Non-Lorentzian Geometry in Field Theory and Gravity

Workshop on Geometry and Physics (in memoriam of **Ioannis Bakas**)

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based on work:

1409.1519 (PLB), 1409.1522 (PRD), 1502.00228 (JHEP) (Hartong,Kiritsis,NO)

1504.0746 (JHEP) (Hartong,NO) (special thanks to Ioannis !)

1604.08054 (PRD) & in progress (Hartong,Lei,NO)

1606.09644 (JHEP) (Hartong NO,Sanchioni)

1607.01753 (PRD), 1607.01926 (JHEP) (Festuccia, Hansen, Hartong NO)

in progress: (de Boer,Hartong,NO,Sybesma,Vandoren)

(Grosvenor,Hartong,Keeler,NO)

1311.4794 (PRD) & 1311.6471 (JHEP) (Christensen,Hartong,NO,Rollier)

Outline

- Motivation why non-Lorentzian geometry ?

- non-relativistic geometry: Newton-Cartan (NC) geometry

- Dynamical NC geometry = Horava-Lifshitz gravity

- 3D Chern-Simons theories as non-relativistic gravity theories
  - extended Bargmann/Newton-Hooke
  - extended Schroedinger

- new asymptotic symmetry groups for CS Schroedinger gravity

- Outlook

Black hole entropy and holography

- black hole entropy/quantum aspects of black holes

Hawking/Bekenstein

$$S_{\mathrm{BH}} = \frac{A}{4G}$$

- need a theory of quantum gravity to explain
- black hole entropy suggests holographic principle

't Hooft/Susskind

- string theory promising candidate

- -> concrete realization of holography: AdS/CFT correspondence

Maldacena

but presently only understood for relatively special spacetimes
& holographic principle expected to be universal

AdS/CFT is special

has Einstein gravity in the "bulk", and spacetimes asymptotically AdS

local bulk symmetries map to global on boundary

-> field theories on the boundary are relativistic (& conformal),
i.e. Lorentzian

power of AdS/CFT: strong/weak coupling duality

- learn about quantum BHs from strongly coupled FTs
- use (classical) gravity to learn about strongly coupled relativistic FTs

e.g. AdS/CFT at long-wave lengths -> fluid/gravity correspondence
-> many new insights into relativistic hydrodynamics

More general gravity/QFT correspondence

- understand universality of holography

also: we would like holographic dualities capable of:

- describing realistic BHs (and hence non-AdS)

  ….. our universe is not AdS

- fluid/gravity for more general classes of quantum field theories

  … many physical systems in nature (condensed matter) do not have relativistic symmetries

- feature tractable toy models of quantum gravity

  compute quantum effects of gravity, quantum properties of BHs

have learned in recent years that we can make progress using non-Lorentzian geometry

Non-Lorentzian geometry: first look

very generally: take some symmetry algebra that includes space and time translations and spatial rotations (assume isotropic): Aristotelian symmetries

gauge the symmetry and turn space/time translations into local diffeormphisms

Poincare -> Lorentzian(pseudo-Riemannian) geometry (relativistic)

Galilean/Bargmann -> torsional Newton-Cartan geometry (non-relativistic)

Carroll -> Carrollian geometry (ultra-relativistic)

crucial difference -> type of boosts		geometry
L: Lorentz	$t \to \gamma(t + \vec{v}\vec{x}/c^2) \quad , \quad \vec{x} \to \gamma(\vec{x} + \vec{v}t)$	$g_{\mu\nu}$
G/B: Galilean/Bargmann	$t \to t \quad , \quad \vec{x} \to \vec{x} + \vec{v}t$	$\tau_\mu \quad , \quad h^{\mu\nu} \quad , \quad m_\mu$
C: Carroll	$t \to t + \vec{\tilde{v}}\vec{x} \quad , \quad \vec{x} \to \vec{x}$	$v^\mu \quad , \quad h_{\mu\nu}$

Non-Lorentzian geometry in holography

appears as boundary geometry of class of non-AdS Lifshitz spacetimes known as Lifshitz spactimes

