Interactions for multiple spin-2 fields

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Workshop “Geometry and Strings”
Schloss Ringberg, 26/07/18

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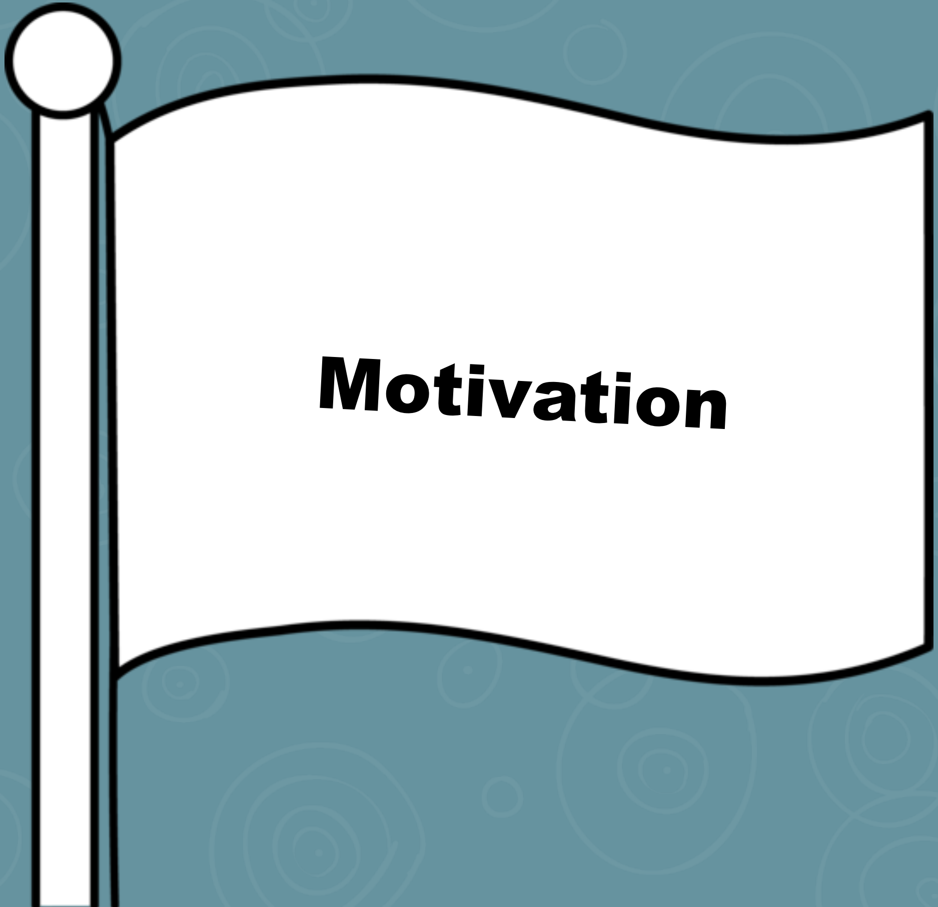


Motivation


http://southernskeptic.com
Gravity
Just a theory.

Consistent Field Theories

Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson ϕ

Spin 1/2: leptons, quarks ψ^a

Spin 1: gluons, photon, W- & Z-boson A_μ

Spin 2: graviton $g_{\mu\nu}$

Consistent Field Theories

Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson ϕ

Spin 1/2: leptons, quarks ψ^a

Spin 1: gluons, photon, W- & Z-boson A_μ

massless & massive

Spin 2: graviton $g_{\mu\nu}$

MASSLESS !

Consistent Field Theories

Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson ϕ

Spin 1/2: leptons, quarks ψ^a

Spin 1: gluons, photon, W- & Z-boson A_μ

multiplets of gauge groups

Spin 2: graviton $g_{\mu\nu}$

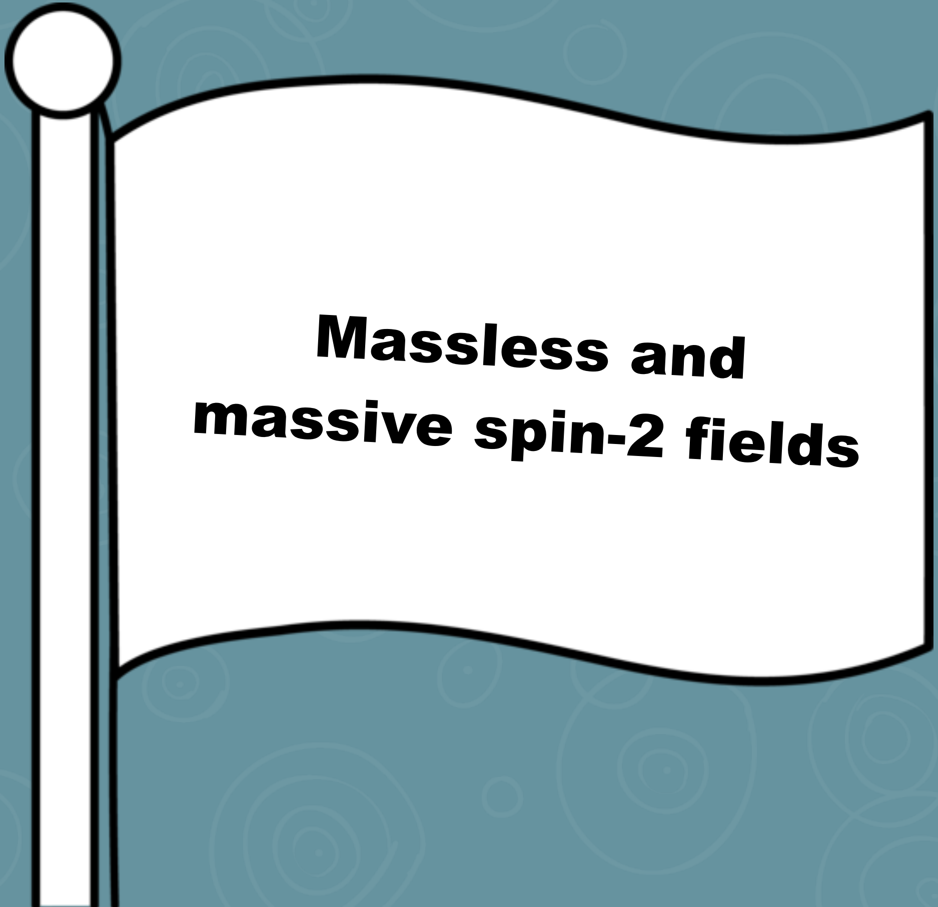
just one field...

