Massive AdS Gravity from **String Theory**

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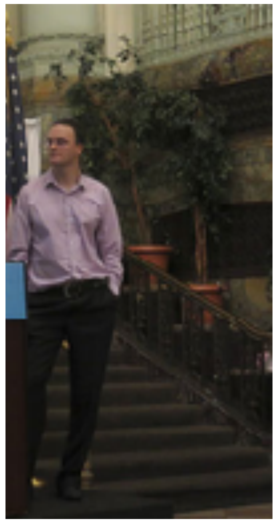
Based on two papers with Ioannis Lavdas

arXiv: 1807.00591

arXiv: 1711.11372

Earlier work with John Estes



arXiv: 1103.2800

If I have time, I may also comment briefly on

arXiv: 1711.06722 with Bianchi & Hanany

Plan of Talk

1. Foreword
2. g-Mass from holography
3. Representation merging 
4. Review of N=4 AdS4/CFT3
5. g-Mass operator
6. `Scottish Bagpipes' 
7. 3 rewritings & bigravity 
8. Final remarks

1 Foreword

An old question: Can gravity be `higgsed' (become massive) ?

Extensive (recent & less recent) literature:

Pauli, Fierz, Proc.Roy.Soc. 1939 . . . .

Nice reviews: Hinterblicher 1105.3735; de Rham 1401.4173
Schmidt-May & von Strauss 1512.00021

The question is obviously interesting, since any sound **IR modification** of General Relativity could have consequences for cosmology

[degravitating dark energy? `mimicking dark matter' ? . . .]

The main messages of this talk :

– Massive AdS Gravity **is** part of the string-theory landscape

– A quasi-universal, quantized formula for the mass

Setting is 10d IIB sugra, and holographic dual CFTs

If $\exists$ time, I will comment on gauged 4d supergravity

Have little to say about the effective 4d theory
around these string-theory vacua

But note that in certain sense massive AdS gravity is an `easier' case:

the limit $m_{\rm g} \to 0$ is smooth

$\nexists$ van Dam-Veltman-Zakharov discontinuity

No need for strong non-linearities of Vainshtein screening

Kogan, Mouslopoulos, Papazoglou '00
Porrati '00

2 G-mass from Holography

Consider $\mathrm{AdS}_4 \leftrightarrow \mathrm{CFT}_3$

For a primary spin-2 operator: $m^2 L_4^2 = \Delta(\Delta - 3)$

(like stress tensor T_{ab})

If conserved, $\partial_a T^{ab} = 0$, the representation is **short**

& an algebraic manipulation gives $\Delta = 3 \implies m_{\mathrm{g}} = 0$

↑ canonical

So to get **G-mass** we must allow 3d energy-momentum to **leak out**

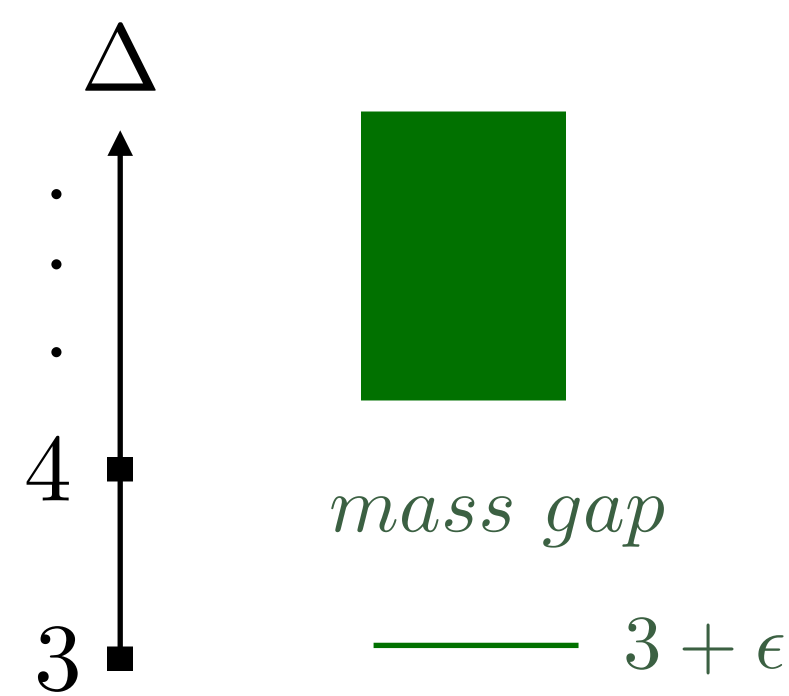
Two options consistent with 3d conformal invariance:

— Couple to another 3d CFT: one massless & one massive graviton (bigravity models)

— Leaking out to a 4d (or higher ?) defect CFT to get a `single' massive-graviton theory

Need **weak leakage**

$\epsilon \ll 1$



Will focus on defect CFT, but bigravity is closely related

Bulk CFT_4 — $\mathcal{N}=4 \quad SU(n)$ super Yang Mills

Bnry or Interface — $1/2$ BPS Gaiotto-Hanany-Witten theories

Unbroken superconformal symmetry: $Osp(4|4) \supset SO(2,3) \times SO(4)_R$

Weak leakage ?

