Aligned Inflation near the Conifold

Florian Wolf

Max Planck Institute for Physics, Munich



Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

Workshop on Geometry and Physics at Castle Ringberg on November 20, 2016

Large-Field Inflation in String Theory

Experiments [BICEP2 '14, Planck '15] motivated to study large-field inflation.

- axions with shift symmetry prevent UV corrections
- axions arise naturally in String Theory as moduli



Large-Field Inflation in String Theory

Experiments [BICEP2 '14, Planck '15] motivated to study large-field inflation.

- axions with shift symmetry prevent UV corrections
- axions arise naturally in String Theory as moduli



Aligned Inflation [Kim, Nilles, Peloso '05]:

- 2 axionic moduli with sub-Planckian decay constants for controlled string compactification
- effective decay constant $f_{\rm eff}$ trans-Planckian to realise large-field inflation

Large-Field Inflation in String Theory

Experiments [BICEP2 '14, Planck '15] motivated to study large-field inflation.

- axions with shift symmetry prevent UV corrections
- axions arise naturally in String Theory as moduli



Aligned Inflation [Kim, Nilles, Peloso '05]:

- 2 axionic moduli with sub-Planckian decay constants for controlled string compactification
- effective decay constant $f_{\rm eff}$ trans-Planckian to realise large-field inflation

Difficulty: control **all** effective theories by correct mass hierarchy!

 $M_{\Theta} < H_{\rm inf} < M_{\rm mod} \sim M_{\rm inf} < M_{\rm KK} < M_{\rm s} < M_{\rm Pl}$

Based on [JHEP 1608 (2016) 110 by Blumenhagen, Herschmann, FW]

Based on [JHEP 1608 (2016) 110 by Blumenhagen, Herschmann, FW] *for more information see talk by Daniela Herschmann*

Mirror manifold of Calabi-Yau threefold $\mathbb{P}_{11226}[12]$ (similar [Conlon, Quevedo '04]):

$$P = z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 - 12\psi z_1 z_2 z_3 z_4 z_5 - 2\phi z_1^6 z_2^6$$

complex structure moduli

conifold locus at $864\psi^6+\phi=1$

 \rightarrow

Based on [JHEP 1608 (2016) 110 by Blumenhagen, Herschmann, FW] *for more information see talk by Daniela Herschmann*

Mirror manifold of Calabi-Yau threefold $\mathbb{P}_{11226}[12]$ (similar [Conlon, Quevedo '04]):

$$P = z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 - 12\psi z_1 z_2 z_3 z_4 z_5 - 2\phi z_1^6 z_2^6$$
complex structure moduli

$$\longrightarrow$$
 conifold locus at $864\psi^6 + \phi = 1$

Derive basis of periods solving Picard-Fuchs equations from fundamental period [Candelas, De La Ossa, Font, Katz, Morrison '94]

$$\varpi_f(\psi,\phi) = -\frac{1}{6} \sum_{n=1}^{\infty} \frac{\Gamma\left(\frac{n}{6}\right) (-12\,\psi)^n \, u_{-\frac{n}{6}}(\phi)}{\Gamma(n)\,\Gamma^2\left(1-\frac{n}{6}\right)\,\Gamma\left(1-\frac{n}{2}\right)}$$

with $u_{-\frac{n}{6}}(\phi)$ a polynomial series.

---> transform to symplectic basis and introduce new inhomogeneous coordinates $\{\psi,\phi\}\to\{Z,Y\}$

Final periods for $\mathbb{P}_{11226}[12]$:

 \rightarrow transform to symplectic basis and introduce new inhomogeneous coordinates $\{\psi, \phi\} \rightarrow \{Z$ Final pe Periods $F_0 = 1 \,,$ $F_1 = Z \,,$ $F_2 = (0.46 + 0.11i) + (1.10 - 2.17i)Y - 0.19Z$ $-(7.34 - 14.73i)Y^{2} + (2.71 + 1.42i)YZ + (0.11 - 1.69i)Z^{2}$ and $X^{0} = (-0.04 + 0.23i) + (1.10 + 0.06i)Y + 0.17Z$ $-(7.34+1.83i) Y^{2} + (0.55+1.42i) YZ + (0.11-0.17i) Z^{2},$ $X^{1} = -\frac{1}{2\pi i} Z \log Z + 0.18 - 0.42 Y - 1.43 i Z + \dots,$ $X^{2} = 0.09 - 2.19Y + 14.67Y^{2} - 2.84iYZ - 0.22Z^{2}$