Boulanger, Damour, Gualteri, Henneaux (2000):

Multiple massless spin-2 fields cannot interact with each other.

Can multiple spin-2 fields interact if we include mass terms ?

How do we make a spin-2 field massive ?



Massless and massive spin-2 fields

General Relativity

- 📌 **Einstein-Hilbert action:** $S_{\mathrm{EH}}[g] = M_{\mathrm{P}}^2 \int \mathrm{d}^4 x \sqrt{g} \left(R(g) - 2\Lambda \right)$
- 📌 **Einstein's equations:** $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0$
- 📌 **Maximally symmetric solutions:** $\bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu}$

General Relativity

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Maximally symmetric solutions: $\bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu}$

Linear perturbations of Einstein's equations, $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$:

$$\bar{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma} \delta g_{\rho\sigma} = 0 \qquad \bar{\mathcal{E}} \sim \nabla\nabla + \Lambda$$

⇨ equation for a massless spin-2 field with 2 degrees of freedom,

tensor analogue of $\Box\phi = 0$

Hamiltonian analysis ⇨ 2 d.o.f. also at the nonlinear level

General Relativity

=

unique description of
self-interacting massless spin-2 field

Linear Massive Gravity

Fierz & Pauli (1939)

Equation for a massive spin-2 field:

$$\bar{\mathcal{E}}_{\mu\nu}{}^{\rho\sigma}\delta g_{\rho\sigma} + \frac{m_{\mathrm{FP}}^2}{2}\left(\delta g_{\mu\nu} - \mathbf{a}\,\bar{g}_{\mu\nu}\delta g\right) = 0$$

tensor analogue of $\Box\phi - m^2\phi = 0$

📌 propagates 5 degrees of freedom for $\mathbf{a = 1}$

Linear Massive Gravity

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tensor analogue of $\Box\phi - m^2\phi = 0$

- propagates 5 degrees of freedom for $\mathbf{a} = 1$
- for $\mathbf{a} \neq 1$ there is an additional scalar mode which gives rise to a ghost instability (negative kinetic energy)

⇨ need extra constraint to remove the ghost

Can we write down a nonlinear mass term ?

Nonlinear Mass Term

... should not contain derivatives nor loose indices.

Examples: scalar (spin 0) vector (spin 1)

$$-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2 \qquad - g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}F_{\mu\nu} - m^2 g^{\mu\nu}A_\mu A_\nu$$

For the spin-2 tensor contracting indices of the metric gives: $g^{\mu\nu}g_{\mu\nu} = 4$

This is not a mass term.

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Simplest way out: Introduce second "metric" to contract indices:

$$g^{\mu\nu}f_{\mu\nu} = \mathrm{Tr}\left(g^{-1}f\right) \qquad f^{\mu\nu}g_{\mu\nu} = \mathrm{Tr}\left(f^{-1}g\right)$$

⇨ Massive gravity action: $S_{\mathrm{MG}}[g] \;=\; \underbrace{S_{\mathrm{EH}}[g]}_{\text{kinetic term}} - \underbrace{\int \mathrm{d}^4x\, V(g,f)}_{\text{mass term}}$

Nonlinear Mass Term

… should n derivatives nor loose indices.

Examples: sc 1)

$-g$ $^{\mu\nu} A_\mu A_\nu$

For the spin-2 t $\nu = 4$

This is not a ma

Simplest way ou :

$g^{\mu\nu} f_{\mu\nu}$ $g)$

⇨ Massive gr action: $S_{\mathrm{MG}}[g] = S_{\mathrm{EH}}[g] - \int \mathrm{d}^4x \, V(g, f)$

kinetic term mass term

What determines $f_{\mu\nu}$?
Shouldn't it be dynamical ?

Bimetric Theory

Nonlinear action for two interacting tensors:

$$\begin{aligned} S_{\mathrm{b}}[g,f] &= m_g^2 \int \mathrm{d}^4x \sqrt{g}\,\Big(R(g) - 2\Lambda\Big) \\ &+ m_f^2 \int \mathrm{d}^4x \sqrt{f}\,\Big(R(f) - 2\tilde{\Lambda}\Big) - \int \mathrm{d}^4x\, V(g,f) \end{aligned}$$

- both metrics are dynamical and treated on equal footing
- should describe massive & massless spin-2 field (5+2 d.o.f.)

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- both metrics are dynamical and treated on equal footing
- should describe massive & massless spin-2 field (5+2 d.o.f.)

This looks good, but what about  ?

The Nonlinear Ghost

Can we extend the Fierz-Pauli mass term
by nonlinear interactions ?

$$\frac{m_{\mathrm{FP}}^2}{2}\left(\delta g_{\mu\nu} - \bar{g}_{\mu\nu}\delta g\right) + \mathbf{c_1}\delta g_{\mu}{}^{\rho}\delta g_{\rho\nu} + \mathbf{c_2}\delta g \delta g_{\mu\nu} + \dots$$

🧩 Can we choose coefficients $\mathbf{c_i}$ such that the 👻 remains absent ?

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🧩 Can we choose coefficients $\mathbf{c}_\mathbf{i}$ such that the 👻 remains absent ?