$g_{\rm YM} \to 0$ will NOT do (decompactification)

instead $n^2/\tilde{F}_3 \ll 1$ **scarce** bulk a.o.t. bnry degrees of freedom

Idea of using defect CFT due to **Karch + Randall** `00, `01

Modelled with thin AdS4 brane in AdS5

<u>Here</u>: proper string theory embedding of the idea;

Thin-brane approximation fails, so KK scale is L_4 NOT L_5

(problem of scale separation in flux vacua)

<u>Other approaches</u>:

Porrati; Duff, Liu, Sati — transparent AdS bnry

Kiritsis; K+Niarchos
Aharony, Clark, Karch — multitrace coupling of two CFTs

I will discuss their relation in the end

3 Representations

AdS Higgsing converts a short rep into a long rep of $SO(2,3)$

At the unitarity threshold $\Delta \to s+1$

$$[s]_\Delta \quad \to \quad [s]_{s+1} \oplus [s-1]_{s+2}$$

For spin 2 : $[2]_{3+\epsilon} \to [2]_3 \oplus [1]_4$ Goldstone = massive vector

With $\mathcal{N}=4$ supersymmetry these fields should belong to reps of $Osp(4|4)$

These are classified completely

Dolan 0811.2740

Cordova, Dumitrescu, Intriligator 1612.00809

Reps come in four series:

$L[s]_{\Delta}^{(j,\tilde{j})}$	$A_1[s]_{s+j+\tilde{j}+1}^{(j,\tilde{j})}$	$A_2[0]_{j+\tilde{j}+1}^{(j,\tilde{j})}$	$B_1[0]_{j+\tilde{j}}^{(j,\tilde{j})}$
$\Delta > s+j+\tilde{j}+1$ long	$s > 0$ short	short	absolutely protected

Higgsing of the graviton amounts to

$$L[0]_{1+\epsilon}^{(0;0)} \xrightarrow{\epsilon \to 0} A_2[0]_1^{(0;0)} \oplus B_1[0]_2^{(1;1)}$$

massive supergraviton (→ $L[0]_{1+\epsilon}^{(0;0)}$)

graviton supermultiplet
(6 vectors, 2 scalars) (→ $A_2[0]_1^{(0;0)}$)

Goldstone mode
112 bosonic d.o.f.
& **spin 3/2 fermion** (→ $B_1[0]_2^{(1;1)}$)

- The spin-3/2 Goldstone multiplet is not part of the 4d gauged-sugra spectrum

  So this corner of the landscape is not accessible by gauged sugra

- That Higgsing is compatible with susy is not automatic.

  For instance $\mathcal{N} = 4$ forbids the Higgsing of normal gauge symmetries

  because gauge fields, in $B_1[0]_1^{(1;0)}$ or $B_1[0]_1^{(0;1)}$, are protected

  Louis, Triendl '14
  Corodova et al '16

$\mathcal{N} = 4$ susy allows e-m to leak out but not flavor charge

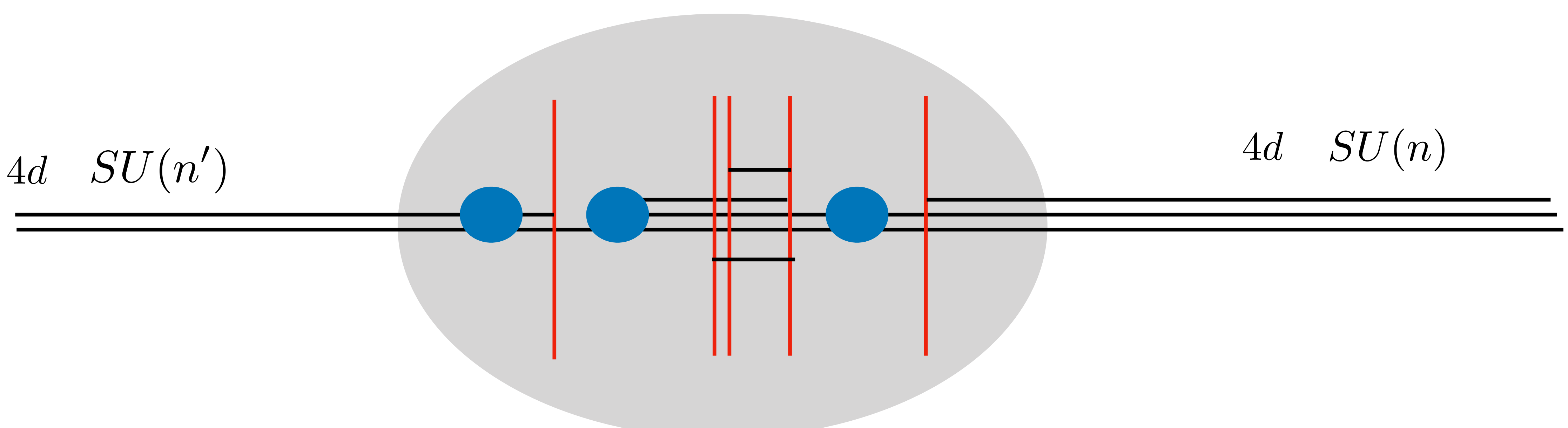
4 Review of $\mathcal{N}=4$ $\mathrm{AdS}_4/\mathrm{CFT}_3$

I will be brief — have talked about this before, only stress some features that we will need below.

