$\mathcal{N} = 3$ four dimensional field theories

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Based on
[1512.06434] with I. García-Etxebarria

Goal

Understand F-theory at singularities with **no supersymmetric smoothing**.

- M-theory on weakly curved spaces, at low energies: 11d supergravity.

- Typically: family of spaces X_ϵ, parametrized by ϵ such that:
  - X_0 is the singular supersymmetric space of interest.
  - X_ϵ is smooth and preserves SUSY, for $\epsilon \neq 0$.
  - Study X_ϵ and take the limit $\epsilon \to 0$.

- What if X_ϵ is always SUSY breaking for $\epsilon \neq 0$?
  - Does this even make sense? Yes! The CFT description on S^1 is fine.

- This kind of singularities have not been classified. We will focus on some simple examples.
  - D3-branes probing codimension 4 terminal singularities in F-theory.

Strategy

We will examine a well-known example of this and then generalize it.

- D3-branes probing an O3-plane from several perspectives:
  - Worldsheet
  - M/F-theory
  - 4d field theory
  - Holographic dual
- Generalize to obtain new setups.

As we will see, the simplest generalization leads to $\mathcal{N} = 3$ four dimensional field theories.

O3s in perturbation theory

- In 2d CFT, O3s are defined as the **quotient** of 10d Type IIB by $\mathcal{I}(-1)^{F_L}\Omega$

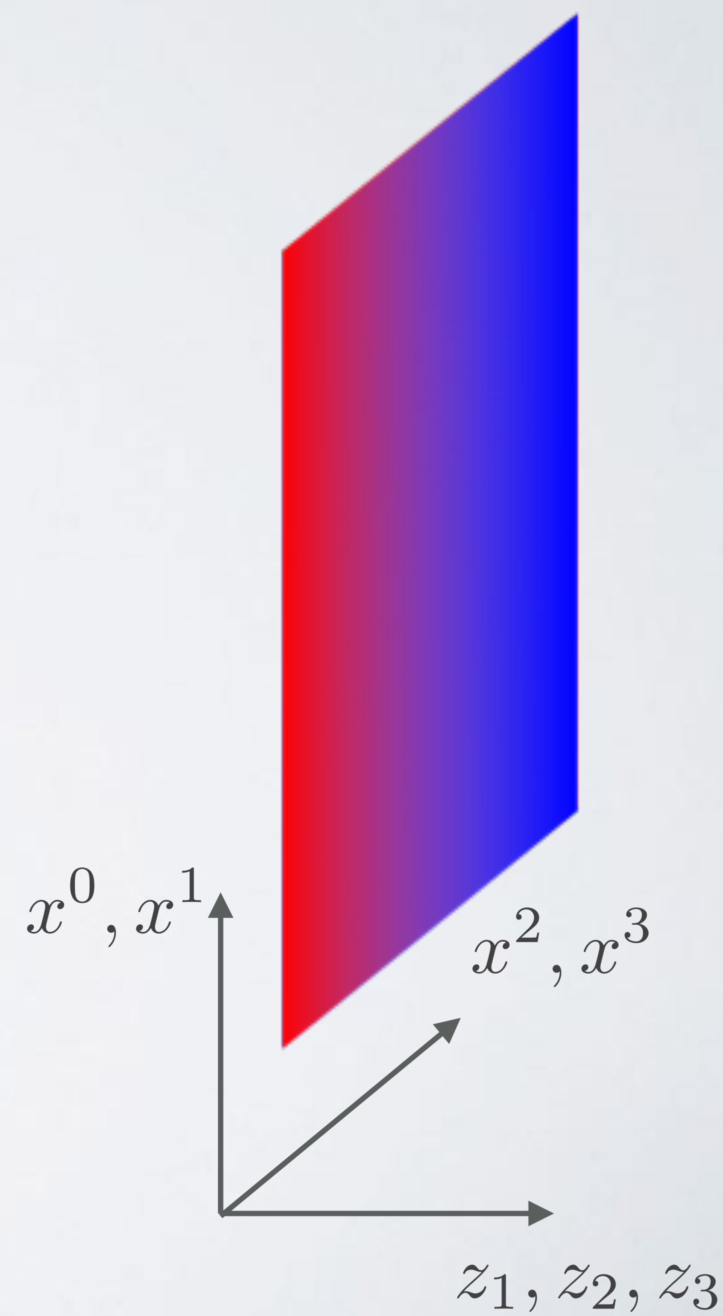
$$\mathcal{I} : (z_1,\, z_2,\, z_3) \to (-z_1, -z_2, -z_3)$$