---> transform to symplectic basis and introduce new inhomogeneous coordinates $\{\psi,\phi\}\to\{Z,Y\}$

Final periods for $\mathbb{P}_{11226}[12]$: $F_1 = Z$, $X^1 = -\frac{1}{2\pi i} Z \log Z + 0.18 - 0.42 Y - 1.43 i Z + \dots$, \vdots

Kähler potential:
$$K_{cs} = -\log\left[-i\left(X^{\Lambda}\bar{F}_{\Lambda} - \bar{X}^{\Lambda}F_{\Lambda}\right)\right]$$

$$= -\log\left[\frac{1}{2\pi}|Z|^{2}\log\left(|Z|^{2}\right) + A + \operatorname{Re}Y + B\left(\operatorname{Re}Y\right)^{2} + C|Z|^{2}\dots\right]$$

---> transform to symplectic basis and introduce new inhomogeneous coordinates $\{\psi,\phi\}\to\{Z,Y\}$

Final periods for $\mathbb{P}_{11226}[12]$: $F_1 = Z$, $X^1 = -\frac{1}{2\pi i} Z \log Z + 0.18 - 0.42 Y - 1.43 i Z + \dots$,

Kähler potential:
$$K_{cs} = -\log\left[-i\left(X^{\Lambda}\bar{F}_{\Lambda} - \bar{X}^{\Lambda}F_{\Lambda}\right)\right]$$
$$= -\log\left[\frac{1}{2\pi}|Z|^{2}\log\left(|Z|^{2}\right) + A + \operatorname{Re}Y + B\left(\operatorname{Re}Y\right)^{2} + C\left|Z\right|^{2}\dots\right]$$

- can change (e.g. linear term in Z) due to possible $Sp(6;\mathbb{Z})$ transf. of periods
- obeys shift symmetries: [Etxebarria, Grimm, Valenzuela '14]

$$\begin{split} \mathrm{Im}(Y) &\to \mathrm{Im}(Y) + \varphi & \text{axionic sym.} \\ & Z \to e^{i\theta}Z & \text{broken by higher orders to discrete sym.} \ \theta \to \theta + 2\pi \end{split}$$

Consider more flexible models inspired by periods and Kähler potential of $\mathbb{P}_{11226}[12]$.

Consider more flexible models inspired by periods and Kähler potential of $\mathbb{P}_{11226}[12]$.

General flux-induced superpotential [Gukov, Vafa, Witten '99]

$$W_{\rm GVW} = -\left(\mathfrak{f}_{\Lambda}X^{\Lambda} - \tilde{\mathfrak{f}}^{\Lambda}F_{\Lambda}\right) + iS\left(h_{\Lambda}X^{\Lambda} - \tilde{h}^{\Lambda}F_{\Lambda}\right)$$
periods
axio-dilaton $S = g_s^{-1} - iC_0$

Consider more flexible models inspired by periods and Kähler potential of $\mathbb{P}_{11226}[12]$.

General flux-induced superpotential [Gukov, Vafa, Witten '99]

$$W_{\rm GVW} = -\left(\mathfrak{f}_{\Lambda}X^{\Lambda} - \tilde{\mathfrak{f}}^{\Lambda}F_{\Lambda}\right) + iS\left(h_{\Lambda}X^{\Lambda} - \tilde{h}^{\Lambda}F_{\Lambda}\right)$$
periods
axio-dilaton $S = g_s^{-1} - iC_0$

Turn on fluxes to generate a superpotential of the form (similar [Giddings, Kachru, Polchinski '01])

$$W \sim f\left(-\frac{1}{2\pi i}Z\log Z + 0.18 - 0.42Y - 1.43iZ + \dots\right)$$
$$+ i\left(hS + \hat{f}Y\right)Z - ih'S - i\hat{f}'Y + \dots$$
$$\underset{\text{complex structure modulus } Y = y + i\zeta$$

Consider more flexible models inspired by periods and Kähler potential of $\mathbb{P}_{11226}[12]$.