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{1}{r^2}\left(dr^2 + d\vec{x}^2\right)$$

characterized by anisotropic (non-relativistic) scaling between time and space

$$t \to \lambda^z t\,, \qquad \vec{x} \to \lambda \vec{x}$$

* introduced originally to study strongly coupled systems with critical exponent z

[Kachru,Liu,Mulligan][Taylor]

near boundary: lightcones open up (for z>1) -> Newton-Cartan (NC) geometry
”c -> infinity limit” of Lorentzian geometry i.e. non-relativistic

more precisely: novel extension of NC that includes torsion (=TNC)

Christensen,Hartong,Rollier,NO (1311)
Hartong,Kiritsis,NO (1409)

Non-Lorentzian geometry in holography (II)

by making the resulting non-Riemannian geometry dynamical one gains access to other bulk theories of gravity (than those based on Riemannian gravity)

- apply holography (e.g. HL gravity)
- interesting in their own right

[Griffin,Horava,Melby-Thompson (1211)
[Janiszweski,Karch (1211)]

- other holographic setups with non-Lorentzian geometry:

Carrollian geometry = ultra-relativistic geometry (light cones close up)

- 2D Carrollian geometry = warped (bdry) geometry to which WCFTs couple
  minimal holographic setup: SL(2,R) x U(1) CS

Hofmann,Rollier (1411)

- 2D Carrollian geometry is seen in 3D flat space holography as the geometry induced on boundary at future null infinity

Hartong(1511)

Non-Lorentzian geometry in Field Theory

- in relativistic FT: very useful to couple to background (Riemannian) geometry
  -> compute EM tensors, study anomalies, Ward identities, etc.

\- background field methods for systems with non-Lorentzian symmetries require non-Lorentzian geometry

-> FTs can have Lorentz/Galilean/Carrollian boost symmetries:
background field methods for systems with:
\- non-relativistic (NR) symmetries require NC geometry (with torsion)
\- ultra-relativistic (UR) symmetries require Carrol. geometry

-> there is full space-time diffeomorphism invariance when coupling to the right background fields

\- Examples

[Son, 2013], [Geracie, Son, Wu, Wu, 2014],…

* effective field theory for the FQHE uses NC geometry

[Jensen,2014] [Kiritsis,Matsuo,2015,2016]
[Hartong,NO,Sanchioni,2016]
[deBoer,Hartong,NO,Sybesma,Vandoren, to appear]

* non-relativistic (NR) hydrodynamics

Non-Lorentzian Geometry in Gravity

- interesting to make non-Lorentzian geometry dynamical
  -> "new" theories of gravity

have shown: $\boxed{\text{dynamical Newton-Cartan (NC) = Horava-Lifshitz (HL) gravity}}$

Hartong,NO (1504)

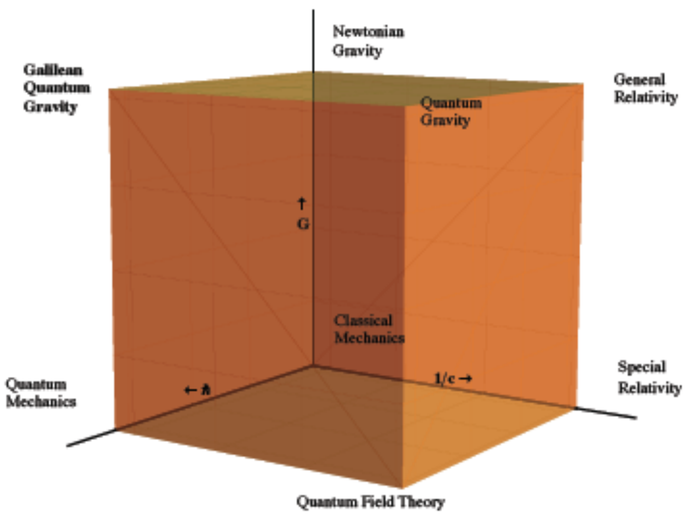
Horava (0812,0901);
Horava,Melby-Thompson (2010)