$(-1)^{F_L}$: left moving spacetime fermion number

Ω : orientation reversal on the worldsheet

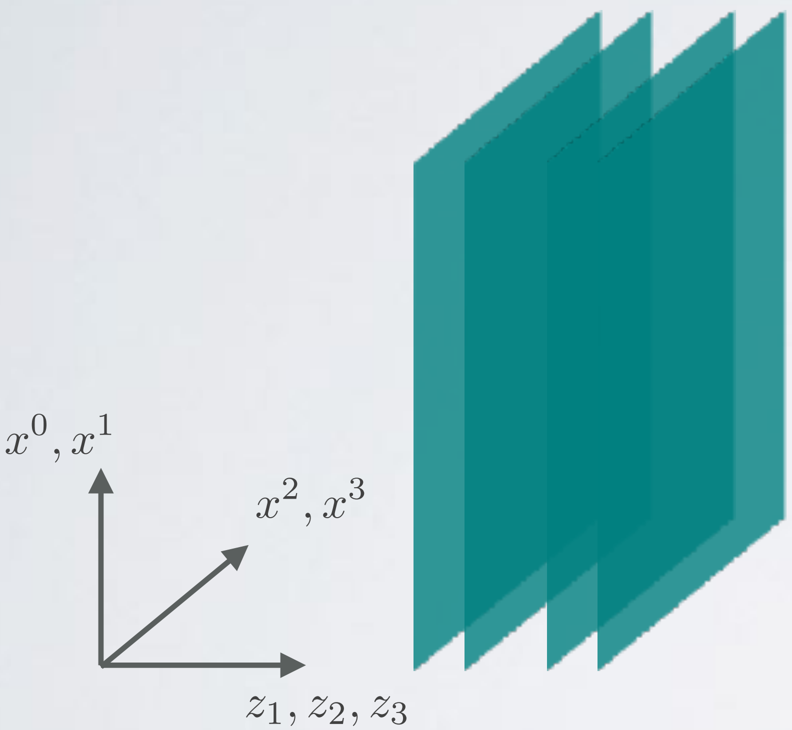
- Two different actions → Purely geometrical / Worldsheet

- Some key properties:
  - Breaks 10d Poincare
  - Breaks half of the supercharges
  - Has no dynamics
  - Comes in different flavors

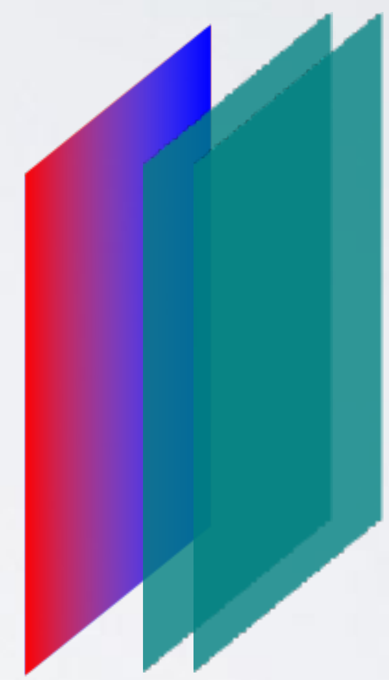


O3s+D3s in perturbation theory

- Since the O3 has no dynamics, we include N parallel D3s to probe it.
- Extra open string sector. The orientifold action also acts on this sector.



$$\xrightarrow{\mathcal{I}(-1)^{F_L}\Omega}$$



Before the quotient

After the quotient

$$4d\ \mathcal{N}=4\ \mathfrak{u}(N) \longrightarrow \begin{array}{l} 4d\ \mathcal{N}=4\ \mathfrak{so}(N) \\ 4d\ \mathcal{N}=4\ \mathfrak{usp}(N) \quad (N\in 2\mathbb{Z}) \end{array}$$

O3s in M/F-theory

What is the F-theory description of an O3-plane? [Hanany, Kol]

- M-theory description of Type IIB (F-theory):

M-theory on $\mathbb{R}^{1,2} \times \mathbb{C}^3 \times T^2 \xrightarrow{T^2 \to 0}$ Type IIB on $\mathbb{R}^{1,3} \times \mathbb{C}^3$