General flux-induced superpotential [Gukov, Vafa, Witten '99]

$$W_{\rm GVW} = -\left(\mathfrak{f}_{\Lambda}X^{\Lambda} - \tilde{\mathfrak{f}}^{\Lambda}F_{\Lambda}\right) + iS\left(h_{\Lambda}X^{\Lambda} - \tilde{h}^{\Lambda}F_{\Lambda}\right)$$
periods
axio-dilaton $S = g_s^{-1} - iC_0$

Turn on fluxes to generate a superpotential of the form (similar [Giddings, Kachru, Polchinski '01])

$$W \sim f\left(-\frac{1}{2\pi i}Z\log Z + 0.18 - 0.42Y - 1.43iZ + \dots\right) + \frac{i\left(hS + \hat{f}Y\right)Z}{i\left(hS + \hat{f}Y\right)Z} - ih'S - i\hat{f}'Y + \dots$$

Conic modulus Z is stabilised via F-term condition $F_Z = 0$:

$$Z \sim \exp\left(-\frac{2\pi}{f}(hS + \hat{f}Y)\right)$$

axio-dilaton

complex structure modulus

Conic modulus Z is stabilised via F-term condition $F_Z = 0$:

$$Z \sim \exp\left(-\frac{2\pi}{f}(hS + \hat{f}Y)\right)$$

axio-dilaton

complex structure modulus

Integrating out heavy conic modulus Z yields effective superpotential

$$W_{\text{eff}} \sim i(f + h'S + \hat{f}'Y) + \not{Z} + \dots$$

----> tree-level superpotential

Conic modulus Z is stabilised via F-term condition $F_Z = 0$:

$$Z \sim \exp\left(-\frac{2\pi}{f}(hS + \hat{f}Y)\right)$$

axio-dilaton

complex structure modulus

Integrating out heavy conic modulus Z yields effective superpotential

$$W_{\text{eff}} \sim i(f + h'S + \hat{f}'Y) + \mathbb{Z} + \dots$$

----> tree-level superpotential

• assume Kähler potential $K_{\text{eff}} = -2\log \mathcal{V} - \log(S + \bar{S}) - \log(A + \kappa \operatorname{Re} Y - (\operatorname{Re} Y)^2)$

Minkowski minimum:

- axion $\Sigma = \hat{f}' \zeta h' C_0$ and saxions stabilised by fluxes
- axionic moduli combination $\Theta = \hat{f}\zeta hC_0$ remains unstabilised

• use unfixed axion Θ for large-field inflation

- use unfixed axion Θ for large-field inflation
- + consider subdominant term to stabilise Θ

$$W_{\text{eff}} = i\alpha \left(f + h'S + \hat{f}'Y\right) + Z + \dots$$

- use unfixed axion Θ for large-field inflation
- consider subdominant term to stabilise Θ

$$W_{\text{eff}} = i\alpha \left(f + h' S + \hat{f}' Y \right) + \left(\frac{f\hat{C}}{2\pi i} \exp \left(-\frac{2\pi}{f} \left(hS + \hat{f}Y \right) \right) \right)$$

 \rightarrow instanton-like term

- use unfixed axion Θ for large-field inflation
- consider subdominant term to stabilise Θ

$$W_{\text{eff}} = i\alpha \left(f + h'S + \hat{f}'Y \right) + \left(\frac{f\hat{C}}{2\pi i} \exp \left(-\frac{2\pi}{f} \left(hS + \hat{f}Y \right) \right) \right)$$

$$\longrightarrow \text{ instanton-like term}$$

backreaction of the inflaton shifts minimum slightly

The inflaton potential in the regime $f/\hat{f}' \gg 1$, $h/h' \gg 1$ and $\kappa = 0$ reads

$$V_{\text{eff}} \sim \frac{4|Z|^2}{A\mathcal{V}^2} \frac{fh^2}{h'} \left(1 - \cos\left(\frac{2\pi}{f}\Theta\right)\right)$$

- use unfixed axion Θ for large-field inflation
- consider subdominant term to stabilise Θ

$$W_{\text{eff}} = i\alpha \left(f + h'S + \hat{f}'Y \right) + \left(\frac{f\hat{C}}{2\pi i} \exp \left(-\frac{2\pi}{f} \left(hS + \hat{f}Y \right) \right) \right)$$

$$\longrightarrow \text{ instanton-like term}$$

backreaction of the inflaton shifts minimum slightly

The inflaton potential in the regime $f/\hat{f}' \gg 1$, $h/h' \gg 1$ and $\kappa = 0$ reads

$$V_{\text{eff}} \sim \frac{4|Z|^2}{A\mathcal{V}^2} \frac{fh^2}{h'} \left(1 - \cos\left(\frac{2\pi}{f}\Theta\right)\right)$$