➡ natural geometric framework with full diffeomorphism invariance

such theories of gravity interesting as
- other bulk theories of gravity in holographic setups
- effective theories (cond mat, cosmology)

Griffin,Horava,Melby-Thompson (2012)
Janiszweski,Karch (2012)

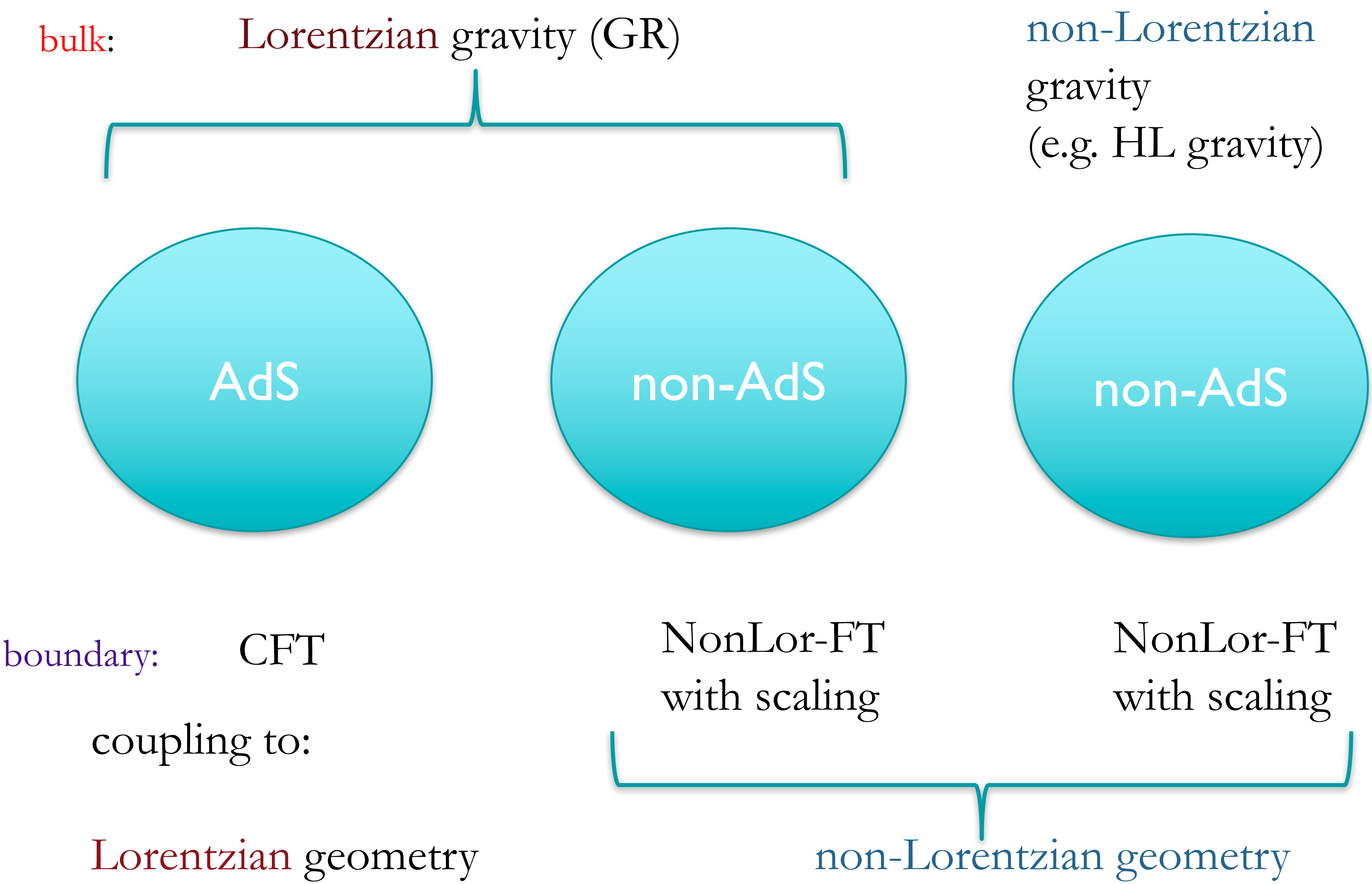
- Galilean quantum gravity
  infinite c limit of $(\hbar, G_N, 1/c)$ cube