(Recall that the O3 is a **quotient** of Type IIB: geometrical + worldsheet)

- With the orientifold

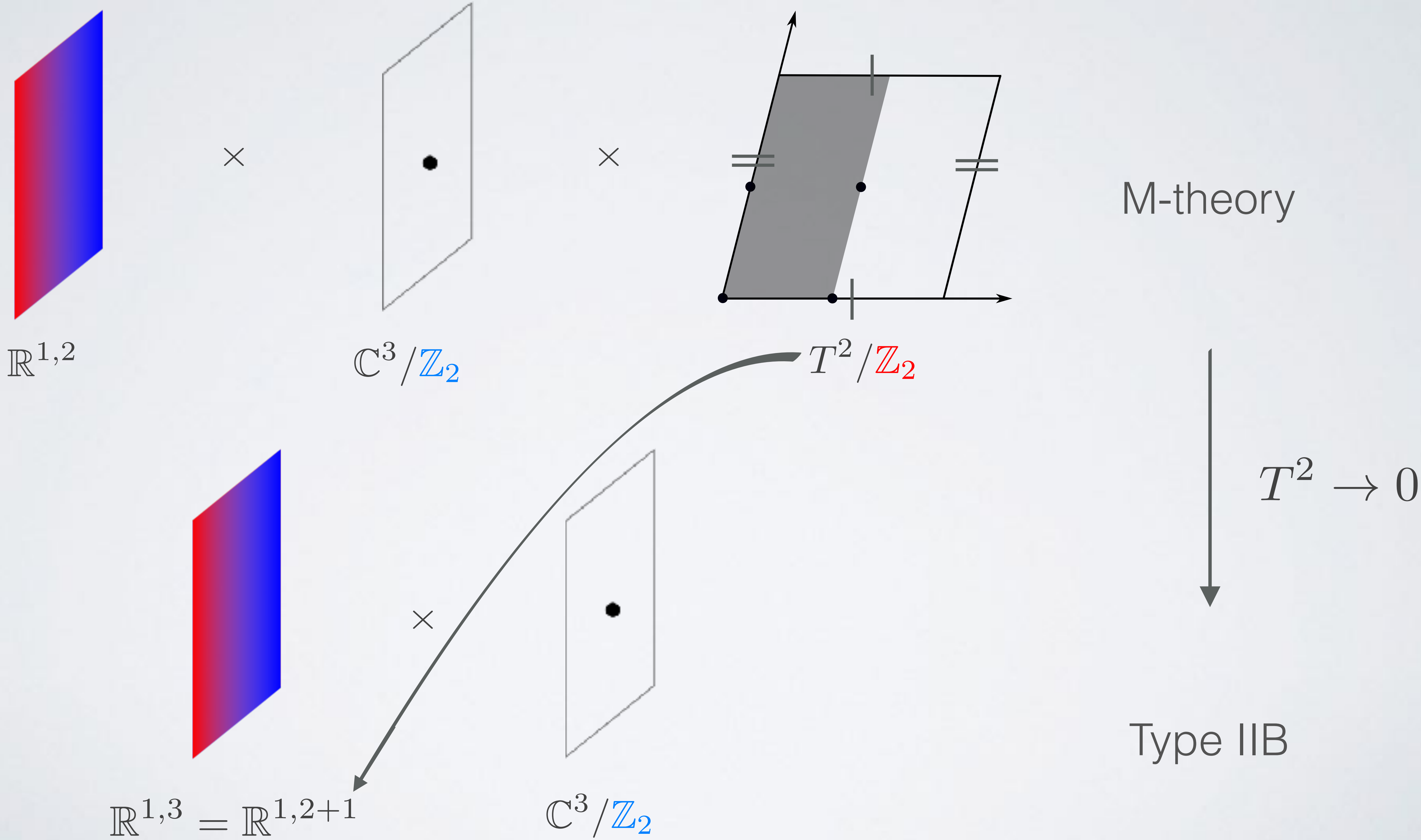
M-th. on $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$ with $(z_1,\, z_2,\, z_3,\, u) \to (-z_1,\, -z_2,\, -z_3,\, -u)$

$(-1)^{F_L}\Omega$ lifts to: $\mathcal{M}_{(-1)^{F_L}\Omega} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \in SL(2,\mathbb{Z})$

The O3-plane is simply an **orbifold** (involving the torus). Only geometry!