Canonical normalisation gives the axion decay constant

$$f_{\tilde{\Theta}} = \frac{f}{2\pi\sqrt{A}} \frac{h'}{h\hat{f}' - h'\hat{f}}$$

- use unfixed axion Θ for large-field inflation
- consider subdominant term to stabilise Θ

$$W_{\text{eff}} = i\alpha \left(f + h'S + \hat{f}'Y \right) + \left(\frac{f\hat{C}}{2\pi i} \exp \left(-\frac{2\pi}{f} \left(hS + \hat{f}Y \right) \right) \right)$$

$$\longrightarrow \text{ instanton-like term}$$

backreaction of the inflaton shifts minimum slightly

The inflaton potential in the regime $f/\hat{f}' \gg 1$, $h/h' \gg 1$ and $\kappa = 0$ reads

$$V_{\text{eff}} \sim \frac{4|Z|^2}{A\mathcal{V}^2} \frac{fh^2}{h'} \left(1 - \cos\left(\frac{2\pi}{f}\Theta\right)\right)$$

Canonical normalisation gives the axion decay constant

$$f_{\tilde{\Theta}} = \frac{f}{2\pi\sqrt{A}} \frac{h'}{h\hat{f}' - h'\hat{f}}$$

 \longrightarrow alignment mechanism for $(h\hat{f}' - h'\hat{f}) < h'$

assume Kähler moduli are stabilised via LVS

• assume Kähler moduli are stabilised via LVS

| Scale | $(Mass)^2$ in $M_{\rm Pl}^2$ |
|---|---------------------------------|
| string scale $M_{\rm s}^2$ | $\frac{1}{f^{1/2}\mathcal{V}}$ |
| Kaluza-Klein scale $M^2_{\rm KK}$ | $rac{1}{\mathcal{V}^{4/3}}$ |
| conic c.s. modulus M_Z^2 | $rac{f}{\mathcal{V}^2 Z ^2}$ |
| inflationary mass scale $M_{\rm inf}^2$ | $rac{f^{1/2} Z }{\mathcal{V}}$ |
| other moduli $M^2_{\rm mod}$ | $rac{f}{\mathcal{V}^2}$ |
| gravitino mass $M_{3/2}^2$ | $rac{f}{\mathcal{V}^2}$ |
| large Kähler modulus $M_{\tau_b}^2$ | $rac{f^{5/2}}{\mathcal{V}^3}$ |
| inflaton $M^2_{\tilde{\Theta}}$ | $rac{ Z ^2}{f\mathcal{V}^2}$ |

• assume Kähler moduli are stabilised via LVS



• assume Kähler moduli are stabilised via LVS



assume K\u00e4hler moduli are stabilised via LVS



• assume Kähler moduli are stabilised via LVS

Summary of moduli masses and scales:

| Scale | $(Mass)^2$ in $M_{\rm Pl}^2$ |
|---|----------------------------------|
| string scale $M_{\rm s}^2$ | $rac{1}{f^{1/2}\mathcal{V}}$ |
| Kaluza-Klein scale $M^2_{\rm KK}$ | $rac{1}{\mathcal{V}^{4/3}}$ |
| conic c.s. modulus M_Z^2 | $rac{f}{\mathcal{V}^2 Z ^2}$ |
| inflationary mass scale $M_{\rm inf}^2$ | $\frac{f^{1/2} Z }{\mathcal{V}}$ |
| other moduli $M_{\rm mod}^2$ | $\frac{f}{\mathcal{V}^2}$ |
| gravitino mass $M_{3/2}^2$ | $\frac{f}{\mathcal{V}^2}$ |
| large Kähler modulus $M_{\tau_b}^2$ | $\frac{f^{5/2}}{\mathcal{V}^3}$ |
| inflaton $M^2_{\tilde{\Theta}}$ | $\frac{ Z ^2}{f \mathcal{V}^2}$ |

correct hierarchy in principle possible!

But generic mass hierarchy is given by

 $\left(M_{\tau_b} < M_{\tilde{\Theta}} < M_{\text{mod}} < M_{\text{inf}} \sim M_Z < M_{\text{KK}} < M_{\text{s}} < M_{\text{Pl}}\right)$

- proper inclusion of Kähler moduli
 - order of integrating out unclear
 - LVS minimum needs dS uplift [Quevedo, Cicoli, Valandro, Kallosh, Wrase, ...]