- opposite case: Carollian gravity

  Hartong (1505)

More General Holographic Setups



Intermezzo: Motivation (SUSY)

- (global) supersymmetric non-relativistic field theories

- susy extensions of Lifshitz field theories

[Orlando,Reffert],[Xue][Chapman,Oz,Raviv-Moshe(1508)]

$$\{Q,Q\} \sim H, \quad [M_{ij}, Q_\alpha] = 0, \quad [D,Q] = i\frac{z}{2}Q$$

- non-relativistic SUSY-CS matter theory (complex scalar, fermion, gauge field) describing FQHE (from non-rel limit of N=2 CS)

Tong,Turner(1508)

Bergshoeff,Rosseel,Zojer (1505,1509,1512)

⇒ non-rel. supergravities (Newton-Cartan SUGRA,..) (so far 2+1 D, from non-rel limit of N=2 SUGRA)

- potentially useful in order to extend localization techniques to non-rel SUSY FTs on curved bgr e.g. [Pestun],[Festuccia,Seiberg]
- need NC bgrs that admit FTs with non-rel SUSY [Knodel.Lisbao,Liu]
- precision tests of non-rel AdS/CFT ?

Newton-Cartan makes Galilean local

- NC geometry originally introduced by Cartan to geometrize Newtonian gravity

Eisenhart,Trautman,Dautcourt,Kuenzle,Duval,Burdet,Perrin,Gibbons,Horvathy,Nicolai,Julia…..

both Einstein's and Newton's theories of gravity admit geometrical formulations which are diffeomorphism invariant

- NC originally formulated in "metric" formulation

more recently: vielbein formulation (shows underlying sym. principle better)

Andringa,Bergshoeff,Panda,de Roo

Riemannian geometry: tangent space is Poincare invariant

Newton-Cartan geometry: tangent space is Bargmann (central ext. Gal.) invariant

- gives geometrical framework and extension to include torsion
i.e. as geometry to which non-relativistic field theories can couple
(boundary geometry in holographic setup is non-dynamical)

* will consider dynamical (torsional) Newton-Cartan later

Mini-review: From Poincare to GR by gauging

- make Poincare local (i.e. gauge the translations and rotations)

vielbein spin connection

$$\mathcal{A}_\mu = P_a e^a_\mu + \frac{1}{2} M_{ab} {\omega_\mu}^{ab}$$

turn local translations into diffeomorphisms

$$\delta A_\mu \to \delta e^a_\mu = \mathcal{L}_\xi e^a_\mu + e^b_\mu {\lambda_b}^a$$

- spin connection expressed in terms of vielbein

- covariant derivative defined via vielbein postulate

- GR is a diff invariant theory whose tangent space invariance group is the Poincaré group
* Einstein equivalence principle -> local Lorentz invariance

Relevant non-relativistic algebras

$$\overbrace{\underbrace{H, P_a, J_{ab}, G_a}_{}\quad N}^{}$$

Galilean: H, P_a, J_{ab}, G_a

Bargmann: $H, P_a, J_{ab}, G_a \quad N$

(Galilean algebra is c to infinity limit of Poincare)

$$[H, G_a] = P_a \qquad [P_a, G_b] = 0$$

$$[P_a, G_b] = N\delta_{ab}$$

Lifshitz (aka "Aristotelian & scaling"): $H, P_a, J_{ab}, D,$

Schrödinger: $H, P_a, J_{ab}, D, \quad G_a, N, K (z = 2)$

$$[D, H] = zH \qquad [D, P_a] = P_a$$

$$[D, N] = (2 - z)N$$

Schrödinger = Bargmann + dilatations (+ special conformal for z=2)