O3s in M/F-theory

$$\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$$



O3s+D3s in M/F-theory

- D3-branes parallel to the O3-plane lift to M2-branes.

- Four fixed points, which locally look like $\mathbb{C}^4/\mathbb{Z}_2$.
  This singularity has **no susy smoothings**: no low-energy dynamics associated to the O3.

[Morrison, Stevens; Anno]

- In M-theory, this is precisely **ABJM** (at level $k = 2$).

[Aharony, Bergman Jafferis, Maldacena]

The F-theory limit provides the 4d lift of ABJM.

$k = 1$: $4d$ $\mathcal{N} = 4$ $\mathfrak{u}(N)$ (Without orientifold)

$k = 2$: $4d$ $\mathcal{N} = 4$ $\mathfrak{so}(N), \mathfrak{usp}(N)$ (With orientifold)

D3s in field theory

Let's see how all this looks like from the perspective of the D3-branes.

- Before the quotient we have $4d\ \mathcal{N}=4\ \mathfrak{u}(N)$ on the probe D3s, with coupling constant $\tau_{\mathrm{YM}} = \tau_{\mathrm{IIB}}$.
  - It has R-symmetry group $SO(6)_R$
  - Montonen-Olive duality:

$$\tau \to \tau' = \frac{a\tau + b}{c\tau + d} \qquad \text{for} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

The duality group is **not a symmetry** of the theory!

A theory T_τ is defined by specifying a particular τ, then: $T_\tau \xrightarrow{SL(2,\mathbb{Z})} T_{\tau'}$

However, if for $\Gamma \subset SL(2,\mathbb{Z})$, $\Gamma(\tau^*) = \tau^*$, then Γ is a symmetry of T_{τ^*},

$$T_{\tau^*} \xrightarrow{\Gamma} T_{\tau^*}$$

O3s+D3s in field theory

We have to identify the blue and the red quotients in field theory terms.
The F-theory construction gives us the answer.

- Rotations around the O3 map to R-symmetry, so $\mathcal{I}$ maps to $\mathbb{Z}_2^R \subset SO(6)_R$.
- We have seen that $(-1)^{F_L}\Omega$ maps to $\mathbb{Z}_2^S \subset SL(2,\mathbb{Z})$.

  $SL(2,\mathbb{Z})$ is a duality, not a symmetry: $\tau \to \dfrac{a\tau + b}{c\tau + d}$

  However, $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \in \mathbb{Z}_2^S$ is a symmetry: $\tau \to \tau$

- Therefore, the orientifold corresponds to gauging $\mathbb{Z}_2^{O3} = \mathbb{Z}_2^R \cdot \mathbb{Z}_2^S$.
- Supercharges: $Q_{\alpha a}$ is charged under both $\mathbb{Z}_2^R$ and $\mathbb{Z}_2^S$. [Kapustin, Witten]

  $\mathbb{Z}_2^{O3} : Q_{\alpha a} \to Q_{\alpha a}$ (the O3 does not break SUSY further)

Holographic dual (O3)

[Witten]

- The holographic dual can be obtained from the IIB construction.

- Before introducing the O3, we have $N \gg 1$ D3-branes, whose near horizon limit is $AdS_5 \times S^5$. [Maldacena]

- The geometric action $\mathbb{Z}_2$ gives IIB on $AdS_5 \times S^5/\mathbb{Z}_2$ (which is not supersymmetric by itself).

- There is also an $\underset{\substack{\uparrow \\ \mathbb{Z}_2}}{SL(2,\mathbb{Z})}$ bundle on $S^5/\mathbb{Z}_2$ (now supersymmetric).

- The near horizon geometry in F-theory is $AdS_5 \times (S^5 \times T^2)/\mathbb{Z}_2$.

- The different O3 variants are given by turning on discrete fluxes

$$[H_3, F_3] \in H^3(S^5/\mathbb{Z}_2, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$$

Recap

Different ways to look at O3+D3s:

- Worldsheet: quotient by $\mathcal{I}(-1)^{F_L}\Omega$.
- M/F-theory: F-theory limit of $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$.
- 4d gauge theory: quotient by R-symmetry ($\mathbb{Z}_2^R$) and $SL(2,\mathbb{Z})$ ($\mathbb{Z}_2^S$).
- Holographic dual: F-theory on $AdS_5 \times (S^5 \times T^2)/\mathbb{Z}_2$

Beyond the O3

What if we take instead $\mathbb{Z}_2 \longrightarrow \mathbb{Z}_k$?

