A Route to Non-Perturbative String Theory[1]

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What is String Theory?

Perturbatively, our best understanding is in terms of Superstring Perturbation Theory (SPT) (type IIA & IIB, Heterotic SO(32) & $E_8 \times E_8$, type I). More generally, most general 2D CFT with zero central charge and possibly a negative norm field (time) (but unitary otherwise). These ingredients have led to the notion of a "String Landscape", (where we imagine the 2D CFT's embedded in space of most general 2D QFT).

There is also Superstring Field Theory (SFT) (SPT reorganised to look like a QFT), but still perturbative. It does not fully utilise worldsheet duality and so requires "artificial" partitioning of moduli space making (even simple) calculations very difficult. For certain questions SFT is nevertheless powerful (gauge invariance, mass renormalisation, vacuum shifts).

Non-perturbatively (in some special cases) we have holographic descriptions (AdS/CFT), but these seemingly require special (unrealistic) boundary conditions and local bulk physics hard to probe. There are also toy models (A & B model topological strings) and also matrix models (BFSS).

$\Rightarrow$ *A satisfactory bulk definition of String Theory is lacking*

What is String Theory?

What is String Theory?

Since this is a (somewhat) ill-defined question, we will make it sharper by considering an analogy to QFT. In QFT we often define a theory by a path integral (+Wilson), so would like something analogous to this:

A Non-Perturbative Path Integral Definition of String Theory

HERE: we present a proposal for a non-perturbative (path integral) definition of String Theory. E.g., I will discuss a sense in which an answer to the above question might best be thought of as emerging from the

partition function of the most general 2D QFT on S_2

I will discuss the derivation, and our approach will also involve a conjecture (introducing RG flow into String Theory).

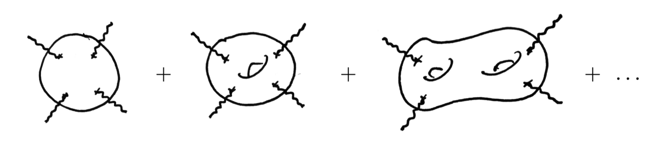
It is not clear whether the construction will be useful in any meaningful way, but I think it is correct to regard it as a step in the right direction. E.g., should be able to *derive* old and *new* dualities (and I will discuss how these might arise), leading to a deeper understanding of string landscape.

As a by-product, we will also understand how to interpret NLSM's and geometry (within non-perturbative string theory), discuss some of their (deep) limitations, derive the Fischler-Susskind mechanism, and discuss bootstrap methods in this context. We focus on the *bosonic string* for simplicity (but in general backgrounds).

The Textbook Approach to SPT

In SPT we usually:
(1) choose a 2D CFT (with $c = 0$);
(2) construct vertex operators, $\hat{\mathscr{V}}_j$, ($\hat{\mathscr{V}}_j \in \ker Q_B/\mathrm{Image}\, Q_B$);
(3) compute correlation functions of $\mathfrak{n}$ vertex operators on a genus-$\mathfrak{g}$ Riemann surface, $\Sigma_\mathfrak{g}$;
(4) integrate over moduli space, $\mathcal{M}_{\mathfrak{g},\mathfrak{n}}$, leading to amplitudes, $\mathcal{A}_{\mathfrak{g},\mathfrak{n}}$;
(5) sum over topologies, $\sum_\mathfrak{g} \mathcal{A}_{\mathfrak{g},\mathfrak{n}}$.

Schematically:



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More precisely (denoting by $g_s = e^{\Phi}$ the string coupling):

$$S = \sum_{\mathfrak{g}=0}^{\infty} g_s^{2\mathfrak{g}-2} \int_{\mathcal{M}_{\mathfrak{g},\mathfrak{n}}} \frac{d^{2m}\tau}{n_R} \Big\langle \prod_{k=1}^{m} B_k \tilde{B}_k \prod_{i=1}^{\mathfrak{n}} \hat{\mathscr{V}}_i \Big\rangle_{\Sigma_{\mathfrak{g}}}$$