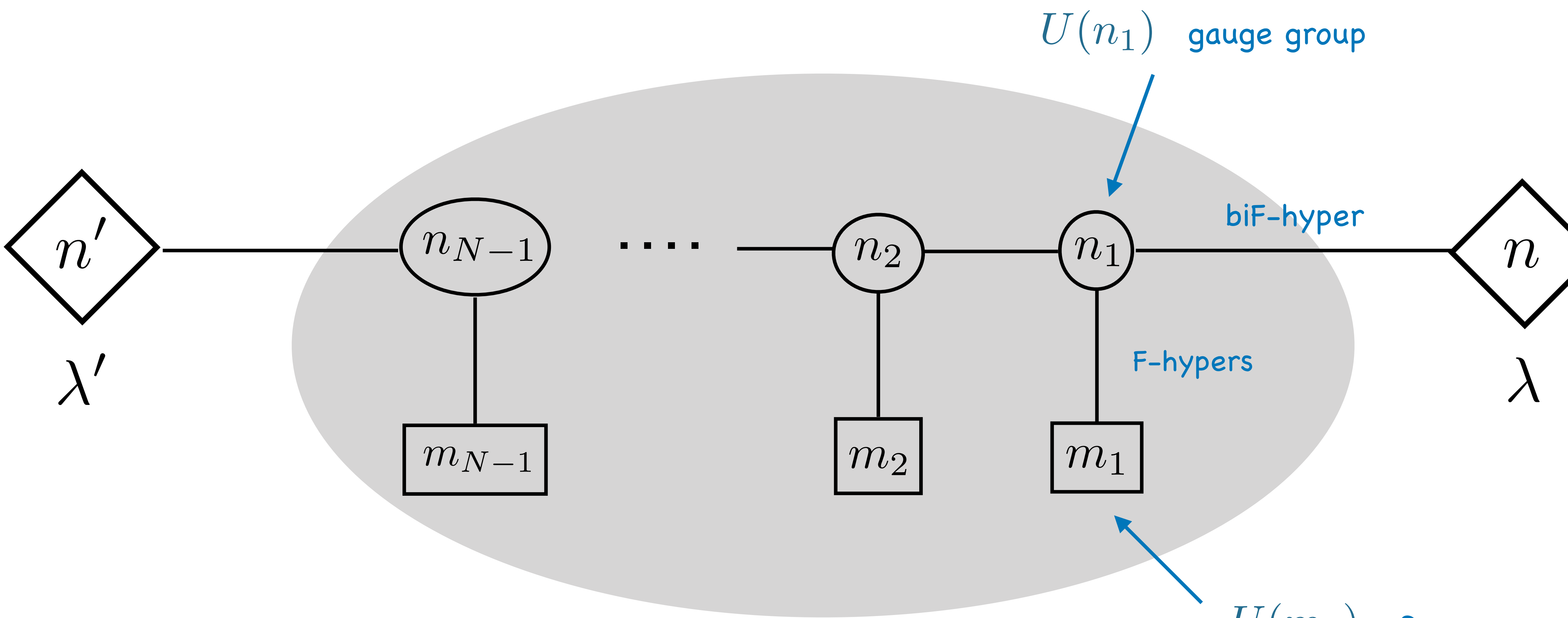
CFT side Brane engineering using D3-D5-NS5 branes:

Hanany, Witten '96

$3d$ Gaiotto, Witten '08



Quiver gauge theory



In `good' IR SCFT the interface depends only on **discrete data**

masses = FI terms = CS terms = 0 ; 3d YM couplings $\sim [mass]^{1/2} \longrightarrow \infty$

NB: magnetic quiver by exchanging D5 & NS5 has its own flavor symmetry

The general type-IIB solution with $Osp(4|4)$ symmetry was found by

D'Hoker, Estes, Gutperle arXiv: 0705.0022 ; 0705.0024

The map to the Gaiotto-Witten interface CFTs was derived in

Assel, CB, Estes, Gomis arXiv: 1106.4253 ; 1210.2590

The geometry has the fibered form $AdS_4 \times_w M_6$ where, in order to realize the R symmetry, $M_6 = (\mathrm{S}_2 \times \hat{\mathrm{S}}_2) \times_w \Sigma$

All backgrounds can be written in terms of two harmonic functions $h, \hat{h}$ which have singularities on the boundary of Σ

the explicit expressions are

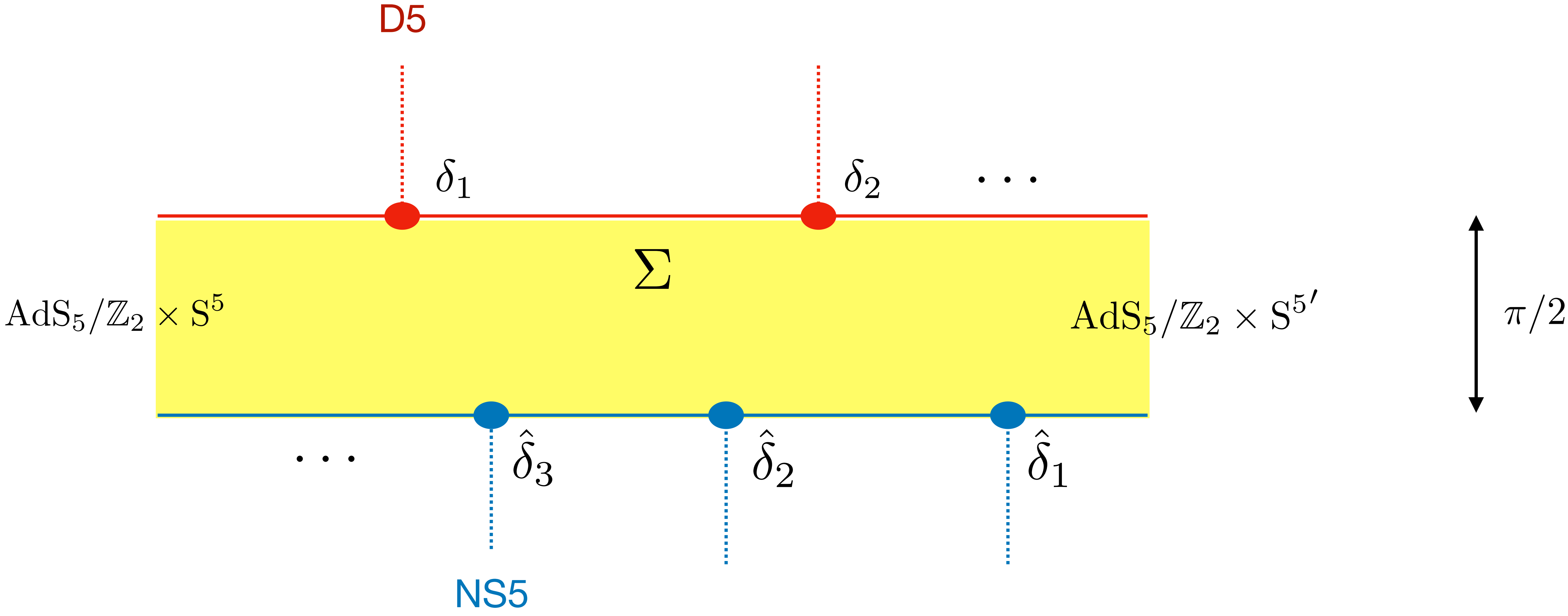
$$ds^2_{10} = L_4^2 ds^2_{\mathrm{AdS}_4} + f^2 ds^2_{\mathrm{S}^2} + \hat{f}^2 ds^2_{\hat{\mathrm{S}}^2} + 4\rho^2 dz d\bar{z}$$