- Worldsheet: not very promising…

However, the other three seem to make sense a priori.

- M/F-theory: F-theory limit of $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$.
- 4d gauge theory: quotient by R-symmetry ($\mathbb{Z}_k^R$) and $SL(2,\mathbb{Z})$ ($\mathbb{Z}_k^S$).
- Holographic dual: F-theory on $AdS_5 \times (S^5 \times T^2)/\mathbb{Z}_k$

We call the associated objects $\mathrm{OF3}_k$-planes. $(\mathrm{OF3}_2 = \mathrm{O3})$

OF3s in M/F-theory

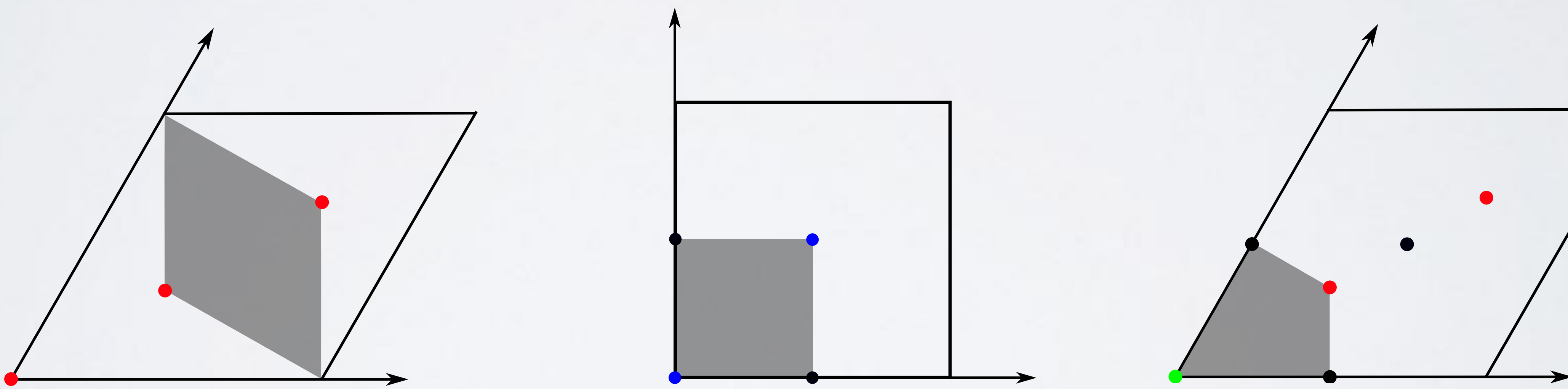
- We want to consider M/F-theory on $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$

$$(z_1, z_2, z_3, u) \to (\zeta_k z_1, \bar{\zeta}_k z_2, \zeta_k z_3, \bar{\zeta}_k u) \quad \text{with} \quad \zeta_k = e^{2\pi i/k} \qquad (k = 2, 3, 4, 6)$$

$\mathrm{OF3}_k$-planes exist only for some values of k.

Only well-defined for special values of the complex structure τ (g_s^{IIB}).

$T^2/\mathbb{Z}_k$:



$k = 3$, $\tau = e^{i\pi/3}$

$k = 4$, $\tau = i$

$k = 6$, $\tau = e^{i\pi/3}$

- This already explains why we can't do this in the worldsheet!

OF3s in M/F-theory

- Similarly to $k=2$, these do not have supersymmetric smoothings.

[Morrison, Stevens ; Anno]

- Preserve twelve supercharges, $\mathcal{N}_{3d}=6$ or $\mathcal{N}_{4d}=3$. $(k>2)$

- ABJM at level $k>2$ preserves $\mathcal{N}_{3d}=6$. The lift only works for some values of k, because there has to be a torus in M-theory.

  Is there a 4d theory such that, when we put it on a circle, it flows to ABJM? Yes, at least for $k=1,2,3,4,6$.