Boulware & Deser (1972): Beyond linear order this is impossible!

No consistent nonlinear massive gravity / bimetric theory ? ☹

The ghost-free theory

- free Bimetric Theory

de Rham, Gabadadze, Tolley (2010);
Hassan, Rosen, ASM, von Strauss (2011)

$$\begin{aligned} S_{\mathrm{b}}[g,f] &= m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\ &+ m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f) \end{aligned}$$

$$\boxed{V\left(g,f\right) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n \left(\sqrt{g^{-1}f}\right)}$$

- arbitrary spin-2 mass scale m
- 3 interaction parameters β_n
- square-root matrix S defined through $S^2 = g^{-1}f$

👻 - free Bimetric Theory

de Rham, Gabadadze, Tolley (2010);
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$$\begin{aligned} S_{\mathrm{b}}[g,f] \;&=\; m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\ &+\; m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) \;-\; \int \mathrm{d}^4x\, V(g,f) \end{aligned}$$

$$\boxed{V\left(g,f\right) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n\left(\sqrt{g^{-1}f}\right) = m^4 \sqrt{f} \sum_{n=0}^{4} \beta_{4-n}\, e_n\left(\sqrt{f^{-1}g}\right)}$$

elementary symmetric polynomials:

$$e_1(S) = \mathrm{Tr}[S] \qquad e_2(S) = \tfrac{1}{2}\left((\mathrm{Tr}[S])^2 - \mathrm{Tr}[S^2]\right)$$

$$e_3(S) = \tfrac{1}{6}\left((\mathrm{Tr}[S])^3 - 3\,\mathrm{Tr}[S^2]\mathrm{Tr}[S] + 2\,\mathrm{Tr}[S^3]\right)$$

Ghost-free bimetric theory

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unique description of
massless + massive spin-2

Proportional solutions

Hassan, ASM, von Strauss (2012)

Ansatz: $\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$ **with** $c =$ const.

- gives two copies of Einstein's equations $(\alpha \equiv m_f/m_g)$:

$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_g(\alpha,\beta_n,c)\bar{g}_{\mu\nu} = 0$$
$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_f(\alpha,\beta_n,c)\bar{g}_{\mu\nu} = 0$$

- consistency condition: $\Lambda_g(\alpha,\beta_n,c) = \Lambda_f(\alpha,\beta_n,c)$ determines c

⇨ Maximally symmetric backgrounds with $R_{\mu\nu}(\bar{g}) = \Lambda_g \bar{g}_{\mu\nu}$

Mass spectrum

Hassan, ASM, von Strauss (2012)

Perturbations around proportional backgrounds:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad f_{\mu\nu} = c^2 \bar{g}_{\mu\nu} + \delta f_{\mu\nu}$$

Can be diagonalised into mass eigenstates:

$$\delta G_{\mu\nu} \quad \propto \quad \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu} \qquad \text{massless (2 d.o.f.)}$$

$$\delta M_{\mu\nu} \quad \propto \quad \delta f_{\mu\nu} - c^2 \delta g_{\mu\nu} \qquad \text{massive (5 d.o.f.)}$$

Linearised equations:

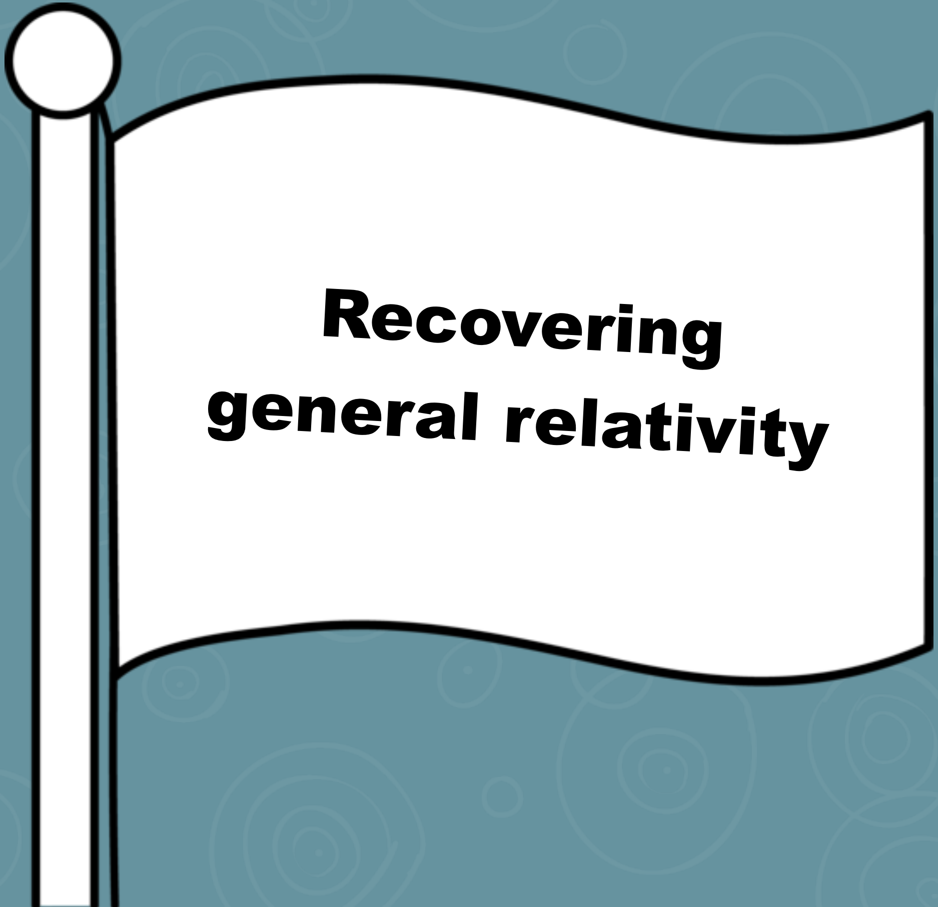
$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} \delta G_{\rho\sigma} = 0$$

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} \delta M_{\rho\sigma} + \frac{m_{\rm FP}^2}{2} \left(\delta M_{\mu\nu} - \bar{g}_{\mu\nu} \delta M \right) = 0$$

with Fierz-Pauli mass $m_{\rm FP} = m_{\rm FP}(\alpha, \beta_n, c)$

What is the physical metric ?

