Strings and Branes are Waves and Monopoles

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Outline

Motivation

One way of looking at Double Field theory is of a lift of the NS-NS sector of supergravity to a purely *geometric* theory. That is, it is a sort of Kaluza Klein theory that gives ordinary gravity and 2-form gauge theory under reduction. Its local symmetries must be a combination of diffeomorphisms with the gauge tranformations of the 2-form potentials and its action and equations of motion must contain the usual SUGRA ones once one removes dependences on any extra dimensions we have added.

Kaluza Klein modes

- Start with massless, thus null states in the full theory, with momentum directed in the extra dimensions
- These states from the perspective of the reduced theory have mass and charge
- The mass will be given by momentum in KK direction

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M-theory Example

- Null wave solution in M-theory gives D0-brane
- D0-brane is momentum mode in 11th direction
- Mass and charge given by momentum - BPS state

Standard Solutions

- wave

D0-Brane

$$
\begin{aligned}
\mathrm{d}s^2 &= -H^{-1}\mathrm{d}t^2 + H\left[\mathrm{d}z - (H^{-1}-1)\mathrm{d}t\right]^2 + \mathrm{d}\vec{y}^2_{(D-2)} \\
B_{\mu\nu} &= 0, \qquad\qquad e^{-2\phi} = e^{-2\phi_0}
\end{aligned}
$$

- F1-string

$$
\begin{aligned}
\mathrm{d}s^2 &= -H^{-1}\left[\mathrm{d}t^2 - \mathrm{d}z^2\right] + \mathrm{d}\vec{y}^2_{(D-2)} \\
B_{tz} &= -(H^{-1}-1), \qquad\qquad e^{-2\phi} = He^{-2\phi_0}
\end{aligned}
$$

- Harmonic Function

$$
H = 1 + \frac{h}{|\vec{y}_{(D-2)}|^{D-4}}, \qquad \nabla^2 H = 0
$$

Introduction to Double Field Theory

Novel formulation of string theory

- Bosonic NS-NS sector: $g_{\mu\nu}$, $B_{\mu\nu}$ and ϕ
- Makes $O(D, D; R)$ a manifest symmetry of the action
- Metric and B-field on equal footing - geometric unification

Double the dimension of space but require a global $O(D, D)$ structure

- $O(D, D)$ structure $\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Geometric Framework

Doubling the dimension of space to $2D$

- Introduce new coordinates $\tilde{x}_\mu$
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Unification of two concepts

- Metric and B-field $\rightarrow$ generalized metric
- Diffeos and gauge transformations $\rightarrow$ generalized diffeos
- Generated by generalized Lie derivative

The Doubled Formalism

Generalized coordinates

- Combine x^μ and $\tilde{x}_\mu$ into

$$X^M = (x^\mu, \tilde{x}_\mu)$$

- $\mu = 1, \ldots, D$ and $M = 1, \ldots, 2D$

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Generalized metric

- Combine metric $g_{\mu\nu}$ and Kalb-Ramond field $B_{\mu\nu}$ into

$$\mathcal{H}_{MN} = \begin{pmatrix} g_{\mu\nu} - B_{\mu\rho} g^{\rho\sigma} B_{\sigma\nu} & B_{\mu\rho} g^{\rho\nu} \\ -g^{\mu\sigma} B_{\sigma\nu} & g^{\mu\nu} \end{pmatrix}$$

- Rescale the dilaton $\quad e^{-2d} = \sqrt{g} e^{-2\phi}$

The DFT Action

The action integral

$$S = \int \mathrm{d}^{2D} X e^{-2d} R$$

The *generalized Ricci scalar*

$$\begin{aligned}R = &\frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL}\\ &+ 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN}\\ &- 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d\end{aligned}$$

Equations of Motion

Since $\mathcal{H}$ is constrained, get projected EoMs

$$P_{MN}{}^{KL} K_{KL} = 0$$

where

$$K_{MN} = \delta R / \delta \mathcal{H}^{MN}$$

$$P_{MN}{}^{KL} = \frac{1}{2}(\delta_M{}^{(K} \delta_N{}^{L)} - \mathcal{H}_{MP} \eta^{P(K} \eta_{NQ} \mathcal{H}^{L)Q})$$

Dilaton equation

$$R = 0$$

The DFT Wave Solution

$$X^M = (t, z, y^m, \tilde{t}, \tilde{z}, \tilde{y}_m)$$

Generalized metric

$$\begin{aligned}
\mathrm{d}s^2 &= \mathcal{H}_{MN}\mathrm{d}X^M\mathrm{d}X^N \\
&= (H-2)\left[\mathrm{d}t^2 - \mathrm{d}z^2\right] - H\left[\mathrm{d}\tilde{t}^2 - \mathrm{d}\tilde{z}^2\right] \\
&\quad + 2(H-1)\left[\mathrm{d}t\mathrm{d}\tilde{z} + \mathrm{d}\tilde{t}\mathrm{d}z\right] \\
&\quad + \delta_{mn}\mathrm{d}y^m\mathrm{d}y^n + \delta^{mn}\mathrm{d}\tilde{y}_m\mathrm{d}\tilde{y}_n
\end{aligned}$$