- M-theory geometry admits discrete flux $\longrightarrow$ Different $\mathrm{OF3}_k$

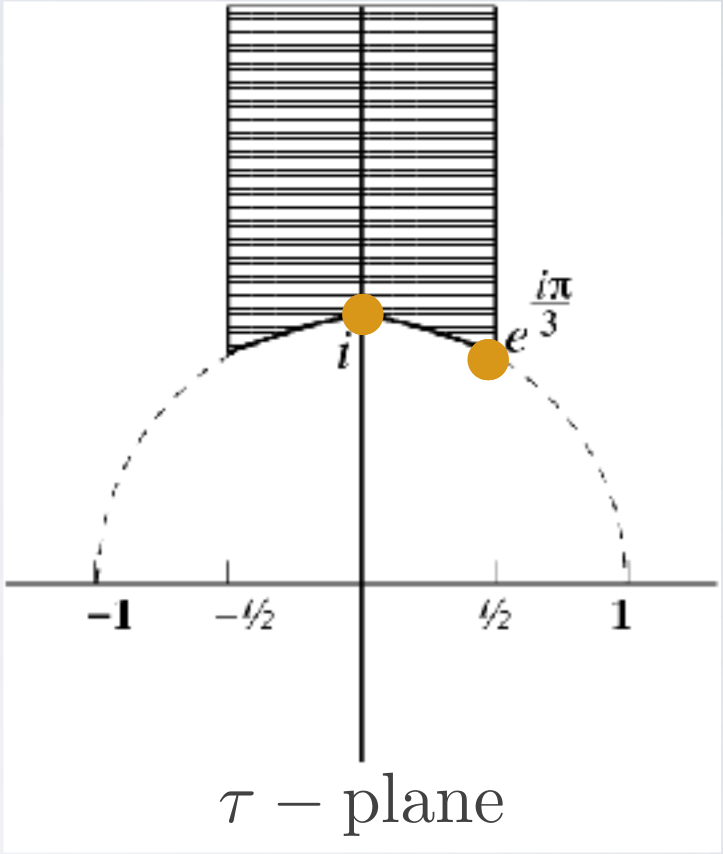
[García-Etxebarria, DR; Aharony, Tachikawa]

OF3s in field theory

- The theory on N D3s probing an $\mathrm{OF3}_k$ should arise as a $\mathbb{Z}_k$ quotient of $4d$ $\mathcal{N}=4$ $\mathfrak{u}(N)$ SYM.

- Just like before, $\mathbb{Z}_k^{\mathrm{OF}} = \mathbb{Z}_k^R \cdot \mathbb{Z}_k^S$ with

  $\mathbb{Z}_k^R \subset SO(6)_R$ and $\mathbb{Z}_k^S \subset SL(2,\mathbb{Z})$



- Again, $SL(2,\mathbb{Z})$ is not a symmetry, $\tau \to \dfrac{a\tau+b}{c\tau+d}$.

  However, if for $\Gamma \subset SL(2,\mathbb{Z})$, $\Gamma(\tau^*) = \tau^*$, then Γ is a symmetry of T_{τ^*}.

$$(\tau^*, \Gamma) \to (i, \mathbb{Z}_4) \text{ and } (e^{i\pi/3}, \mathbb{Z}_6)$$

  Restriction in both k and τ. The theory is stuck at **strong coupling**.
  [similar quotient: Ganor et al]

- The action on the supercharges shows that $\mathcal{N}=3$.

Holographic dual (OF3)

[Ferrara, Porrati, Zaffaroni]

- Just like for the usual O3, we can derive it from the IIB construction.
- Before introducing the OF3, we have N D3-branes, whose near horizon limit is $AdS_5 \times S^5$.
- In the presence of an OF3, we have Type IIB on $AdS_5 \times S^5/\mathbb{Z}_k$ with an $SL(2,\mathbb{Z})$ bundle. Or F-theory on $AdS_5 \times (S^5 \times T^2)/\mathbb{Z}_k$.

$\mathbb{Z}_k$

[García-Etxebarria, DR; Aharony, Tachikawa]

- We see that:
  - Smooth, weakly curved geometry.
  - Stuck at strong string coupling. No marginal deformation in the CFT.

Conclusions

- We have built the first examples of $\mathcal{N}=3$ field theories in 4d as quotients of $\mathcal{N}=4$ SYM by particular R-symmetry and $SL(2,\mathbb{Z})$ symmetries.
- Only works for specific values of the coupling. Isolated field theories.
- The worldvolume theory of D3s probing OF3s (generalized orientifolds).
- Can be thought of as the 4d version of ABJM (only for some k).
- Large N limit as a quotient of $AdS_5 \times S^5$ acting on the IIB coupling.

  [see also Aharony, Tachikawa]
- These were just simple examples of isolated singularities.

Thank you!