$$L_4^8 = 16\,\frac{\mathcal{U}\hat{\mathcal{U}}}{W^2}\ , \quad f^8 = 16\,h^8\,\frac{\hat{\mathcal{U}}W^2}{\mathcal{U}^3}\ , \quad \hat{f}^8 = 16\,\hat{h}^8\,\frac{\mathcal{U}W^2}{\hat{\mathcal{U}}^3}\ , \quad \rho^8 = \frac{\mathcal{U}\hat{\mathcal{U}}W^2}{h^4\hat{h}^4}\ , \quad e^{4\phi} = \frac{\hat{\mathcal{U}}}{\mathcal{U}}$$

$$\text{with} \qquad \mathcal{U} = 2h\hat{h}|\partial_z h|^2 - h^2 W\ , \quad \hat{\mathcal{U}} = 2h\hat{h}|\partial_z \hat{h}|^2 - \hat{h}^2 W\ , \quad \text{and} \quad W = \partial_z\partial_{\bar{z}}(h\hat{h})\ .$$

There are also 3-form and 5-form fluxes, sourced by the 5-branes and D3-branes whose expressions we will not need today

The explicit harmonic functions with Σ the infinite strip are



$$h = -i\alpha \sinh(z-\beta) \quad - \sum_{a=1} \gamma_a \log \tanh\left(\frac{i\pi}{4} - \frac{z}{2} + \frac{\delta_a}{2}\right) \quad + c.c.$$

$$\hat{h} \;=\; \hat{\alpha} \cosh(z-\hat{\beta}) \quad - \sum_{b=1}^{\hat{N}} \hat{\gamma}_b \log \tanh\left(\frac{z}{2} - \frac{\hat{\delta}_b}{2}\right) \quad + c.c.$$

Of particular interest is the supersymmetric Janus solution:

no 5-brane charges, only the dilaton (gauge coupling) varies with $\mathrm{Re}\, z$

parameters: $\alpha, \hat{\alpha}, \beta, \hat{\beta} \;\to\; L_5, \phi_{-\infty}, \phi_{\infty}$

The detailed Janus geometry enters in the calculation of the mass, but most other features are irrelevant; indeed what I describe below should carry over to other solutions (lower $\mathcal{N}$, other d?)

5 Mass operator

The spectrum of spin-2 excitations from any 2-derivative gravity action depends only on geometry (not on scalar fields & fluxes)

Csaki, Erlich, Hollowood, Shirman hep-th/0001033
CB, Estes arXiv: 1103.2800

For any warped compactification

$$ds^2_{10} = L_4^2(y)\, ds^2_{\mathrm{AdS}_4} + \sum_{i,j=1}^{6} g_{ij}(y)\, dy^i dy^j$$

the mass-eigenstate wavefunctions factorize:

$$h_{\mu\nu}(x,y) = \psi(y)\, \chi_{\mu\nu}(x)$$

$\psi(y)$ ↑ M_6 $\quad$ $\chi_{\mu\nu}(x)$ ↑ AdS_4

spin-2 wave operator

$$\mathcal{L}^{\mathrm{AdS}}_{(2)} \chi_{\mu\nu} = \lambda \chi_{\mu\nu}$$

$$\lambda + 2 = m^2(y) L_4^2(y) \quad = \Delta(\Delta - 3)$$

invariant mass

Reducing the 10d linearized Einstein equations & norm leads to:

$$\mathcal{M}^2\psi := -\frac{L_4^{-2}}{\sqrt{g}}\,\partial_i\left(L_4^4\sqrt{g}g^{ij}\,\partial_j\,\psi\right) = (\lambda+2)\,\psi$$

$$\langle\psi_1|\psi_2\rangle = \int_{\mathrm{M}_6} d^6y\,\sqrt{g}\,L_4^2\psi_1^*\psi_2$$