<u>note:</u> $B_k \tilde{B}_k$ factors encode path integral measure and specify gauge slice in moduli space, $\mathcal{M}_{\mathfrak{g},\mathfrak{n}}$, with coordinates $\{\tau^k, \bar{\tau}^k\}$, $k = 1, \ldots, m$, and

$$m \equiv \dim_{\mathbb{C}} \mathcal{M}_{\mathfrak{g},\mathfrak{n}} = 3\mathfrak{g} - 3 + \mathfrak{n}$$

Since the early days (Shenker 1990) it has been known that this approach leads to an asymptotic (non-Borel summable) series, so it is hard to make sense of non-perturbatively (Zograf, Mirzhakani),

$$\mathcal{A}_{g,n} \sim (2g - 3 + n)!$$

So we need new ideas, we adopt a different approach. Recall that sums and integrals generally do not commute (when conditionally convergent),

$$\int dt \sum_{g} A_g(t) \neq \sum_{g} \int dt\, A_g(t),$$

and an asymptotic series often arises from (incorrectly) interchanging one or more sums and integrals.

There is a large and vibrant community trying to understand how to sum asymptotic series (aka resurgence, with dedicated workshops), but the above suggests an alternative possibility: *the order* of doing things in textbook SPT might be wrong.

Question: can we interchange the (aforementioned) order of calculations in textbook SPT?

The relevant sum in SPT is the sum over topologies ($\sum_{g=0}^{\infty}$) and the relevant integral is the integral over moduli, $\int_{\mathcal{M}_{g,n}}$. This suggests we try to *first* sum over topologies and *then* integrate over moduli.

To make sense of this (seemingly naive) statement a *good* first step is to understand how to cut and glue worldsheet path integrals. An additional clue is that the *framework* for a non-perturbative definition of String Theory should be independent of the underlying 2D CFT: we should search the CFT toolbox for statements that are independent of the specific CFT.

Outline:

- Riemann Surfaces and CFT
- Cutting and Gluing (Worldsheet) Path Integrals
- Moduli Deformations
- Summing Loops
- Proposal for Non-Perturbative String Theory
- Duality
- Fischler-Susskind Mechanism

Riemann Surfaces and CFT

A Riemann surface, Σ_g, is a complex manifold $\Rightarrow$ completely specified by a choice of charts, $\{(U_j, z_j)\}$, (s.t. $\cup_j U_j = \Sigma_g$ and $z_j : U_j \to \mathbb{C}$) with holomorphic transition functions, f_{ij}, on chart overlaps:

$$\boxed{\text{On} \quad U_i \cap U_j : \qquad z_i = f_{ij} \circ z_j}$$

On any one such chart, (U_j, z_j), we can go to conformal gauge, where $ds^2 \propto dz_j d\bar{z}_j$. Choose a 2D CFT (with $c = 0$). Then the various primaries of our CFT can be mode-expanded (in coordinates z_j around $z_j = 0$),

$$\phi_j(z_j) = \sum_n \frac{\phi_n^{(z_j)}}{z_j^{n+h}} \qquad \leftrightarrow \qquad \phi_n^{(z_j)} = \oint_C \frac{dz_j}{2\pi i z_j} z_j^{n+h} \phi_j(z_j).$$

For example, the modes $\phi_n^{(z_j)}$ can be identified with: ghost modes, $b_n^{(z_j)}$, $c_n^{(z_j)}$, matter modes, $\alpha_n^{(z_j)}$, Virasoro generators, $L_n^{(z_j)}$, BRST charge, $Q_B^{(z_1)}$, etc., with appropriate commutation relations, etc.

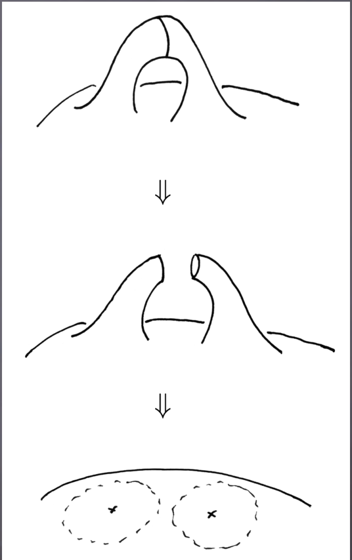
Cutting and Gluing (Worldsheet) Path Integrals

Consider a Riemann surface, Σ_g, with fixed complex structure.

