$SL(2) \times \mathbb{R}^+$ Exceptional Field Theory
An Action for F-Theory

Geometry and Physics

Schloss Ringberg

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Outline

What is Exceptional Field Theory?

$\mathrm{SL}(2) \times \mathbb{R}^+$ Exceptional Field Theory

Relation to Supergravity, Type IIB and F-Theory

What is Exceptional Field Theory?

- "Like Double Field Theory for U-duality and M-theory"
- U-duality covariant formulation of supergravity
- Higher dimensional origin of exceptional symmetries

General features

- Fields and coordinates in representations of G
  $\Rightarrow$ manifest duality symmetry
- Introduce extra coordinates $\Rightarrow$ extended space
- Combines metric and forms
  $\Rightarrow$ generalized diffeomorphism symmetry
- For closure & consistency $\Rightarrow$ section condition
  (reduces coordinate dependence)
- After imposing section condition
  $\Rightarrow$ 11-dim. SUGRA or 10-dim. IIB SUGRA

U-duality

- Reduce maximal 11-dim. SUGRA on a D-torus
- Enhanced (hidden) symmetry G - scalars $\mathcal{M} \in G/H$ coset
- Make manifest by extending the geometry

11-D	D	G	H
9	2	$\mathrm{SL}(2) \times \mathbb{R}^+$	$\mathrm{SO}(2)$
8	3	$\mathrm{SL}(3) \times \mathrm{SL}(2)$	$\mathrm{SO}(3) \times \mathrm{SO}(2)$
7	4	$\mathrm{SL}(5)$	$\mathrm{SO}(5)$
6	5	$\mathrm{SO}(5,5)$	$\mathrm{SO}(5) \times \mathrm{SO}(5)$
5	6	E_6	$\mathrm{USp}(8)$
4	7	E_7	$\mathrm{SU}(8)$
3	8	E_8	$\mathrm{SO}(16)$

Constructive explanation

- Take 11-dim. SUGRA $\Rightarrow$ split coordinates $11 = (11 - D) + D$

$$\hat{x}^{\hat{\mu}} \to (x^{\mu}, y^i)$$

- Introduce new dual coordinates

$$\tilde{y}_{ij}, \ \tilde{z}_{ijklm}, \ \dots$$

associated with brane charges (wrapping modes)

- Combine with the original coordinates $\Rightarrow$ extended coordinates

$$Y^M = (y^i, \tilde{y}_{ij}, \tilde{z}_{ijklm}, \dots)$$

form a representation of G

Constructive explanation

Classify d.o.f. under splitting $\mathrm{SO}(1, 10-D) \times \mathrm{SO}(D)$

(like KK reduction) e.g.:

$$\hat{g}_{\hat{\mu}\hat{\nu}} \to g_{\mu\nu},\ A_{\mu i},\ g_{ij}$$
$$\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} \to C_{\mu\nu\rho},\ C_{\mu\nu\ i},\ C_{\mu\ ij},\ C_{ijk}$$

Repackage fields into representations of G

- Metric $g_{\mu\nu}$
- Vector field ${A_\mu}^M$
- Forms ${B_{\mu\nu}}^\alpha, {C_{\mu\nu\rho}}^{[\alpha\beta]}, \ldots$ (after dualization)
- Scalars $\mathcal{M}_{MN} \in G/H$

Constructive explanation

Repackage symmetries

- Diffeomorphisms plus gauge transformations give generalized diffeomorphisms

$$\delta_\Lambda V^M = \Lambda^N \partial_N V^M - V^N \partial_N \Lambda^M + Y^{MN}{}_{PQ} \partial_N \Lambda^P V^Q$$

(Y-tensor is G-invariant)

Also have

- Tensor hierarchy of forms
- External diffeomorphisms (covariantized under gen. diffeos)

$$\partial_\mu \to D_\mu \equiv \partial_\mu - \mathcal{L}_{A_\mu}$$

Consistency of symmetries

Generalized diffeomorphism form symmetry algebra

- Closure requires section condition

$$Y^{MN}{}_{PQ}\partial_M\mathcal{O}_1\partial_N\mathcal{O}_2 = 0, \qquad Y^{MN}{}_{PQ}\partial_M\partial_N\mathcal{O} = 0$$