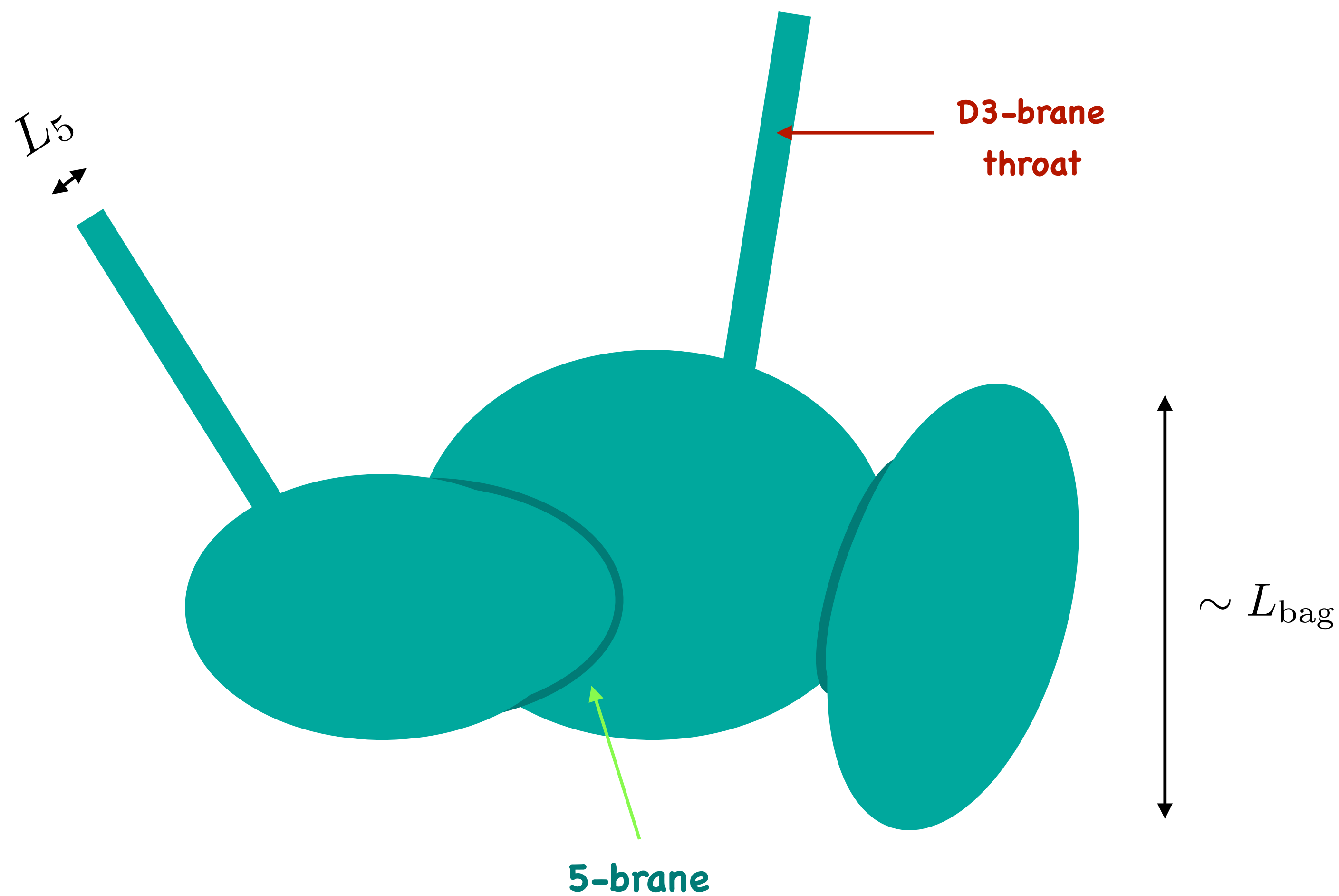
- For direct-product reductions $\mathcal{M}^2$ is the Laplace operator on M_6
- Integrating by parts gives $\langle\psi|\mathcal{M}^2|\psi\rangle = \int_{\mathrm{M}_6} d^6y\,\sqrt{g}\,L_4^4\,(g^{ij}\partial_i\psi^\star\partial_j\psi)$

  $\mathcal{M}^2$ is (hermitean and) non-negative

- When M_6 is compact and L_4 finite $\exists$ massless graviton with constant wavefunction ψ (universality of 4d gravity)

6 Bagpipes manifolds

The `slightly' non-compact manifolds that give massive gravity have the shape of 'Scottish Bagpipes'



$$\frac{L_5}{L_{\rm bag}} \ll 1$$

They are obtained by taking $\alpha, \hat{\alpha} \to 0$ with other parameters held fixed

The limit is smooth in supergravity but not in string theory

Here comes now the key idea:

To find the lowest-lying 4d graviton, replace the eigenvalue- by a <u>minimization</u> problem

$$\lambda_0 + 2 = \ \mathbf{min}_\psi \left[\int_{M_6} d^6y \sqrt{g}\, L_4^4 \, (g^{ij}\partial_i \psi^\star \partial_j \psi) \right] \quad \text{with} \quad \int_{M_6} d^6y \sqrt{g}\, L_4^2 \, |\psi|^2 = 1 \ .$$

If we were to truncate the pipes the (massless) graviton would be

$$\psi_0(y) = \left(\int_{\overline{M}_6} d^6y \sqrt{g}\, L_4^2 \right)^{-1/2} := \psi_{\rm bag} \qquad = \left(V_6 \langle L_4^2 \rangle \right)^{-1/2}$$

bag

In the presence of the pipes $\psi \simeq \psi_{\rm bag}$ in the bag, and goes to zero at the the bottom of the pipes where L_4 is minimal.

The problem is now a minimization problem in the Janus-throat geometry with boundary conditions:

$$\lambda_0 + 2 \simeq \ \mathbf{min}_\psi \left[\int_{\rm throats} d^6y\, \sqrt{g}\, L_4^4 g^{ij} \partial_i \psi^\star \partial_j \psi \right] \quad \text{with} \quad \psi \to \begin{cases} \psi_{\rm bag} & \text{in matching region,} \\ 0 & \text{at infinity .} \end{cases}$$