How much does the theory differ from GR ?



Recovering general relativity

Matter Coupling

Yamashita, de Felice, Tanaka;
de Rham, Heisenberg, Ribeiro (2015)

$$\begin{aligned} S_{\mathrm{b}}[g,f] &= m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\ &+ m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f) \\ &\qquad + \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\mathrm{matter}}(g,\phi) \end{aligned}$$

Absence of ghosts: only one metric can couple to matter!

⇨ $g_{\mu\nu}$ is gravitational metric

Mass Eigenstates

Baccetti, Martin-Moruno, Visser (2012);
Hassan, ASM, von Strauss (2012/14);
Akrami, Hassan, Koennig, ASM, Solomon (2015)

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(linearised) gravitational metric:

$$\delta g_{\mu\nu} \propto \underbrace{\delta G_{\mu\nu}}_{\text{massless}} - \alpha^2 \underbrace{\delta M_{\mu\nu}}_{\text{massive}} \qquad (\alpha \equiv m_f/m_g)$$

The gravitational metric is not massless but a superposition of mass eigenstates.

Mass Eigenstates

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(linearised) gravitational metric:

$$\delta g_{\mu\nu} \propto \underbrace{\delta G_{\mu\nu}}_{\text{massless}} - \alpha^2 \underbrace{\delta M_{\mu\nu}}_{\text{massive}} \qquad (\alpha \equiv m_f/m_g)$$

- for small $\alpha = m_f/m_g$ gravity is dominated by the massless mode
- the massive spin-2 field interacts only weakly with matter

Mass Eigenstates

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$$\alpha = m_f / m_g \to 0$$

is the General Relativity limit of bimetric theory

Structure of Vertices

Hassan, ASM, von Strauss (2012)

(bimetric action expanded in mass eigenstates)

Quadratic (Fierz-Pauli)

δG^2	$\delta G \delta M$	δM^2
$1\,,\Lambda$	0	$1\,,\Lambda\,,m_{\mathrm{FP}}^2$

$$S_{(2)} = \tfrac{1}{2}\int \mathrm{d}^4x\Big[\delta G_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta G_{\rho\sigma} + \delta M_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta M_{\rho\sigma} \\ - \tfrac{m_{\mathrm{FP}}^2}{2}(\delta M^{\mu\nu}\delta M_{\mu\nu} - \delta M^2) - \tfrac{1}{m_{\mathrm{Pl}}}\Big(\delta G^{\mu\nu} - \alpha\,\delta M^{\mu\nu}\Big)T_{\mu\nu}\Big]$$

Structure of Vertices

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what about higher orders?

$$S_{(2)}=\tfrac{1}{2}\int \mathrm{d}^4x\Big[\delta G_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta G_{\rho\sigma}+\delta M_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta M_{\rho\sigma} -\tfrac{m_{\mathrm{FP}}^2}{2}(\delta M^{\mu\nu}\delta M_{\mu\nu}-\delta M^2)-\tfrac{1}{m_{\mathrm{Pl}}}\Big(\delta G^{\mu\nu}-\alpha\,\delta M^{\mu\nu}\Big)T_{\mu\nu}\Big]$$