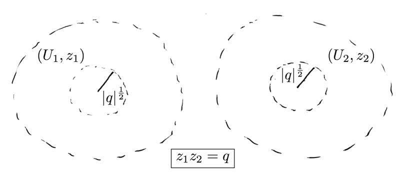
Pick a canonical intersection homology basis, $\{A_I, B_I\}$, $I = 1, \ldots, g$, and consider an A_I-cycle handle.

Cut open the path integral across this A_I-cycle. Summing over all states on the resulting boundary circles reproduces the original Riemann surface.

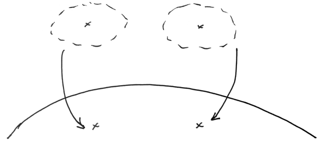
Use operator/state correspondence to map these states to bilocal operators, $\mathscr{A}_a^{(z_1)}(0)$, $\mathscr{A}^a_{(z_2)}(0)$, using charts (U_1, z_1), (U_2, z_2), with identifications $z_1 \sim z_2$ when $z_1 z_2 = q$.



More precisely, we map states to local operators, $\mathscr{A}_a^{(z_1)}(0)$, $\mathscr{A}^a_{(z_2)}(0)$, and when, e.g., $z_1 < |q|^{\frac{1}{2}}$ use z_2 coordinate by identifying $z_1 \sim z_2$ with transition function $z_1 z_2 = q$ (and similarly for $z_2 < |q|^{\frac{1}{2}}$ use z_1 coordinate).

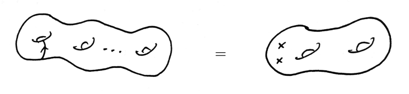


We proceed similarly for all g A_I-cycles. Then map these $2g$ charts onto the (almost) global (U, z) chart of (the stereographically projected) S_2 using $\mathrm{SL}(2, \mathbb{C})$ transformations,



We rescale z_1, z_2 such that $|q| \leq 1$.

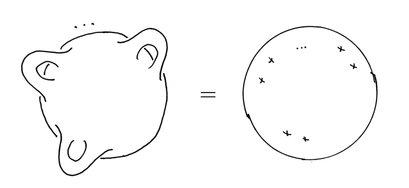
Schematically, we effectively end up with a correspondence (for each of the A_I cycles):



and this is exact (other channels come from infinite sums and OPE associativity). In terms of correlation functions,

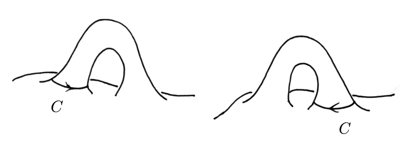
$$e^{-\chi(\Sigma_{\mathrm{g}})\Phi}\Big\langle \dots \Big\rangle_{\Sigma_{\mathrm{g}}} \quad = \quad e^{-\chi(\Sigma_{\mathrm{g}-1})\Phi}\Big\langle \sum\!\!\!\!\!\!\int_a \mathscr{A}_a^{(z_1)}\mathscr{A}^a_{(z_2)}\dots \Big\rangle_{\Sigma_{\mathrm{g}-1}}$$

enabling us to represent correlation functions on Σ_{g} in terms of correlation functions on $\Sigma_0 \equiv S_2$, for all g,



We determine the operators, $\mathscr{A}_a^{(z_1)}, \mathscr{A}^a_{(z_2)}$, and corresponding interpretation for $\sum\!\!\!\!\!\!\int_a$, by the following consistency conditions:

1) We should be able to deform all mode contours, C, across a handle without obstruction:



In terms of the bilocal operator insertions this is equivalent to demanding that, for *any* mode operators, $\phi_n^{(z_j)}$, (such that the BRST charge, Virasoro generators, etc.):

$$\boxed{\Big(\phi_n^{(z_1)} - (-)^h q^n \phi_{-n}^{(z_2)}\Big) \sum\!\!\!\!\!\!\!\int_a q^{h_a} \bar{q}^{\tilde{h}_a} \mathscr{A}_a^{(z_1)} \mathscr{A}^a_{(z_2)} = 0}$$

which is an operator statement and so holds inside the path integral, and recall that:

$$\phi_n^{(z_1)} = \oint_C \frac{dz_1}{2\pi i z_1}\, z_1^{n+h} \phi_1(z_1).$$

2) The phase of chart coordinate, z_j, is not globally defined (the obstruction being the Euler number), so will always take this phase to be integrated. E.g., demand that $(L_0^{(z_1)} - \tilde{L}_0^{(z_1)}) \cdot \mathscr{A}_a^{(z_1)} = 0$.

3) Since the $\mathscr{A}_a^{(z_1)}$ represent string states in loops they are offshell, hence not primaries (nor $\mathrm{SL}(2,\mathbb{C})$ primaries) or BRST invariant. We must demand that BRST-exact contributions decouple from cuts.

4) Consistent factorisation requires the normalisation be chosen such that (with transition functions $z_1 z_2 = z_2 z_3 = 1$):

$$\mathscr{A}_c^{(z_1)} = \sum\!\!\!\!\!\!\int_a \mathscr{A}_a^{(z_1)} e^{-2\Phi} \left\langle \mathscr{A}_{(z_2)}^{a} \mathscr{A}_c^{(z_3)} \right\rangle_{S_2}$$

These conditions can easily be solved in a coherent state basis, where the $\{a\}$ are comprised of continuous quantum numbers (in addition to momenta and possibly discrete quantum numbers).

(The operator $\mathscr{A}_{(z_2)}^{a}$ is the *dual* of $\mathscr{A}_a^{(z_1)}$.)

For illustration, the chiral half of $\mathscr{A}_a^{(z_1)}$ in a coherent state basis in the standard bosonic string reads:

$$
\begin{aligned}
:\exp\Bigg[&\sum_{n\geq 2} e^{in\phi}\Big(\frac{\mathbf{b}_n}{(n-2)!}\partial^{n-2}b\Big)e^{-inq\cdot x_{\rm L}}\\
&+\sum_{n\geq 0} e^{in\phi}\Big(\frac{\mathbf{c}_n}{(n+1)!}\partial^{n+1}c\Big)e^{-inq\cdot x_{\rm L}}\\
&+\sum_{n\geq 1}\frac{1}{n}e^{in\phi}\frac{i\mathbf{a}_n}{(n-1)!}\cdot\sqrt{\frac{2}{\alpha'}}\partial^n x_{\rm L}\, e^{-inq\cdot x_{\rm L}}\Bigg]\\
&\times\big(ce^{ip\cdot x_L}+e^{i\phi}\mathbf{b}_1\, e^{i(p-q)\cdot x_{\rm L}}\big)(0):_{z_1}
\end{aligned}
$$

After inclusion of the anti-chiral half, an integral over the phase ϕ, and an overall coupling g_D, we obtain $\mathscr{A}_a^{(z_1)}$. Given p^μ, the quantity q^μ is such that $p\cdot q=1$ and $q^2=0$. In this basis,

$$
\sum\!\!\!\!\!\!\!\int_a = \frac{\alpha'}{8\pi i}\int\frac{d^Dp}{(2\pi)^D}\int d\mu_{\mathbf{abc}}\Big(\equiv\frac{\alpha'}{8\pi i}\sum\!\!\!\!\!\!\!\int_a{}'\Big)
$$

Also, ghost number is *indefinite* and *all* $\mathscr{A}_a^{(z_1)}$ have: $\boxed{h_a=\frac{\alpha'}{4}p^2-1}$.