Inserting the Janus solution gives

$$\lambda_0 + 2 \ = \ \mathbf{min}_\psi \left[\frac{\pi^3}{4} L_5^8 \int_{x_c}^{\infty} dx\, \mathcal{G}(x) \left(\frac{d\psi}{dx} \right)^2 \right] \quad \text{with} \quad \psi(x) \to \begin{cases} \psi_{\rm bag} & \text{at } x = x_c, \\ 0 & \text{at } x = \infty \ , \end{cases}$$

$$\mathcal{G}(x) := \left(\frac{\cosh 2x + \cosh \delta\phi}{\cosh \delta\phi} \right)^2$$

pipe param

bag param

$$\delta\phi = \phi_{\rm bag} - \phi_\infty$$

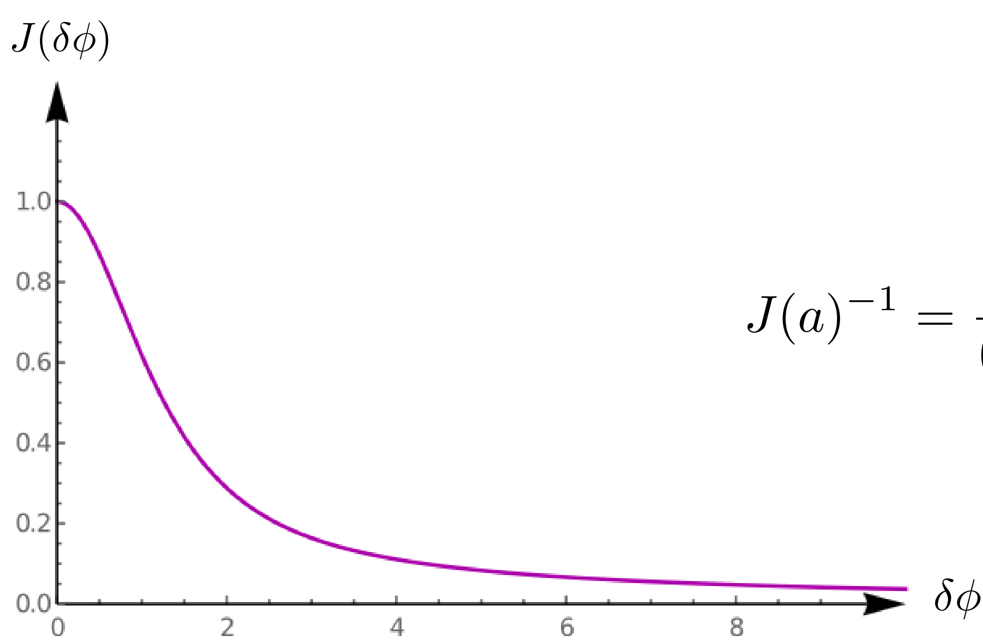
The problem can be integrated analytically with the result

$$\psi_0(x,a) \simeq \frac{1}{2}\psi_{\text{bag}}\left[1 - \frac{I(x,a)}{I(\infty,a)}\right]$$

$$x = \text{Re}z$$
$$a = \cosh\delta\phi$$

$$I(x,a) = \frac{a^3}{2(a^2-1)^{3/2}}\log\left[\frac{\sqrt{a+1}+\sqrt{a-1}\,\tanh x}{\sqrt{a+1}-\sqrt{a-1}\,\tanh x}\right] - \frac{a^2}{(a^2-1)}\frac{\tanh x}{[(a+1)-(a-1)\tanh^2 x]}$$

$$\lambda_0 + 2 = \frac{3\pi^3}{4}L_5^8\,\psi_{\text{bag}}^2\,J(a)$$



Janus correction factor

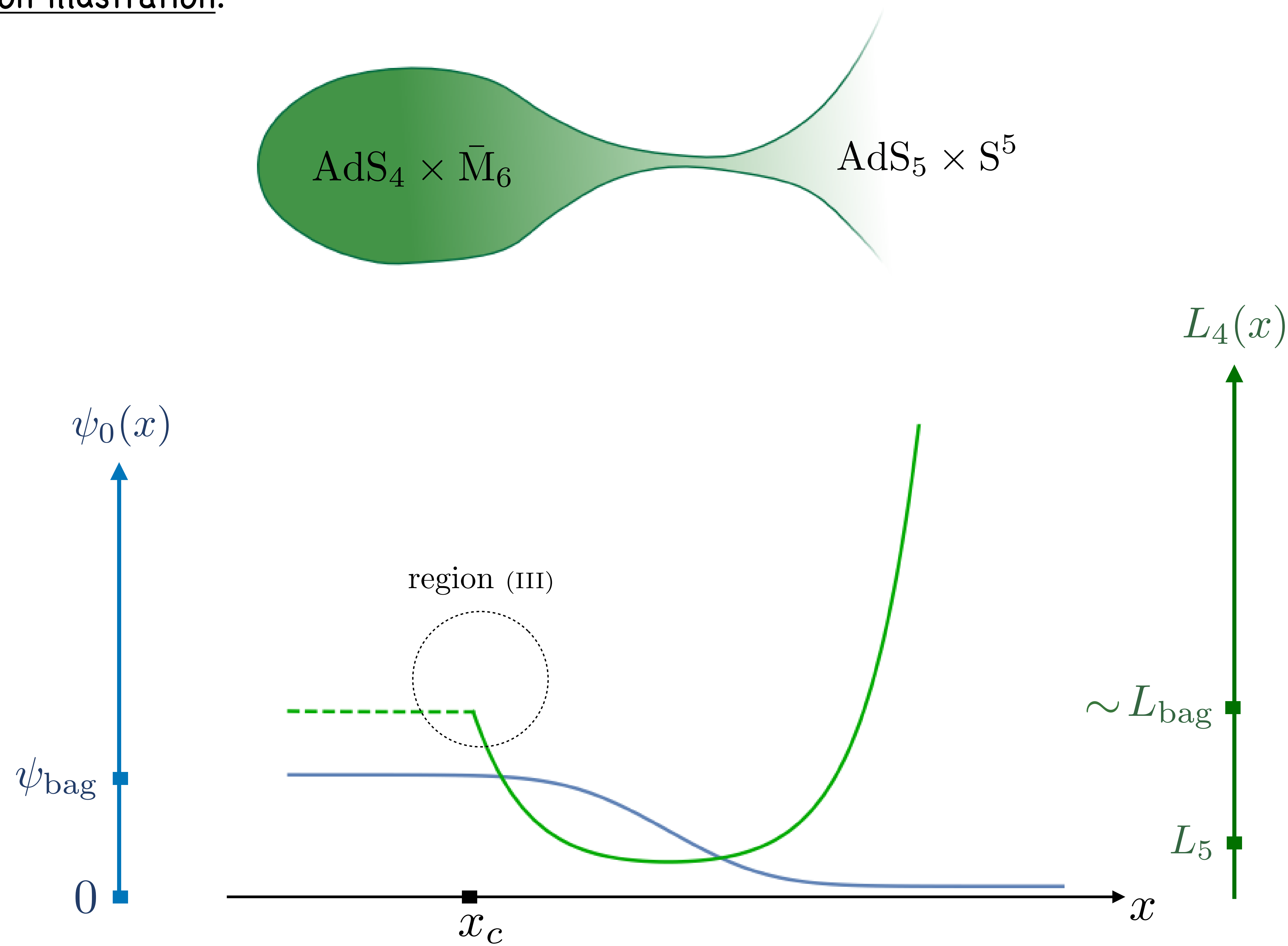
$$J(a)^{-1} = \frac{3a^3}{(a^2-1)^{3/2}}\log\left[a+\sqrt{a^2-1}\right] - \frac{3a^2}{(a^2-1)}$$