$$
\begin{aligned}
\mathcal{L}^{(3)}_{\mathrm{GM}} = &-\frac{m_{\mathrm{FP}}^2(1+\alpha^2)(\beta_1+\beta_2)}{4\alpha\,\mu^2}\,e_3(\delta M) \\
&-\frac{m_{\mathrm{FP}}^2}{24\alpha}\Big[-2[\delta M]^3+9[\delta M][\delta M^2]-7[\delta M^3] \\
&\qquad +\alpha\left(-3[\delta G][\delta M]^2+12[\delta M][\delta G\delta M]+3[\delta G][\delta M^2]-12[\delta G\delta M^2]\right) \\
&\qquad +\alpha^2\left([\delta M]^3-6[\delta M][\delta M^2]+5[\delta M^3]\right)\Big] \\
&-\frac{\Lambda}{4}\Big[[\delta G][\delta M]^2-4[\delta M][\delta G\delta M]-2[\delta G][\delta M^2]+8[\delta G\delta M^2]\Big] \\
&+\frac{1}{4}\Big[\delta G^{\mu\nu}\Big(\nabla_\mu\delta M_{\rho\sigma}\nabla_\nu\delta M^{\rho\sigma}-\nabla_\mu\delta M\nabla_\nu\delta M+2\nabla_\nu\delta M\nabla_\rho\delta M_\mu{}^\rho+2\nabla_\nu\delta M_\mu{}^\rho\nabla_\rho\delta M \\
&\quad -2\nabla_\rho\delta M\nabla^\rho\delta M_{\mu\nu}+2\nabla_\rho\delta M_{\mu\nu}\nabla_\sigma\delta M^{\rho\sigma}-4\nabla_\nu\delta M_{\rho\sigma}\nabla^\sigma\delta M_\mu{}^\rho-2\nabla_\rho\delta M_{\nu\sigma}\nabla^\sigma\delta M_\mu{}^\rho \\
&\quad +2\nabla_\sigma\delta M_{\nu\rho}\nabla^\sigma\delta M_\mu{}^\rho\Big) \\
&\quad +\frac{1}{2}\delta G\left(\nabla_\rho\delta M\nabla^\rho\delta M-\nabla_\rho\delta M_{\mu\nu}\nabla^\rho\delta M^{\mu\nu}-2\nabla_\rho\delta M\nabla_\mu\delta M^{\mu\rho}+2\nabla_\rho\delta M_{\mu\nu}\nabla^\nu\delta M^{\mu\rho}\right)\Big] \\
&+\frac{1}{2}\Big[\delta M^{\mu\nu}\Big(\nabla_\mu\delta G_{\rho\sigma}\nabla_\nu\delta M^{\rho\sigma}-\nabla_\mu\delta G\nabla_\nu\delta M+\nabla^\rho\delta G_{\rho\mu}\nabla_\nu\delta M+\nabla_\nu\delta G_{\mu\rho}\nabla^\rho\delta M \\
&\quad -\nabla_\rho\delta G_{\mu\nu}\nabla^\rho\delta M+\nabla_\rho\delta G^{\rho\sigma}\nabla_\sigma\delta M_{\mu\nu}-2\nabla_\mu\delta G^{\rho\sigma}\nabla_\sigma\delta M_{\nu\rho}+\nabla_\mu\delta G\nabla^\rho\delta M_{\rho\nu} \\
&\quad +\nabla^\rho\delta G_{\mu\nu}\nabla^\sigma\delta M_{\rho\sigma}-2\nabla_\rho\delta G_{\mu\sigma}\nabla_\nu\delta M^{\rho\sigma}-2\nabla^\rho\delta G_{\mu\sigma}\nabla^\sigma\delta M_{\nu\rho}+2\nabla^\rho\delta G_{\mu\sigma}\nabla_\rho\delta M_\nu{}^\sigma \\
&\quad +\nabla^\rho\delta G\nabla_\nu\delta M_{\mu\rho}-\nabla^\rho\delta G\nabla_\rho\delta M_{\mu\nu}\Big) \\
&\quad +\frac{1}{2}\delta M\Big(\nabla_\rho\delta G\nabla^\rho\delta M-\nabla_\rho\delta G_{\mu\nu}\nabla^\rho\delta M^{\mu\nu}-\nabla_\rho\delta G\nabla_\sigma\delta M^{\rho\sigma} \\
&\quad -\nabla_\rho\delta G^{\rho\sigma}\nabla_\sigma\delta M+2\nabla_\rho\delta G_{\mu\nu}\nabla^\nu\delta M^{\mu\rho}\Big)\Big]
\end{aligned}
$$

Higher-Order Vertices

Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)

Quadratic (Fierz-Pauli)

δG^2	$\delta G \delta M$	δM^2
$1\,,\Lambda$	0	$1\,,\Lambda\,,m_{\rm FP}^2$

Cubic (suppressed by $m_{\rm Pl}^{-1}$)

δG^3	$\delta G^2 \delta M$	$\delta G \delta M^2$	δM^3
$1\,,\Lambda$	0	$1\,,\Lambda\,,m_{\rm FP}^2$	$\alpha\,,\alpha\Lambda\,,\alpha m_{\rm FP}^2$ $\frac{1}{\alpha}\,,\frac{1}{\alpha}\Lambda\,,\frac{1}{\alpha}m_{\rm FP}^2$

$$m_{\rm Pl} = m_g\sqrt{1+\alpha^2}$$

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- self-interactions of massless spin-2 sum up to Einstein-Hilbert action

$$S(g,f)|_{\delta M=0} = m_{\rm Pl}^2 \int \mathrm{d}^4x\,\sqrt{|G|}\,\left(R(G) - 2\Lambda\right) \qquad G_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{m_{\rm Pl}}\delta G_{\mu\nu}$$

- massive mode becomes heavy and strongly coupled for small $\alpha = m_f/m_g$

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$$S(g,f)|_{\delta M=0} = m_{\mathrm{Pl}}^2 \int \mathrm{d}^4x\,\sqrt{|G|}\,\left(R(G)-2\Lambda\right) \qquad G_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{m_{\mathrm{Pl}}}\delta G_{\mu\nu}$$

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→ Dark Matter ?

Ghost-free bimetric theory

=

Massive spin-2 in background
set by massless spin-2

Curvature corrections

Gording & ASM (2018)

Equations for $f_{\mu\nu}$ can be solved perturbatively in $\alpha = m_f/m_g$

- Higher-derivative action for $g_{\mu\nu}$:

$$S_{\text{eff}}[g] = \int \mathrm{d}^4x \, \sqrt{g} \Big[\underbrace{m_{\text{Pl}}^2 \big(-2\Lambda + R \big)}_{\text{general relativity}} + \alpha^4 c_{RR} \underbrace{\big(\tfrac{1}{3} R^2 - R^{\mu\nu} R_{\mu\nu} \big)}_{\text{(Weyl tensor)}^2 \text{ + total derivative}} \Big] + \mathcal{O}(\alpha^6)$$