Consistency check: we show that BRST-exact states decouple from cuts, that the basis is correctly normalised, and that it provides a resolution of unity, by computing the Virasoro-Shapiro amplitude by gluing two 3-pt amplitudes, $S_{S_2} = S_{S_2}^{\infty_s} + S_{S_2}^{\infty_u}$, with:

$$S_{S_2}^{\infty_s} = g_D^4 \sum\!\!\!\!\!\!\!\int_a e^{-2\Phi} \Big\langle \prod_{j=1,2} :\tilde{c}ce^{ik_j\cdot x}(z_j,\bar{z}_j): \mathscr{A}_a^{(z)} \Big\rangle_{S_2}$$
$$\times \int d^2q\, q^{h_a-1}\bar{q}^{\tilde{h}_a-1} e^{-2\Phi} \Big\langle \big[b_0\tilde{b}_0\cdot \mathscr{A}^a_{(u)}\big] \prod_{j=3,4} :\tilde{c}ce^{ik_j\cdot x}(u_j,\bar{u}_j): \Big\rangle_{S_2'}$$

By worldsheet duality, the $s+u$ channels reproduce the full amplitude. Computing the imaginary part (away from t-channel poles and writing $S = i(2\pi)^D\delta^D(k)\mathcal{A}$) leads to:

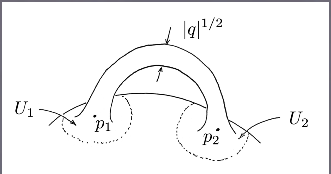
$$\mathrm{Im}\,\mathcal{A}_{S_2}(s,t,u) =$$
$$= \Big(\frac{8\pi g_D}{\alpha'}\Big)^2 \sum_{n=0}^{\infty}\Big(\frac{\Gamma(2+n+\frac{\alpha'}{4}t)}{\Gamma(n+1)\Gamma(2+\frac{\alpha'}{4}t)}\Big)^2 \pi\delta\big(s-\tfrac{4}{\alpha'}(n-1)\big) + s\leftrightarrow u,$$

in precise agreement with the known result.

Moduli Deformations

Everything so far has been for fixed complex structure moduli.

To incorporate moduli deformations we need to pick a gauge slice in moduli space, $\mathcal{M}_{g,n}$. We do so by associating 3 of the $3g-3+n$ moduli to every handle:



This may accomplished by attaching 3 terms of the form $B_k\tilde{B}_k$ to every bilocal operator, where:

$$B_k = \frac{1}{2\pi i}\sum_{(ij)}\int_{C_{ij}}\left(dz_i\frac{\partial z_i}{\partial\tau^k}\Big|_{z_j} b_{z_iz_i} - d\bar{z}_i\frac{\partial\bar{z}_i}{\partial\tau^k}\Big|_{\bar{z}_j}\tilde{b}_{\bar{z}_i\bar{z}_i}\right)$$

The sum is over overlapping patches $U_i\cap U_j$ and the contour C_{ij} is along the $U_i\cap U_j$ (in a counter-clockwise sense from viewpoint of U_i). For position moduli the relevant transition functions map the centred discs to the global (U,z) chart of S_2, and for pinch moduli the transition functions can be identified with $z_1z_2=q$.

More explicitly, for every position modulus, u, the infinitesimal SL(2,$\mathbb{C}$) transition functions (mapping charts (U_j, z_j) to global S_2 z coordinate) read (Nelson 1989):

$$z = z_j + u + \frac{1}{8}R_{(2)}\bar{u}z_j^2 + \mathcal{O}(u^2)$$

leading to an insertion into the path integral (Polchinski 1988):

$$B_u \to \hat{b}_{-1} = b_{-1} + \frac{1}{8}R_{(2)}\tilde{b}_1 + \ldots$$

(The shape of the pinch changes as it is translated across the worldsheet due to curvature.) More generally, we integrate this SL(2,$\mathbb{C}$) to obtain finite transition functions. This leads to the *integrated vertex operator picture*. Similarly, for every pinch modulus q we obtain an insertion into the path integral:

$$B_q\tilde{B}_q \to \frac{b_0\tilde{b}_0}{q\bar{q}}$$

The remaining $n-3$ moduli are precisely the number of moduli of a sphere with n vertex operators.