- proper inclusion of Kähler moduli
 - order of integrating out unclear
 - LVS minimum needs dS uplift [Quevedo, Cicoli, Valandro, Kallosh, Wrase, ...]
- suppression of Y polynomial terms in $W_{\rm eff}$

e.g.
$$W_{\text{eff}} = i\alpha \left(f + h'S + \hat{f}'Y\right) + iBY^2 + \frac{f\hat{C}}{2\pi i} \exp\left(-\frac{2\pi}{f}(hS + \hat{f}Y)\right)$$

- proper inclusion of Kähler moduli
 - order of integrating out unclear
 - LVS minimum needs dS uplift [Quevedo, Cicoli, Valandro, Kallosh, Wrase, ...]
- suppression of Y polynomial terms in $W_{\rm eff}$

e.g.
$$W_{\text{eff}} = i\alpha \left(f + h'S + \hat{f}'Y \right) + iBY^2 + \frac{f\hat{C}}{2\pi i} \exp\left(-\frac{2\pi}{f}(hS + \hat{f}Y)\right)$$





Do instanton-like terms in $W_{\rm eff}$ satisfy the weak gravity conjecture (WGC)?

Do instanton-like terms in $W_{\rm eff}$ satisfy the weak gravity conjecture (WGC)?

 $S_{\text{inst}} \cdot f_{\text{inst}} \leq 1$

WGC for instantons:

instanton action

axion decay constant

Do instanton-like terms in $W_{\rm eff}$ satisfy the weak gravity conjecture (WGC)?



Loop-Hole to fulfill mild WGC:

[Brown, Cottrell, Shiu, Soler '15, Hebecker, Mangat, Rompineve, Witkowski '15]

• add second instanton $S_{\rm inst}^{(2)}$ (e.g. D(-1)) with $S_{\rm inst} < S_{\rm inst}^{(2)}$

$$V \sim e^{-2 S_{\text{inst}}} \left(1 - \cos\left(\frac{\tilde{\Theta}}{f_{\tilde{\Theta}}}\right) \right) + e^{-2 S_{\text{inst}}^{(2)}} \left(1 - \cos\left(\frac{k \tilde{\Theta}}{f_{\tilde{\Theta}}}\right) \right)$$

Do instanton-like terms in $W_{\rm eff}$ satisfy the weak gravity conjecture (WGC)?



[Brown, Cottrell, Shiu, Soler '15, Hebecker, Mangat, Rompineve, Witkowski '15] • add second instanton $S_{inst}^{(2)}$ (e.g. D(-1)) with $S_{inst} < S_{inst}^{(2)}$

$$V \sim e^{-2 S_{\text{inst}}} \left(1 - \cos\left(\frac{\tilde{\Theta}}{f_{\tilde{\Theta}}}\right)\right) + e^{-2 S_{\text{inst}}^{(2)}} \left(1 - \cos\left(\frac{k \tilde{\Theta}}{f_{\tilde{\Theta}}}\right)\right)$$

large-field inflation

Do instanton-like terms in $W_{\rm eff}$ satisfy the weak gravity conjecture (WGC)?

large-field inflation

satisfies WGC

Summary:

- integrating out conic modulus mimics instanton-like term in the superpotential
- ignoring Kähler moduli can lead to aligned inflation
- exponential hierarchies give control over effective theories

Summary:

- integrating out conic modulus mimics instanton-like term in the superpotential
- ignoring Kähler moduli can lead to aligned inflation
- exponential hierarchies give control over effective theories

Open questions:

- construction of a realistic model with concrete flux choice
- further analysis of conifold locus [Bizet, Loaiza-Brito, Zavala '16]
- other special points in moduli space: Gepner point, colliding conifolds [Curio, Klemm, Lüst, Theisen '01]

Summary:

- integrating out conic modulus mimics instanton-like term in the superpotential
- ignoring Kähler moduli can lead to aligned inflation
- exponential hierarchies give control over effective theories

Open questions:

- construction of a realistic model with concrete flux choice
- further analysis of conifold locus [Bizet, Loaiza-Brito, Zavala '16]
- other special points in moduli space: Gepner point, colliding conifolds [Curio, Klemm, Lüst, Theisen '01]

Thank you!