- curvature corrections to GR capture effects of heavy spin-2 field

Ghost-free bimetric theory

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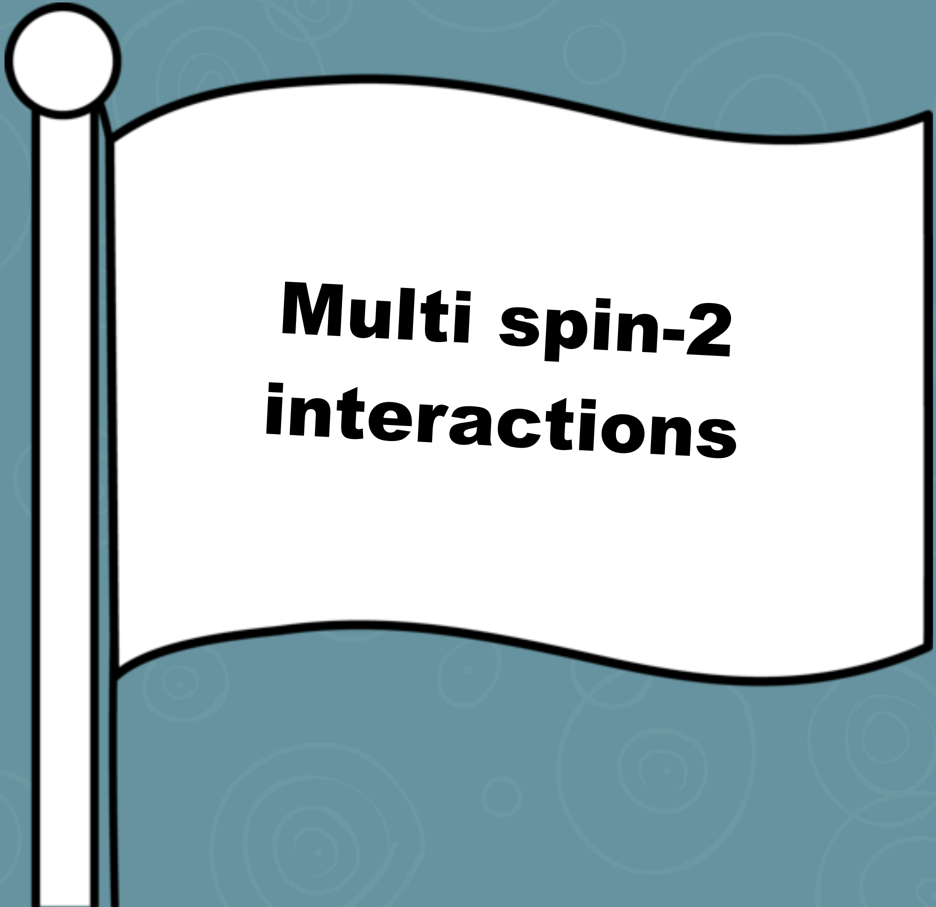
General Relativity + corrections
induced by additional tensor field

Can we have more than two spin-2 fields ?



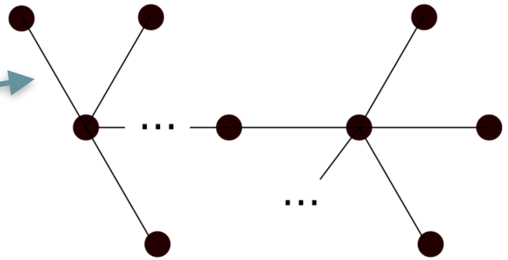
Multi spin-2 interactions

Multimetric Interactions

Hinterbichler & Rosen;
Nomura & Soda (2012)

Absence of ghosts: only pairwise bimetric interactions

$$V\left(g,f\right)=m^4\sqrt{g}\sum_{n=0}^{4}\beta_n\,e_n\left(\sqrt{g^{-1}f}\right)$$



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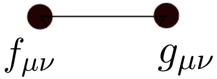
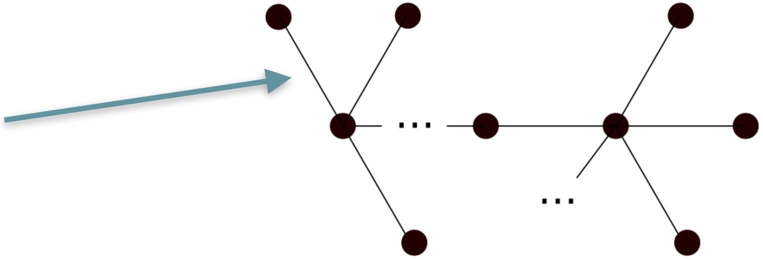
Nomura & Soda (2012)

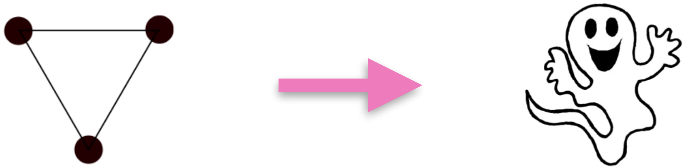
Absence of ghosts: only pairwise bimetric interactions



$$V\left(g,f\right)=m^4\sqrt{g}\sum_{n=0}^{4}\beta_n\,e_n\left(\sqrt{g^{-1}f}\right)$$



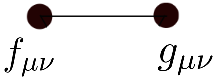
No loops !!!



Multimetric Interactions

Hinterbichler & Rosen;
Nomura & Soda (2012)

Absence of ghosts: only pairwise bimetric interactions



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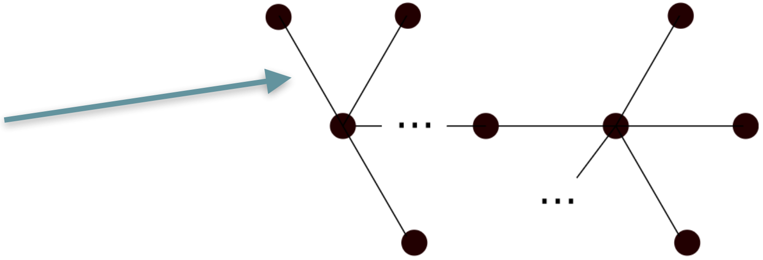
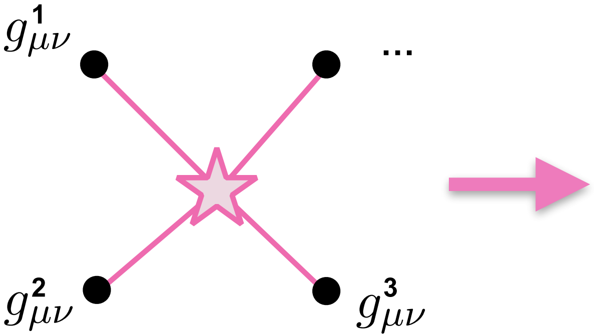


- theory with N metrics describes 1 massless and $(N-1)$ massive spin-2

Multi Spin-2 Vertices

Can we have interactions beyond the pairwise bimetric couplings?