Rescaled dilaton

$$d = const.$$

The DFT Wave Solution

Properties

- Null
- Carries momentum in $\tilde{z}$ direction
- Interprete as null wave in DFT
- Smeared over dual directions $\rightarrow$ obeys section condition

Reducing the Solution

Examine from the point of view of the reduced theory

- Get fundamental string solution
- Extended along z
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If z and $\tilde{z}$ are exchanged

- Get pp-wave in z direction
- Expected as wave and string are T-dual

Key Result

The fundamental string is a massless wave in doubled space with momentum in a dual direction.

Goldstone Mode Analysis

Zero modes

- Symmetry breaking
- Moduli $\rightarrow$ collective coordinates
- Generated by large gauge transformations / diffeos
- Make local on worldvolume $\rightarrow$ get zero modes

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Number of modes

- String: $D-2$ modes
- Doubled wave / string: ???

Constructing the Zero Modes

Transformations of $\mathcal{H}$ and d

$$h_{MN} = \mathcal{L}_\xi \mathcal{H}_{MN} \qquad\qquad \lambda = \mathcal{L}_\xi d$$

- gauge parameter $\xi^M = (0, H^\alpha \hat{\phi}^m, 0, H^\beta \hat{\tilde{\phi}}_m)$
- $\hat{\phi}^m$ and $\hat{\tilde{\phi}}_m$ are constant moduli

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Allow dependece on $x^a = (t, z)$ to get zero modes

$$\hat{\phi}^m \to \phi^m(x) \qquad \hat{\tilde{\phi}}_m \to \tilde{\phi}^m(x)$$

Equations of motion

- Insert into DFT EoMs (two derivatives, first order)
- Find $\Box\phi = 0$ and $\Box\tilde{\phi} = 0$
- Also get self-duality relation for $\Phi^M = (0, \phi^m, 0, \tilde{\phi}_m)$

$$\mathcal{H}_{MN}\mathrm{d}\Phi^N = \eta_{MN} \star \mathrm{d}\Phi^N$$

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Duality symmetric string in doubled space (Tseytlin)

- Can be written as (anti-)chiral equation for $\psi_\pm = \phi \pm \tilde{\phi}$

$$\mathrm{d}\psi_\pm = \pm \star \mathrm{d}\psi_\pm$$

Summary

Wave solution in DFT

- Solution unifies pp-wave and F1-string (T-duals)
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Goldstone modes

- Find chiral zero modes of the wave solution
- Gives the correct degrees of freedom for the string in doubled space with manifest $O(d,d)$

Extension to M-Theory

Extended theories

- Make U-duality manifest
- Include brane wrapping directions
- Geometrically unify metric and C-field(s)

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Example: $SL(5)$

- Duality group for M-theory in 4 dimensions x^μ
- Combine with 6 wrapping directions $y_{\mu\nu}$
- Wave in extended space gives M2-brane

Extend space to include *dual* membrane winding modes, $y_{\mu\nu}$ along with usual x^μ coordinates. No longer a simple doubling. Now the generalised tangent space is:

$$\Lambda^1(M) \oplus \Lambda^{*2}(M)\,. \tag{1}$$

The metric for the Sl_5 case is given by:

$$M_{IJ} = \begin{pmatrix} g_{ab} + \frac{1}{2}C_a{}^{ef}C_{bef} & \frac{1}{\sqrt{2}}C_a{}^{kl} \\ \frac{1}{\sqrt{2}}C^{mn}{}_b & g^{mn,kl} \end{pmatrix}, \tag{2}$$

where $g^{mn,kl} = \frac{1}{2}(g^{mk}g^{nl} - g^{ml}g^{nk})$ and has the effect of raising an antisymmetric pair of indices.

We can construct the Lagrangian with all the right properties:

$$
\begin{aligned}
L \;&=\; \Bigg(\frac{1}{12}M^{MN}(\partial_M M^{KL})(\partial_N M_{KL}) - \frac{1}{2}M^{MN}(\partial_N M^{KL})(\partial_L M_{MK}) \\
&+\; \frac{1}{12}M^{MN}(M^{KL}\partial_M M_{KL})(M^{RS}\partial_N M_{RS}) \\
&+\; \frac{1}{4}M^{MN}M^{PQ}(M^{RS}\partial_P M_{RS})(\partial_M M_{NQ})\Bigg)
\end{aligned}
$$

The SL5 Wave Solution

Properties

- Wave solution as before with:
- momentum in y_{zw} direction
- this is a null wave in extended geometry
- Smeared over dual direction obeying section condition
- Interpretation in reduced theory is as a membrane stretched over the zw directions!