Gauging the Bargmann algebra

symmetry	generators	gauge field	parameters	curvatures
time translations	H	τ_μ	$\zeta(x^\nu)$	$R_{\mu\nu}(H)$
space translations	P_a	$e_\mu{}^a$	$\zeta^a(x^\nu)$	$R_{\mu\nu}{}^a(P)$
boosts	G_a	$\omega_\mu{}^a$	$\lambda^a(x^\nu)$	$R_{\mu\nu}{}^a(G)$
spatial rotations	J_{ab}	$\omega_\mu{}^{ab}$	$\lambda^{ab}(x^\nu)$	$R_{\mu\nu}{}^{ab}(J)$
central charge transf.	N	m_μ	$\sigma(x^\nu)$	$R_{\mu\nu}(N)$

impose curvature constraints:

$R_{\mu\nu}(H) \quad R^a_{\mu\nu}(P) \quad R_{\mu\nu}(N)$ (e.g. =0)

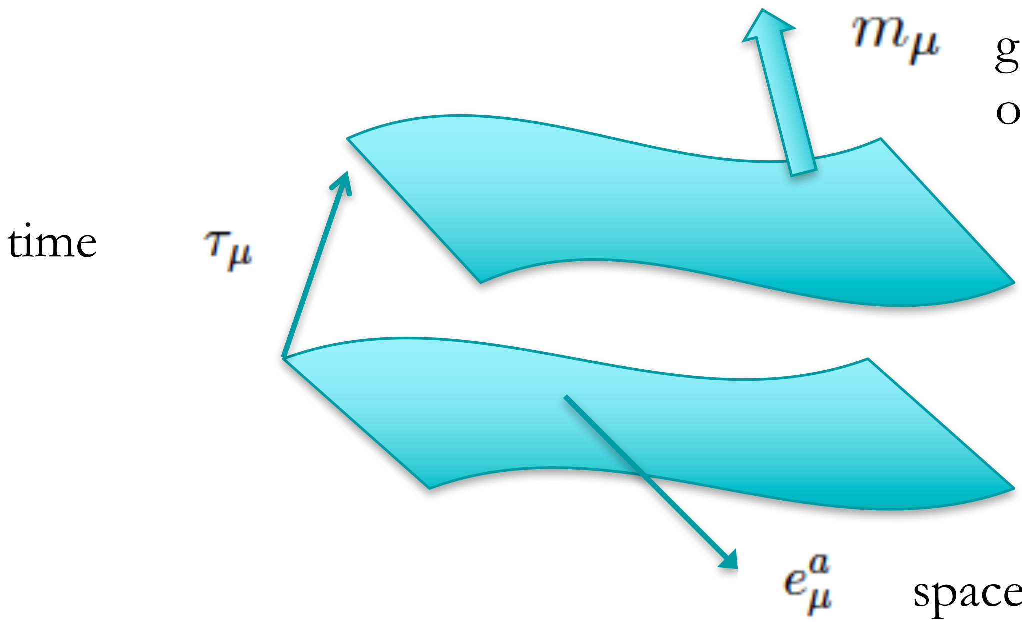
independent fields: $\tau_\mu,\ e^a_\mu,\ m_\mu$

= gauge fields of Hamiltonian, spatial translations and central charge

transformations:

$$\begin{aligned} \delta\tau_\mu &= \mathcal{L}_\xi \tau_\mu \\ \delta e^a_\mu &= \mathcal{L}_\xi e^a_\mu + \lambda^a \tau_\mu + \lambda^a{}_b e^b_\mu \\ \delta m_\mu &= \mathcal{L}_\xi m_\mu + \partial_\mu \sigma + \lambda_a e^a_\mu \end{aligned}$$

Newton-Cartan geometry



gauge field of central extension of Galilean algebra (Bargmann)

(inverse) spatial metric:

$$h^{\mu\nu} = e^\mu_a e^\nu_b \delta^{ab}$$

NC geometry = no torsion $\longrightarrow$ $\tau_\mu = \partial_\mu t$ notion of absolute time