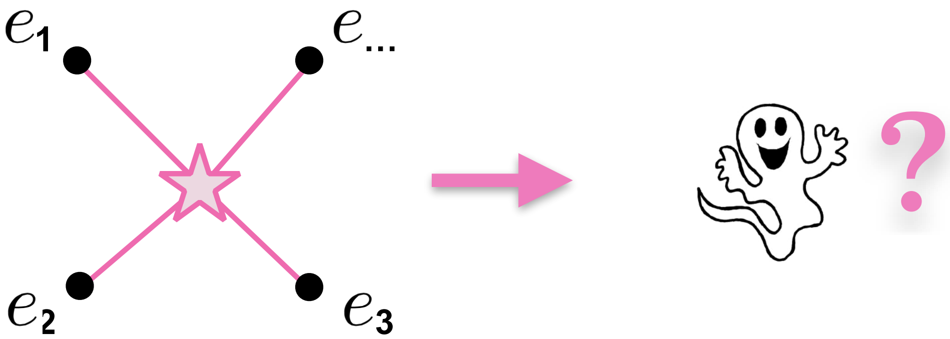
Vertices including more than two tensor fields:



Multi Spin-2 Vertices

Can we have interactions beyond the pairwise bimetric couplings?

Vertices including more than two ~~tensor~~ vierbein fields:



Vierbein Formulation

Introduce two sets of vierbein fields:

$$g_{\mu\nu} = e^a{}_\mu \eta_{ab} e^b{}_\nu \qquad f_{\mu\nu} = \tilde{e}^a{}_\mu \eta_{ab} \tilde{e}^b{}_\nu$$

Einstein-Hilbert terms: $S_{\mathrm{EH}} = m_g^2 \epsilon_{abcd} \int \left(R^{ab} - \Lambda\, e^a \wedge e^b\right) \wedge e^c \wedge e^d$

- a general vierbein has 16 components; including 6 degrees of freedom that can be removed by a Lorentz transformation

➩ constraints in bimetric equations will remove the extra components

Vierbein Interactions

Hinterbichler & Rosen (2012)

$$g_{\mu\nu} = e^a{}_\mu \eta_{ab} e^b{}_\nu \qquad f_{\mu\nu} = \tilde{e}^a{}_\mu \eta_{ab} \tilde{e}^b{}_\nu$$

interaction terms:

$$S_{\rm int} = -m^4 \epsilon_{abcd} \int \left[\bar{\beta}_1 \, e^a \wedge e^b \wedge e^c \wedge \tilde{e}^d + \bar{\beta}_2 \, e^a \wedge e^b \wedge \tilde{e}^c \wedge \tilde{e}^d + \bar{\beta}_3 \, e^a \wedge \tilde{e}^b \wedge \tilde{e}^c \wedge \tilde{e}^d \right]$$

Vierbein Interactions

Hinterbichler & Rosen (2012)

$$g_{\mu\nu} = e^a{}_\mu \eta_{ab} e^b{}_\nu \qquad f_{\mu\nu} = \tilde{e}^a{}_\mu \eta_{ab} \tilde{e}^b{}_\nu$$

interaction terms:

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- equivalent to bimetric theory and ghost-free only if $e^a{}_\mu \eta_{ab} \tilde{e}^b{}_\nu = e^a{}_\nu \eta_{ab} \tilde{e}^b{}_\mu$
- solves the dynamical constraints, removes additional components

⇨ existence of square-root and intersection of light cones (Hassan & Kocic, 2017)

New interactions

Hassan & ASM (2018)

- start from ghost free multivierbein action with only pair wise interactions
- integrate out a non-dynamical field to obtain new vertices:

$$S_{\text{multi}} = -M^4 \int \mathrm{d}^4x \ \det\Big(\sum_{I=1}^{\mathcal{N}} \beta^I e_I\Big)$$

⇨ up to 4 different vierbeine in one vertex !

⇨ interactions not always expressible in terms of metrics.

Most general interactions

Hassan & ASM (2018)

Interaction potential: $$\sum_{I,J,K,L=1}^{\mathcal{N}} \beta^{IJKL}\, \epsilon_{ABCD}\, e_I^A \wedge e_J^B \wedge e_K^C \wedge e_L^D$$

- bimetric interactions: $\beta^{IJKL} \sim \beta_3\, \delta^{1I} \delta^{2J} \delta^{2K} \delta^{2L}$