$\delta\phi \to \infty$ is decompactifion limit

no continuous Higgsing from this

CB, Estes '11

Cartoon illustration:



7 Rewritings and bigravity

cf Karch-Randall $\sim (L_5/L_4)^2$

geometric:

$$m_g^2 L_4^2 = \frac{3\pi^3 L_5^8}{4V_6 \langle L_4^2 \rangle_{\rm bag}} \times J(\cosh \delta\phi)$$

$\sim (L_5/L_4)^8$

quantized — eff coupling

gravitational:

$$m_g^2 L_4^2 = \frac{3\, n^2}{16\pi^2} \frac{\kappa_4^2}{\langle L_4^2 \rangle_{\rm bag}} \times J(\cosh \delta\phi)$$

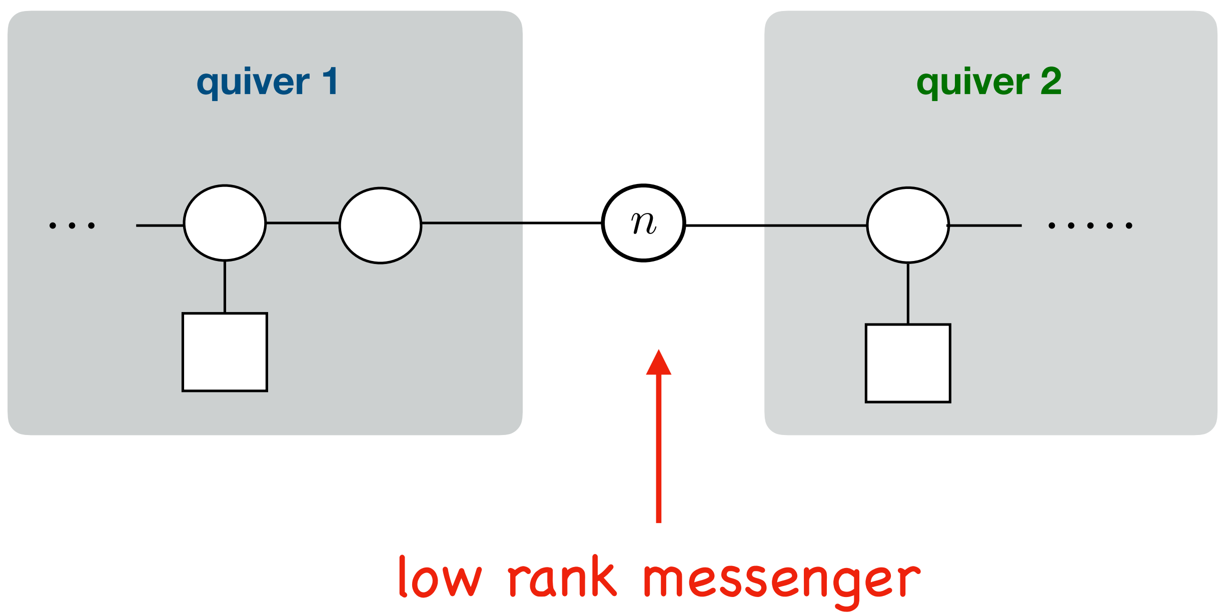
CFT:

$$m_g^2 L_4^2 = \frac{3\pi \tilde{F}_4}{32 \tilde{F}_3} \times J(\cosh \delta\phi)$$