- Restricts Y^M coordinate dependence

Two inequivalent solutions not related by G

- **M-theory section**: at most D of Y^M coordinates
  $\Rightarrow$ in total $(11 - D) + D = 11$
- **IIB section**: at most $D - 1$ of Y^M coordinates
  $\Rightarrow$ in total $(11 - D) + (D - 1) = 10$

Action

The (bosonic) action is fixed by the (bosonic) symmetries

$$S = \int \mathrm{d}x\mathrm{d}Y \sqrt{g} \left[\hat{R}(g) + \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_{\mathrm{pot}} + \frac{1}{\sqrt{g}} \mathcal{L}_{\mathrm{top}} \right]$$

- Ricci scalar $\hat{R}(g) \sim (D_\mu g)^2$
- Kinetic and gauge field terms $\mathcal{L}_{kin} \sim (D_\mu \mathcal{M})^2 + \mathcal{F}^2$
- Scalar potential $\mathcal{L}_{\mathrm{pot}} = V(M, g) \sim (\partial_M \mathcal{M})^2 + (\partial_M g)^2$
- Chern-Simons $\mathcal{L}_{top}$
- Invariant under local G by construction, global G manifest
- Input section condition choice $\Rightarrow$ equivalent to 11-dim. SUGRA / 10-dim. IIB SUGRA

Different Cases

- The full EFT has been constructed for
  - E_8, E_7, E_6 [Hohm, Samtleben 2013; Cederwall, Rosabal 2015]
  - $\mathrm{SO}(5,5)$ [Abzalov, Bakhmatov, Musaev 2015]
  - $\mathrm{SL}(5)$ [Musaev 2015]
  - $\mathrm{SL}(3) \times \mathrm{SL}(2)$ [Hohm, Wang 2015]
  - $+$ SUSY E_7, E_6 [Godazgar, Godazgar, Hohm, Nicolai, Samtleben 2014; Musaev, Samtleben 2014]
- So what about $\mathrm{SL}(2) \times \mathbb{R}^+$?
  Duality group for 11-dim. SUGRA on T^2 / 10-dim. SUGRA on S^1 / F-theory

$\mathrm{SL}(2) \times \mathbb{R}^+$ EFT - simplest extended geometry

Generalized Coordinates

- External: x^μ, $\mu = 1, \ldots, 9$
- Internal: $Y^M = (y^\alpha, y^s)$ in $\mathbf{2}_2 \oplus \mathbf{1}_{-1}$ of $\mathrm{SL}(2) \times \mathbb{R}^+$
- Covariant derivative: $\partial_\mu \to D_\mu \equiv \partial_\mu - \mathcal{L}_{A_\mu}$

Section Condition

$$\partial_\alpha \otimes \partial_s = 0$$

- **M-theory section** $\partial_s = 0$: coordinate dependence on (x^μ, y^α)
- **IIB section** $\partial_\alpha = 0$: coordinate dependence on (x^μ, y^s)

Field Content

Metric and Scalars

- External metric $g_{\mu\nu}$
- Coset valued generalized metric $\mathcal{M}_{MN} \in \mathrm{SL}(2) \times \mathbb{R}^+/\mathrm{SO}(2)$

$$\mathcal{H}_{\alpha\beta} \in \mathrm{SL}(2)/\mathrm{SO}(2)$$
$$\mathcal{M}_{ss} \in \mathbb{R}^+$$

$$\mathcal{M}_{\alpha\beta} = \mathcal{M}_{ss}^{-3/4}\mathcal{H}_{\alpha\beta} \qquad \mathcal{H}_{\alpha\beta} = \frac{1}{\operatorname{Im}\tau}\begin{pmatrix} |\tau|^2 & \operatorname{Re}\tau \\ \operatorname{Re}\tau & 1 \end{pmatrix}$$