Other duality groups. eg $SO(5,5)$

$$\Lambda^1(M) \to \Lambda^{*2}(M) \oplus \Lambda^{*5}(M) \tag{3}$$

So we have coordinates

$$Z^I = (x^a, y_{ab}, y_{abcde}) \tag{4}$$

with $a = 1..5$, $ab = 6..15$, $abcde = 16$. Thus the space is 16 dimensional corresponding to the **16** of SO(5,5). The y_{abcde} correspond to fivebrane winding mode.

The SO(5,5) generalized metric is (upper case latin indices run from 1 to 16):

$$M_{IJ} = \begin{pmatrix} g_{ab} + \frac{1}{2}C_a{}^{ef}C_{bef} + \frac{1}{16}X_aX_b & \frac{1}{\sqrt{2}}C_a{}^{mn} + \frac{1}{4\sqrt{2}}X_aV^{mn} & \frac{1}{4}X_a \\ \frac{1}{\sqrt{2}}C^{kl}{}_b + \frac{1}{4\sqrt{2}}V^{kl}X_b & g^{kl,mn} + \frac{1}{2}V^{kl}V^{mn} & \frac{1}{\sqrt{2}}V^{kl} \\ \frac{1}{4}X_b & \frac{1}{\sqrt{2}}V^{mn} & 1 \end{pmatrix} \quad (5)$$

where we have defined:

$$V^{ab} = \frac{1}{6}\eta^{abcde}C_{cde}\,, \quad (6)$$

with η^{abcde} being the totally antisymmetric permutation symbol (it is only a tensor density and thus distinguished from the usual ϵ^{abcde} symbol) and

$$X_a = V^{de}C_{dea}\,. \quad (7)$$

We can attempt to reconstruct the dynamical theory out of this generalized metric. We have the following Lagrangian with manifest $SO(5,5)$,

$$\begin{aligned} L = \frac{1}{16} M^{MN}(\partial_M M^{KL})(\partial_N M_{KL}) - \frac{1}{2} M^{MN}(\partial_N M^{KL})(\partial_L M_{MK}) \\ + \frac{3}{128} M^{MN}(M^{KL}\partial_M M_{KL})(M^{RS}\partial_N M_{RS}) \\ - \frac{1}{8} M^{MN} M^{PQ}(M^{RS}\partial_P M_{RS})(\partial_M M_{NQ}) \end{aligned} \tag{8}$$

where $\partial_M = \left(\frac{\partial}{\partial x^a}, \frac{\partial}{\partial y_{ab}}, \frac{\partial}{\partial z}\right)$.

By now it is no surprise that a null wave in the y_{abcde} direction is an M5-brane stretched in the $abcde$ directions.
But is there another way to view this?

Fivebranes and monopoles

In Kaluza-Klein theory, once we have waves along the KK directions and see that these allow electric charges we can ask how to produce a monopoles. The gives us the Kaluza-Klein monopole, essentially a nontrivial bundle that is an S^1 over S^2 with total space S^3.
In terms of M-theory, this is the D6 brane.
Can we do the same trick for DFT or other extended geometries and find monopole like solutions?

Yes:
in DFT the monopole whose KK circle is $\tilde{z}$ is an NS5-brane from the reduced perspetive.
In exceptional case, the monopole whose KK circle is in the y_{ab} direction is an M5-brane.
The monopole whose KK circle is in the y_{abcde} direction is the M2-brane.

Summary

In DFT and other extended exceptional geometries, waves with momentum along the novel directions are strings or branes.

A monopoles whose KK direction is along one of those novel directions is also a brane but S-dual to the one given by a wave with monentum in those directions.

Thus all branes in excpetional geomerties are simulataneously waves and monopoles.

Other Solutions

- D0-brane

Back to Standard Solutions

$$ds^2 = -H^{-1}dt^2 + d\vec{y}^2_{(d-1)}, \qquad A_t = -(H^{-1} - 1)$$

- KK-monopole

Back to Other Solutions

$$ds^2 = -dt^2 + d\vec{x}^2_{(d-5)} + H^{-1}\left[dz + A_i dy^i\right]^2 + H d\vec{y}^2_{(3)}$$
$$\partial_{[i}A_{j]} = \frac{1}{2}\epsilon_{ij}{}^k\partial_k H, \qquad e^{-2\phi} = e^{-2\phi}$$

- NS5-brane

$$ds^2 = -dt^2 + d\vec{x}^2_{(d-5)} + H d\vec{y}^2_{(4)}, \quad B_{zi} = A_i, \quad e^{-2\phi} = H^{-1}e^{-2\phi_0}$$