Summarising, to include moduli deformations we include measure contributions, $\{B_k, \tilde{B}_k\}$, by replacing every bilocal operator,

$$\sum\!\!\!\!\!\!\int_a \mathscr{A}_a^{(z_1)} \mathscr{A}^a_{(z_2)},$$

by:

$$\sum\!\!\!\!\!\!\int_a' \int_{|q|<1} d^2q \underbrace{\Big(\int d^2u \sqrt{g}\, \hat{b}_{-1} \hat{\tilde{b}}_{-1} \cdot \mathscr{A}_a^{(z_1)} \Big)}_{\equiv \int_u \mathscr{A}'_a} \underbrace{\Big(\frac{\alpha'}{8\pi i}\, q^{h_a - 1} \bar{q}^{\tilde{h}_a - 1} \Big)}_{\equiv \mathscr{G}_{\mathscr{A}}}$$

$$\times \underbrace{\Big(\int d^2v \sqrt{g}\, \hat{b}_{-1} \hat{\tilde{b}}_{-1} b_0 \tilde{b}_0 \cdot \mathscr{A}^a_{(z_2)} \Big)}_{\equiv \int_u \mathscr{A}'^a}$$

To cover moduli space once note that we already fixed invariance under global diffeomorphisms (by picking a homology basis). The u^k, v^k moduli integrals are then such that for fixed q^k, we integrate over all handle locations on S_2 such that no two handles overlap, and then integrate over $|q^k| < 1$.

Summing Loops

According to the above, genus-g amplitudes take the form:

$$e^{-2\Phi}\Big\langle \sum_{\mathrm{g}=0}^{\infty}\prod_{k=1}^{\mathrm{g}}\Big[\sum\!\!\!\!\!\!\int_a^{\prime}\int_{|q^k|<1} d^2q^k\Big(\int_{u^k}\mathscr{A}^{\prime}_a\Big)\mathscr{G}_{\mathscr{A}}\Big(\int_{v^k}\mathscr{A}^{\prime\,a}\Big)\Big]\dots\Big\rangle_{S_2}$$

The OPE between any two operators has a finite radius of convergence (corresponding to closest other operator). In order to sum over topologies we wish to decouple the integration domains and do so by allowing the range of operator insertions to range of the full S_2, and hence explicitly introduce (field theory) infinities by replacing the above by:

$$e^{-2\Phi}\Big\langle \sum_{\mathrm{g}=0}^{\infty}\frac{1}{\mathrm{g}!}\Big[\sum\!\!\!\!\!\!\int_a^{\prime}\Big(\int_u\mathscr{A}^{\prime}_a\Big)\underbrace{\Big(\int_{|q|<1}d^2q\,\mathscr{G}_{\mathscr{A}}\Big)}_{G_{\mathscr{A}}}\Big(\int_v\mathscr{A}^{\prime\,a}\Big)\Big]^{\mathrm{g}}\dots\Big\rangle_{S_2}$$

To reproduce the original amplitude we must subtract these infinities and compute observables that are independent of the subtraction scheme.

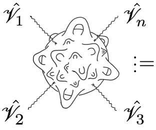
Then, summing over g, the *bilocal operators exponentiate* and we reach a non-perturbative representation for the full string amplitude.

This exponentiation leads to:

$$S = e^{-2\Phi}\Big\langle \exp\Big[{\sum_a}' \Big(\int_u \mathscr{A}'_a\Big) G_{\mathscr{A}} \Big(\int_v \mathscr{A}'^{\,a}\Big)\Big] \prod_{k=1}^{n-3} B_k \tilde{B}_k \prod_{j=1}^{n} \hat{\mathscr{V}}_j \Big\rangle_{S_2}$$

Next introduce a path integral representation for the exponential of the bilocal operator, leading to *non-perturbative string amplitudes*:

$$\sum_{\text{topologies}}$$



$$:= \int \mathcal{D}(\bar{\phi}, \phi)\, e^{\frac{i}{g_D^2}{\sum_a}' \bar{\phi}_a G_{\mathscr{A}}^{-1} \phi^a} \frac{1}{g_s^2} \Big\langle e^{\Psi} \prod_{k=1}^{n-3} \int_k B_k \tilde{B}_k \prod_{i=1}^{n} \hat{\mathscr{V}}_i \Big\rangle_{S_2}$$

with Ψ the integrated-picture string field (generalising the NLSM):

$$\boxed{\Psi \equiv {\sum_a}' \int d^2u \sqrt{g} \Big(\phi^a \mathscr{A}'^{(z)}_a + \bar{\phi}_a \mathscr{A}'^{\,a}_{(z)} \Big)}$$