Field Content

Tensor Hierarchy

Representation	Gauge potential	Field strength
$\mathbf{2}_1 \oplus \mathbf{1}_{-1}$	$A_\mu{}^M$	$\mathcal{F}_{\mu\nu}{}^M$
$\mathbf{2}_0$	$B_{\mu\nu}{}^{\alpha s}$	$\mathcal{H}_{\mu\nu\rho}{}^{\alpha s}$
$\mathbf{1}_1$	$C_{\mu\nu\rho}{}^{[\alpha\beta]s}$	$\mathcal{J}_{\mu\nu\rho\sigma}{}^{[\alpha\beta]s}$
$\mathbf{1}_0$	$D_{\mu\nu\rho\sigma}{}^{[\alpha\beta]ss}$	$\mathcal{K}_{\mu\nu\rho\sigma\lambda}{}^{[\alpha\beta]ss}$
$\mathbf{2}_1$	$E_{\mu\nu\rho\kappa}{}^{\gamma[\alpha\beta]ss}$	$\mathcal{L}_{\mu\nu\rho\sigma\kappa\lambda}{}^{\gamma[\alpha\beta]ss}$
$\mathbf{2}_0 \oplus \mathbf{1}_2$	$F_{\mu\nu\rho\sigma\kappa\lambda}{}^M$	

Action

The (bosonic) action is entirely fixed by the (bosonic) symmetries:

$$S = \int \mathrm{d}^9 x \mathrm{d}^3 Y \sqrt{g} \left[\hat{R}(g) + \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} + \frac{1}{\sqrt{g}} \mathcal{L}_{\text{top}} \right]$$

$$\begin{aligned} \mathcal{L}_{\text{kin}} = & -\frac{7}{32} g^{\mu\nu} D_\mu \ln \mathcal{M}_{ss} D_\nu \ln \mathcal{M}_{ss} + \frac{1}{4} g^{\mu\nu} D_\mu \mathcal{H}_{\alpha\beta} D_\nu \mathcal{H}^{\alpha\beta} \\ & - \frac{1}{2 \cdot 2!} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu N} - \frac{1}{2 \cdot 3!} \mathcal{M}_{\alpha\beta} \mathcal{M}_{ss} \mathcal{H}_{\mu\nu\rho}{}^{\alpha s} \mathcal{H}^{\mu\nu\rho\beta s} \\ & - \frac{1}{2 \cdot 2! \cdot 4!} \mathcal{M}_{ss} \mathcal{M}_{\alpha\gamma} \mathcal{M}_{\beta\delta} \mathcal{J}_{\mu\nu\rho\sigma}{}^{[\alpha\beta]s} \mathcal{J}^{\mu\nu\rho\sigma[\gamma\delta]s} \end{aligned}$$

Action

$$
\begin{aligned}
\mathcal{L}_{\text{pot}} = &\frac{1}{4}\mathcal{M}^{ss}\left(\partial_s\mathcal{H}^{\alpha\beta}\partial_s\mathcal{H}_{\alpha\beta} + \partial_s g^{\mu\nu}\partial_s g_{\mu\nu} + \partial_s \ln|g|\partial_s \ln|g|\right) \\
&+ \frac{9}{32}\mathcal{M}^{ss}\partial_s \ln \mathcal{M}_{ss}\partial_s \ln \mathcal{M}_{ss} - \frac{1}{2}\mathcal{M}^{ss}\partial_s \ln \mathcal{M}_{ss}\partial_s \ln|g| \\
&+ \mathcal{M}_{ss}^{3/4}\left[\frac{1}{4}\mathcal{H}^{\alpha\beta}\partial_\alpha\mathcal{H}^{\gamma\delta}\partial_\beta\mathcal{H}_{\gamma\delta} + \frac{1}{2}\mathcal{H}^{\alpha\beta}\partial_\alpha\mathcal{H}^{\gamma\delta}\partial_\gamma\mathcal{H}_{\delta\beta}\right. \\
&\qquad + \partial_\alpha\mathcal{H}^{\alpha\beta}\partial_\beta \ln\left(|g|^{1/2}\mathcal{M}_{ss}^{3/4}\right) \\
&\qquad + \frac{1}{4}\mathcal{H}^{\alpha\beta}\left(\partial_\alpha g^{\mu\nu}\partial_\beta g_{\mu\nu} + \partial_\alpha \ln|g|\partial_\beta \ln|g|\right. \\
&\qquad \left.\left. + \frac{1}{4}\partial_\alpha \ln \mathcal{M}_{ss}\partial_\beta \ln \mathcal{M}_{ss} + \frac{1}{2}\partial_\alpha \ln g\partial_\beta \ln \mathcal{M}_{ss}\right)\right]
\end{aligned}
$$

