)$ equations. However, we have no Weierstrass maps for most of them. $\Rightarrow$ discard this approach.

Found new algorithm `[work in progress]`:

1. In principle works independently of the ambient space dimension (may take a while, though)
2. Embeds elliptic curves inside $\mathbb{P}_{231}$, $\mathbb{P}_{112}$, $\mathbb{P}^2$, $\mathbb{P}^3$.
3. Works for all but two examples in 3d.

Complete intersection fibers - Mordell-Weil groups

Given the Weierstrass map, one can determine all toric Mordell-Weil groups. One finds:

- MW_T is one of $\{\mathbb{Z}^{\oplus^i}$ with $i = 0, 1, 2, 3, 4$, $\mathbb{Z}^{\oplus^i} \oplus \mathbb{Z}_2$ with $i = 0, 1, 2, 3$, $\mathbb{Z}_3, \mathbb{Z}_4\}$.
- There are 310 nef partitions without sections.
- The two pathological cases have trivial toric Mordell-Weil group.

In principle, the same could be attempted in higher-dimensions. A general scan will probably take too long. However, we plan to implement the algorithm in the general version of Sage.

Complete intersection fibers - tops

Unfortunately, the results by Bouchard & Skarke apply to two-dimensional fibers. In order to *classify* all toric non-Abelian gauge group spectra one would need a similar list of tops. However, we do not yet have a generalization of their results.

Nevertheless, one can easily construct *some* $SU(5)$ tops. $\Rightarrow$ Use these to engineer models with differently charged $\mathbf{10}$ curves for model building purposes.

Complete intersection fibers - Summary

In summary, we find:

- The transition from hypersurface fibers to complete intersection fibers is a technical challenge
- Complete intersection fibers allow more general *toric* gauge groups, both with respect to the rank and the torsion part of the gauge group
- Work needs to be done in order to classify higher-dimensional tops.
- However, *some* $SU(5)$ tops can easily be found and (hopefully) be used to generate multiple $\mathbf{10}$ curves.

Outlook

- Implement functionality in Sage for everybody to use
- Use all of this machinery to do some concrete model building
- Understand the role of massive $U(1)$s
- Use these insights to (partially) classify toric elliptic Calabi-Yau fourfolds

Thank you!

Finding all fibrations

Let the fiber polygon have vertices $\mathbf{f}_1, \ldots, \mathbf{f}_r$, denote the base rays by $\mathbf{v}_1, \ldots, \mathbf{v}_s$ and place the non-Abelian singularity on $\mathbf{v}_1$. Take the top vertices to be τ_j.
Embed into higher-dimensional polytope via

$$\mathbf{f}_i \mapsto (\mathbf{f}_i, \mathbf{0}), \quad \mathbf{v}_1 \mapsto (\tau_j, \mathbf{v}_1), \quad \mathbf{v}_i \mapsto (\mathbf{n}_i, \mathbf{v}_i) \text{ for } i \neq 1\,. \tag{5}$$

The vectors $\mathbf{n}_i$ specify the embedding and $n-2$ of them can be set to zero to eliminate freedom in $GL(n-1, \mathbb{Z})$ transformations.
The convex hull of all points must not add additional points to the fiber polygon:
$\Rightarrow$ linear constraints for remaining $\mathbf{n}_i$

Flatness of the Fibration

If the fiber dimension varies, the fibration is called non-flat.
Phenomenologically, one wants to avoid these cases, as they give rise to infinite towers of fields.
Non-flat fibers have different origins depending on the codimension of the singular locus in the base.

- Codimension 2 (relevant for $n \geq 3$): Base *independent*, occur when top has interior facet points
- Codimension > 2 (relevant for $n \geq 4$): Base *dependent*.

Requiring flatness for $n \geq 4$ imposes additional linear constraints on the $\mathbf{n}_i$ and is *non-generic* in this sense. In particular, certain combinations of top and base are always non-flat.