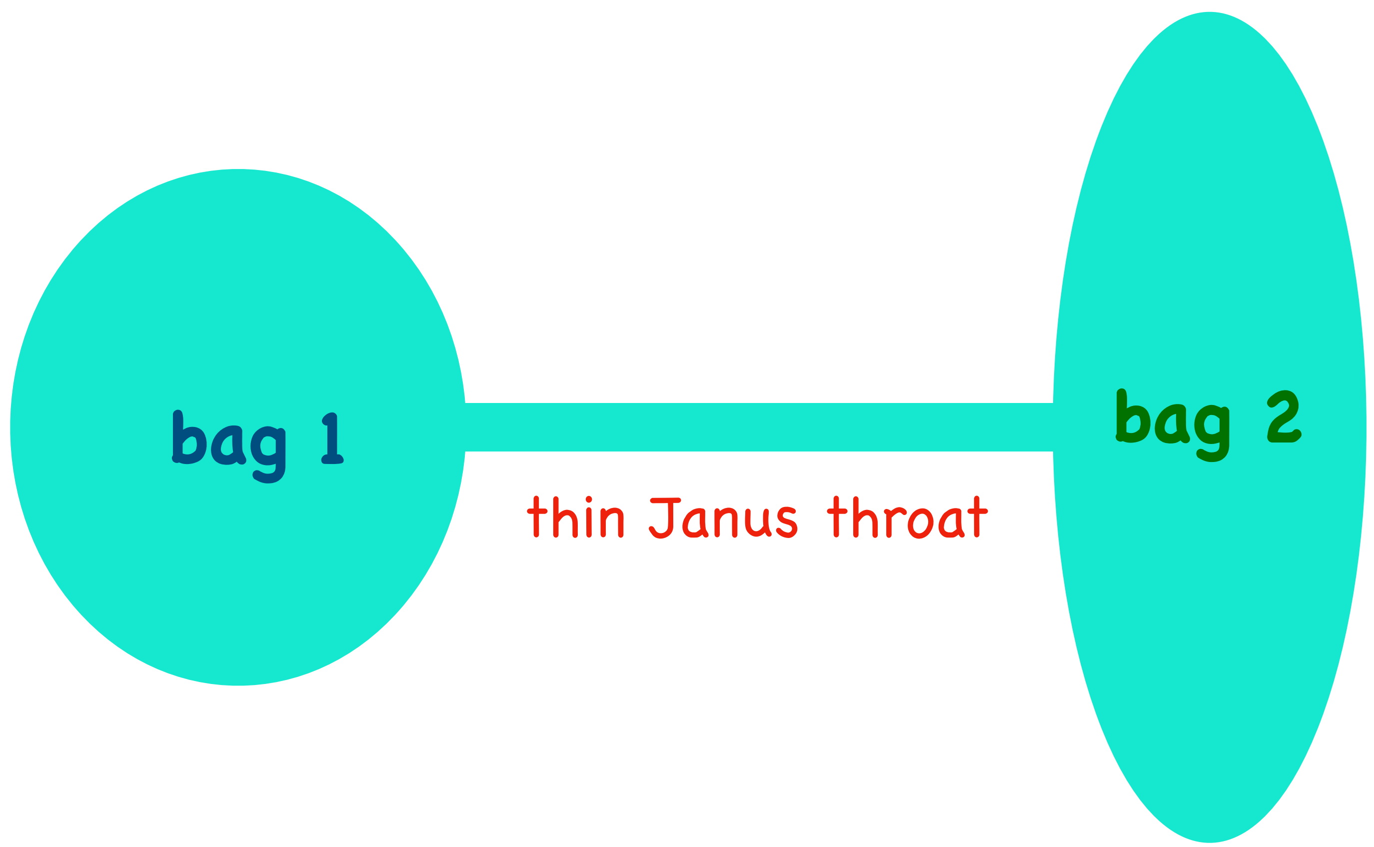
$\tilde{F}_d$: generalized free energy

Giombi + Klebanov '14

Simple to extend to models of bigravity:



Field Theory



Gravity

One massless and one massive graviton

$$m_{\rm g}^2 L_4^2 \;=\; \frac{3\,n^2}{16\pi^2}\left[\frac{\kappa_4^2}{\langle L_4^2\rangle_{\rm bag}} + \frac{\kappa_4^{2\,\prime}}{\langle L_4^2\rangle_{\rm bag'}}\right] \times J(\cosh\delta\phi)$$

Similar to double-trace (one-loop) deformation formula of Aharony, Clark, Karch '06

But:

- the background is exactly conformal (no RG running)
- Integrating-in the messengers restores local geometry at the `expense' of quantization of coupling

8 Last remarks

- Massive AdS gravity is a corner of the string-theory landscape

- The graviton mass is a (non-protected) quantized observable in these models. Can one compute it from CFT and match ?

- Other examples ? Effective low-E theory ? Minkowski ?

Thank You

notation of Cordova et al

$\mathcal{N}=4$ **Multiplet**	**String mode**	**Gauged SUGRA**
$A_2[0]_1^{(0;0)}$	Graviton	yes
$B_1[0]_1^{(1;0)}$	D5 gauge bosons	yes
$B_1[0]_1^{(0;1)}$	NS5 gauge bosons	yes
$B_1[0]_R^{(R>1;0)}$	Open F-strings, $R \in \frac{1}{2}\lvert\ell_a - \ell_b\rvert + \mathbb{N}$ Closed strings, $R \in \mathbb{N}$	only $R=2$
$B_1[0]_{R'}^{(0;R'>1)}$	Open D-strings, $R' \in \frac{1}{2}\lvert\hat{\ell}_{\hat{a}} - \hat{\ell}_{\hat{b}}\rvert + \mathbb{N}$ Closed strings, $R' \in \mathbb{N}$	only $R'=2$
$B_1[0]_{R+R'}^{(R\geq 1;R'\geq 1)}$	Kaluza Klein gravitini $(R,R'\in\mathbb{N})$	no
$A_2[0]_{1+R+R'}^{(R>0;R'>0)}$	Kaluza Klein gravitons $(R,R'\in\mathbb{N})$	no
$A_1[j>0]_{1+j+R+R'}^{(R;R')}$	Stringy excitations	no

scalar monopole harmonics on S2

superpotential

missing (1;1) superpotential

For more on the BPS spectrum see CB, Bianchi, Hanany arXiv: 1711.06722