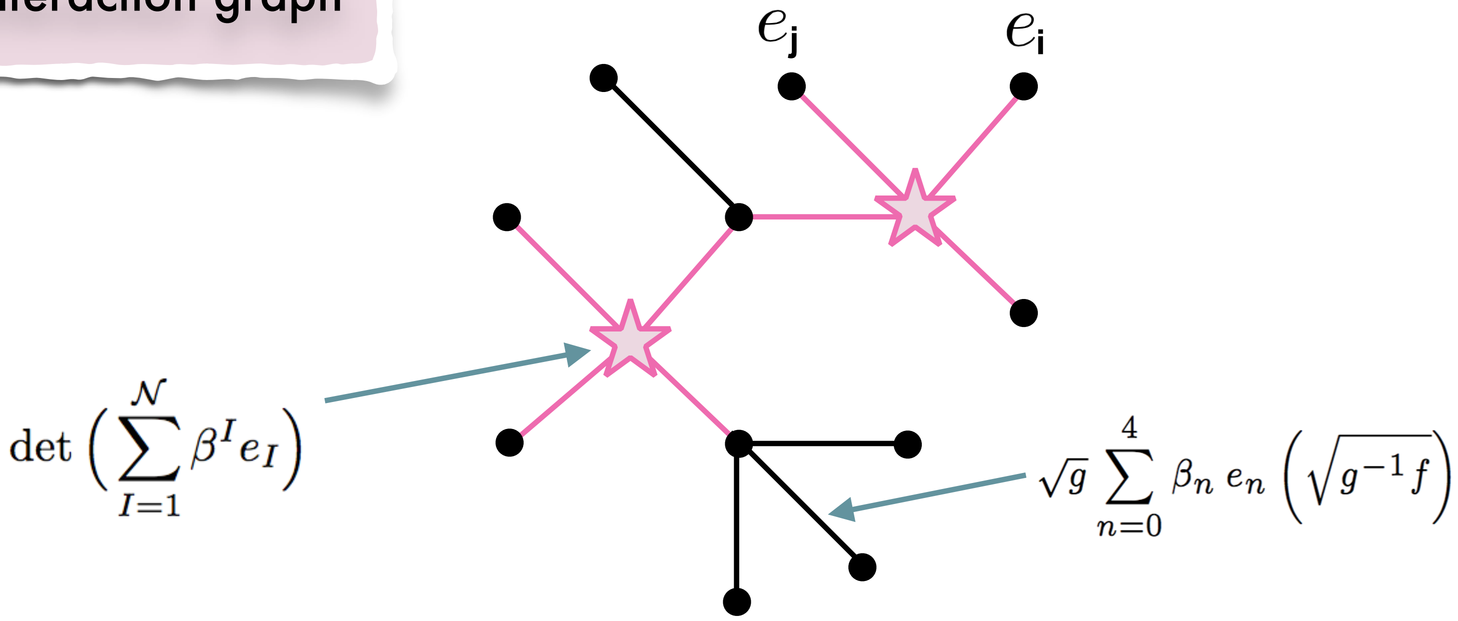
  ⇨ $$\sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n\left(\sqrt{g^{-1} f}\right)$$

- determinant vertex: $\beta^{IJKL} = \beta^I \beta^J \beta^K \beta^L$

  ⇨ $$\det\left(\sum_{I=1}^{\mathcal{N}} \beta^I e_I\right)$$

General interaction graph

Hassan & ASM (2018)

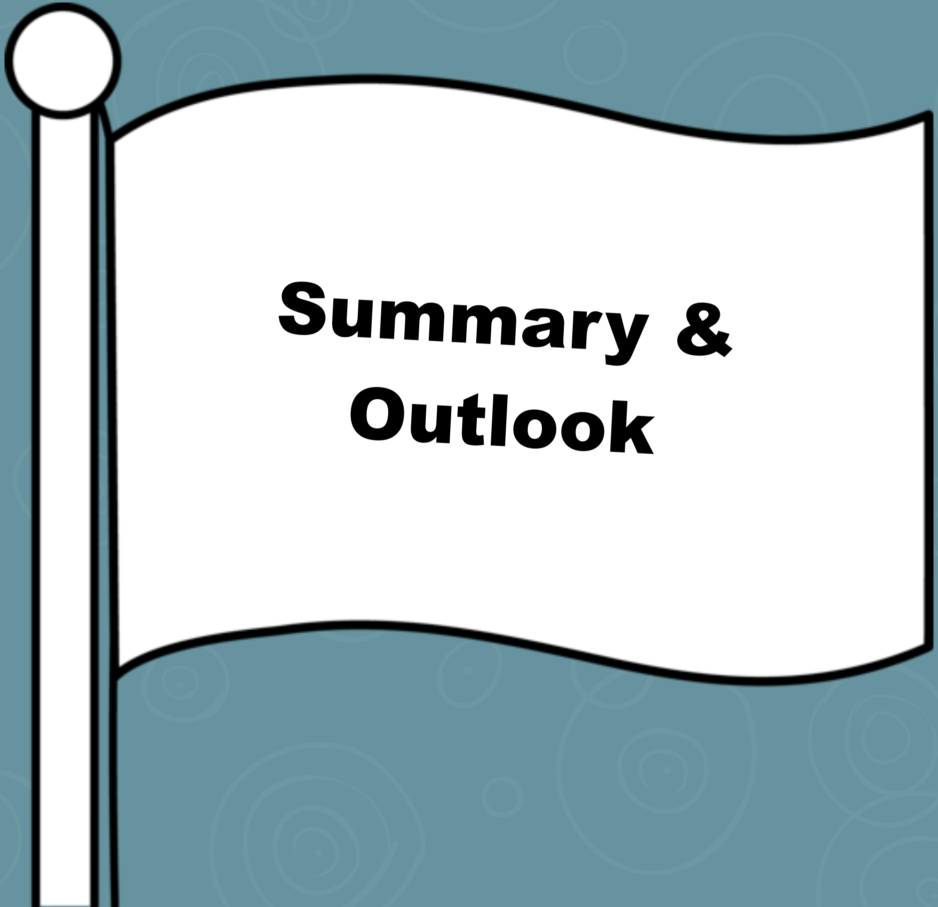


$$\det\left(\sum_{I=1}^{\mathcal{N}} \beta^I e_I\right)$$

$$\sqrt{g}\sum_{n=0}^{4} \beta_n\, e_n\left(\sqrt{g^{-1}f}\right)$$

☆ used to be a (nondynamical) ● which has been integrated out

Multi-spin-2 vertices require the use of vierbein fields.



Summary & Outlook

Ghost- free bimetric theory…

- is one of the few known consistent modifications of General Relativity
- can be interpreted as gravity in the presence of an extra spin-2 field
- can be extended to multiple spin-2 interactions using vierbeine

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review: ASM, Mikael von Strauss; 1512.00021

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- Most general interactions ?
- Additional symmetries ?
- Charged spin-2 fields ?

review: ASM, Mikael von Strauss; 1512.00021