Relation to Supergravity, Type IIB and F-Theory

Embedding Supergravity - EFT Dictionary

EFT field	M-theory	Type IIB
$\mathcal{H}_{\alpha\beta}$	$g^{-1/2} g_{\alpha\beta}$	$\mathcal{H}_{\alpha\beta}$
$\mathcal{M}_{ss}$	$g^{-6/7}$	$(g_{ss})^{8/7}$
${A_\mu}^\alpha$	${A_\mu}^\alpha$	$B_{\mu s}, C_{\mu s}$
${A_\mu}^s$	$C_{\mu\alpha\beta}$	${A_\mu}^s$
${B_{\mu\nu}}^{\alpha s}$	$C_{\mu\nu\alpha}$	$B_{\mu\nu},\ C_{\mu\nu}$
${C_{\mu\nu\rho}}^{\alpha\beta s}$	$C_{\mu\nu\rho}$	$C_{\mu\nu\rho s}$
${D_{\mu\nu\rho\sigma}}^{\alpha\beta ss}$	dual	$C_{\mu\nu\rho\sigma}$

M-Theory / Type IIB Duality

- M-Theory on $T^2(V, \tau)$ gives IIB on $\tilde{S}^1_B(\tilde{R}_B)$ with

$$\tilde{R}_B = V^{-3/4}$$

- Limit of vanishing torus $V \to 0 \Rightarrow$ IIB in 10 dimensions
- Generalization of T-duality: exchange membrane winding with momentum
- Keep track of membrane winding modes!

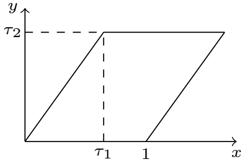
Relation to Supergravity, Type IIB and F-Theory

Type IIB SUGRA in 10 dimensions - $\mathrm{SL}(2)$ invariant

- scalars $\rightarrow$ complex modulus of torus: $\tau = C_0 + ie^{-\phi}$

Geometrical Origin

- Shape of a torus is parametrized by a complex structure τ



$(x, y) \sim (x + 1, y) \sim (x + \tau_1, y + \tau_2),\ \tau = \tau_1 + i\tau_2$

- Introduce an auxiliary torus $T^2 \Rightarrow$ 10-dim. Type IIB with τ varying in spacetime

Higher dimensional origins

Type IIA / M-theory

- Geometric interpretation of dilaton in Type IIA
- Extra circle $\Rightarrow$ in large limit: M-theory (11-dim. SUGRA)

Type IIB / F-theory

- 12-dim. origin of theory? No 12-dim. SUGRA!
- F-theory is a framework for analyzing these fibrations [Vafa '96]

Relationship to F-theory

- Both $\mathrm{SL}(2) \times \mathbb{R}^+$ EFT and F-theory give a 12-dim. perspective on Type IIB
- EFT: Extended space has local $\mathrm{SL}(2) \times \mathbb{R}^+$ symmetry via generalized diffeomorphisms - not conventional geometry
- F-theory auxiliary torus is now part of the EFT extended geometry
- EFT may also be reduced to M-section and when there are two isometries then we have M-theory/IIB duality

M-theory/F-theory duality

- Explicit realization of M-theory/F-theory duality ⇒ direct mapping between fields via EFT dictionary
- Consider just extended directions

$$\text{"}\,\mathrm{d}s^2_{(3)}\,\text{"} = (\mathcal{M}_{ss})^{-3/4}\mathcal{H}_{\alpha\beta}\mathrm{d}y^\alpha \mathrm{d}y^\beta + \mathcal{M}_{ss}(\mathrm{d}y^s)^2$$

- Limit $\mathcal{M}_{ss} \to 0$ ⇒ M-theory directions large

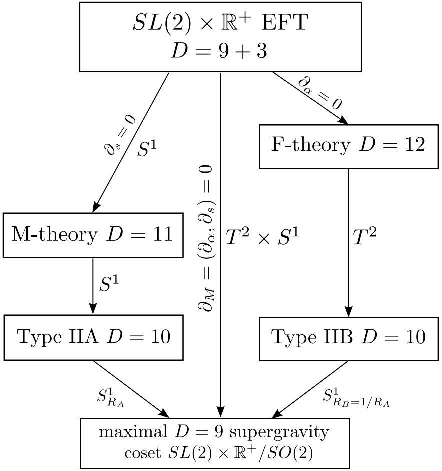
$$\mathrm{d}s^2_M = V\mathcal{H}_{\alpha\beta}\mathrm{d}y^\alpha \mathrm{d}y^\beta$$