Duality

It was important in the above that we *do not* truncate the sum over the spectrum $\{a\}$ in

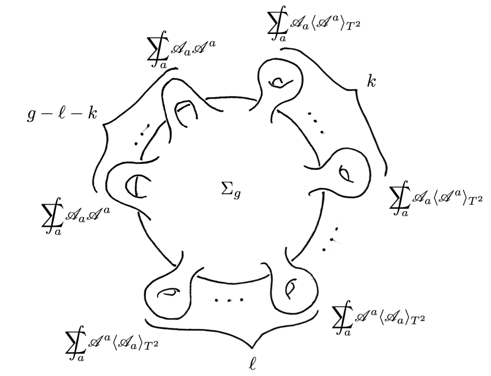
$$\Psi = \sum_a{}' \int d^2u\sqrt{g}\Big(\phi^a \mathscr{A}_a'^{(z)} + \bar{\phi}_a \mathscr{A}_{(z)}'^{\,a}\Big)$$

because otherwise we violate worldsheet duality (akin to Virasoro-Shapiro, where $s+u$ channels also contain t channels provided we sum over all states). Note also that Ψ contains (in an appropriate basis) the NLSM action:

$$\Psi \sim \int d^2u\,\Big(\sqrt{g}\,T(x) + \partial x^\mu \bar{\partial} x^\nu \big(G_{\mu\nu}(x) + B_{\mu\nu}(x)\big) + R_{z\bar{z}}\Phi(x) + \dots\Big),$$

so truncating the massive contributions discards *dual massless contributions* contained in the infinite sum over a, and *truncating to usual NLSM violates duality* and cannot be expected to be correct non-perturbatively.

We can expose dual contributions by cutting open the original path integral across different cycles (as opposed to A_I cycles as we did above),



The result again exponentiates and the "tadpole" contributions effectively shift the target space (background) fields, $\phi_a \to \phi_a + \langle \mathscr{A}_a \rangle_{T^2}$. We can also cut across only tadpole cycles, and then there are no ϕ_a target space fields emerging. All such choices are equivalent due to duality (associativity of the OPE and modular invariance).

Discussion 1

1) We have shown that changing the order of calculation of textbook SPT one can reach a non-perturbative definition of String Theory (in fact various equivalent representations related by duality). The result is independent of the underlying CFT.
2) In order to do so we have introduced field theory infinities, that are to be subtracted in the standard manner. Renormalisation-scheme independent quantities should be independent of the subtraction prescription, so should reproduce the original String Amplitudes.
3) One should not expect to be able to make exact statements about string theory using NLSM. (Truncating to NLSM violates duality, even if only interested in massless sector.)
4) Cutting open the path integral along different cycles and proceeding as discussed (leading to equivalent, dual, path integrals) might allow for a bootstrap approach.

Discussion 2

5) Worldsheet and spacetime instantons, solitons, etc., should all be there (the latter coming from saddle points of the target space path integral and the former included explicitly in the sum over a)
6) Non-Perturbative String Theory is an entirely new beast: we can either think of the e^{Ψ} insertion as the most general 2D QFT (Ψ is *not* Weyl-invariant as it originates from offshell loops) on S_2, or as a coherent state insertion
7) So a good first step towards unravelling non-perturbative string theory is to familiarise oneself with computing string amplitudes with coherent states
8) Making use of 'complete normal ordering' (Ellis,Mavromatos,DS 2016), one can also derive the Fischler-Susskind mechanism (ensuring one is integrating around the true quantum vacuum)

Open questions: What can be said about renormalisability of the resulting prescription? What about gauge invariance? (L_∞ algebra useful?) Generalisation to open strings? Generalisation to the superstring?

thank you!