- Limit $\mathcal{M}_{ss} \to \infty$ ⇒ IIB direction large

$$\mathrm{d}s^2_{IIB} = V^{-3/2}(\mathrm{d}y^s)^2$$

usual relation $\tilde{R}_B \sim V^{-3/4}$.

Schematic Overview



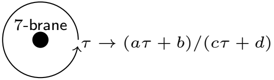
Summary

- Overview of EFT
- Construction of $\mathrm{SL}(2) \times \mathbb{R}^+$ EFT
- Action for F-Theory
- Duality between M-Theory/IIA and F-Theory/IIB

Sevenbranes in F-Theory

Sevenbrane backgrounds $\rightarrow$ codimension 2

- transverse coordinate $z \in \mathbb{C}$: $\tau = \tau(z)$ describes fibration of torus (elliptic fibration)
- At sevenbrane position, $\tau \sim \frac{1}{2\pi i} \log(z - z_{D_7})$. So $\tau \rightarrow i\infty$ at brane position. Paths around the brane have an $\mathrm{SL}(2)$ monodromy $\tau \rightarrow \tau + 1$.



- More general monodromies allowed $\Rightarrow$ non-perturbative (recall $\tau = C_0 + i/g_s$).

Sevenbranes

Metrics

- $ds^2_{(9)} = -dt^2 + d\vec{x}^2_{(6)} + \tau_2 |f|^2 dz d\bar{z}$
- $\text{"}\, ds^2_{(3)} \,\text{"} = \frac{1}{\tau_2}\left[|\tau|^2 (dy^1)^2 + 2\tau_1 dy^1 dy^2 + (dy^2)^2 \right] + (dy^s)^2$

where

$$\tau = j^{-1}(P(z)/Q(z))$$

and $P(z)$ and $Q(z)$ are polynomials in z

- Roots of $Q(z) \rightarrow$ brane locations
- Near brane, solution like "smeared monopole" in extended space

Sevenbranes

On different sections

- D7 in Type IIB

$$\mathrm{d}s^2_{IIB} = -\mathrm{d}t^2 + \mathrm{d}\vec{x}^2_{(6)} + (\mathrm{d}y^s)^2 + \tau_2|f|^2\mathrm{d}z\mathrm{d}\bar{z}$$

- KK7 in M-theory (D6 in Type IIA)

$$\mathrm{d}s^2_M = -\mathrm{d}t^2 + \mathrm{d}\vec{x}^2_{(6)} + \tau_2|f|^2\mathrm{d}z\mathrm{d}\bar{z} + \tau_2(\mathrm{d}y^1)^2 + \frac{1}{\tau_2}\left(\mathrm{d}y^2 + \tau_1\mathrm{d}y^1\right)^2$$

Topological Term

$$S_{top} = \frac{1}{5! \cdot 48} \int d^{10}x \, d^3Y \, \varepsilon^{\mu_1 \ldots \mu_{10}} \frac{1}{4} \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \Big[\frac{1}{5} \partial_s \mathcal{K}_{\mu_1 \ldots \mu_5}{}^{\alpha\beta ss} \mathcal{K}_{\mu_6 \ldots \mu_{10}}{}^{\gamma\delta ss} - \frac{5}{2} \mathcal{F}_{\mu_1\mu_2}{}^{s} \mathcal{J}_{\mu_3 \ldots \mu_6}{}^{\alpha\beta s} \mathcal{J}_{\mu_7 \ldots \mu_{10}}{}^{\gamma\delta} + \frac{10}{3} 2 \mathcal{H}_{\mu_1 \ldots \mu_3}{}^{\alpha s} \mathcal{H}_{\mu_4 \ldots \mu_6}{}^{\beta s} \mathcal{J}_{\mu_7 \ldots \mu_{10}}{}^{\gamma\delta} \Big]$$