TTNC geometry = twistless torsion $\longrightarrow$ $\tau_\mu = \text{HSO}$ preferred foliation in equal time slices

TNC geometry no condition on τ_μ

- in TTNC: torsion measured by $a_\mu = \mathcal{L}_{\hat{v}} \tau_\mu$
  geometry on spatial slices is Riemannian

Adding torsion to NC

Christensen,Hartong,Rollier,NO
Hartong,Kiritsis,NO/Hartong,NO
Bergshoeff,Hartong,Rosseel

- inverse vielbeins (v^μ, e^μ_a)

$$v^\mu \tau_\mu = -1\,, \qquad v^\mu e^a_\mu = 0\,, \qquad e^\mu_a \tau_\mu = 0\,, \qquad e^\mu_a e^b_\mu = \delta^b_a$$

can build Galilean boost-invariants

$$\begin{aligned} \hat{v}^\mu &= v^\mu - h^{\mu\nu} M_\nu\,, \\ \bar{h}_{\mu\nu} &= h_{\mu\nu} - \tau_\mu M_\nu - \tau_\nu M_\mu\,, \\ \tilde{\Phi} &= -v^\mu M_\mu + \tfrac{1}{2} h^{\mu\nu} M_\mu M_\nu\,, \end{aligned}$$

-introduce Stueckelberg scalar chi:
(to deal with broken/unbroken N-sym)

$$M_\mu = m_\mu - \partial_\mu \chi.$$

→ affine connection of TNC (inert under G,J,N)

$$\Gamma^\rho_{\mu\nu} = -\hat{v}^\rho \partial_\mu \tau_\nu + \frac{1}{2} h^{\rho\sigma}\left(\partial_\mu \bar{h}_{\nu\sigma} + \partial_\nu \bar{h}_{\mu\sigma} - \partial_\sigma \bar{h}_{\mu\nu}\right)$$

with torsion $\Gamma^\rho_{[\mu\nu]} = -\frac{1}{2}\hat{v}^\rho(\partial_\mu \tau_\nu - \partial_\nu \tau_\mu)$

$$\nabla_\mu \tau_\nu = 0\,, \qquad \nabla_\mu h^{\nu\rho} = 0\,,$$

analogue of metric compatibility

connection also obtained via Noether: [Festuccia,Hansen,Hartong,NO](1607)

intermezzo: geodesics on NC space-time

- worldline action of non-rel particle of mass m on NC background

$$S = \int d\lambda L = \frac{m}{2}\int d\lambda \frac{\bar{h}_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}{\tau_\rho \dot{x}^\rho}$$

[Kuchar],
[Bergshoeff et al]

- gives the geodesic equation with NC connection $\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\lambda}\frac{dx^\rho}{d\lambda} = 0\,,$

* reduces to Newton's law $\frac{d^2x^i}{dt^2} + \delta^{ij}\partial_j\Phi = 0\,,$

provided we take $$\boxed{\begin{aligned} M_t &= \partial_t M + \Phi\,, \\ M_i &= \partial_i M\,, \end{aligned}}$$

for flat NC space-time: zero Newtonian potential

symmetries of flat NC = conformal Killing vectors (spanning Lifshitz) + extra

Coupling FTs to TNC

[Hartong,Kiritsis,NO]

EM tensor: $T^\mu{}_\nu$

mass current T^μ

- action functional $S(v^\mu, h^{\mu\nu}, m_\mu)$

$$\delta S = \int d^{d+1}x e (T_\mu dv^\mu + \frac{1}{2} T_{\mu\nu}\delta h^{\mu\nu} + J^\mu \delta m_\mu)$$

$$T^\mu{}_\nu = v^\mu T_\mu + h^{\mu\rho} T_{\rho\nu}$$

$T^t{}_t$, $T^t{}_i$, $T^i{}_t$, $T^i{}_j$ $\qquad$ J^t , J^i

energy density, momentum flux, energy flux, spatial stress, mass density, mass flux

* important to have torsion in order to describe the most general energy current !

\- from the various local symmetries:

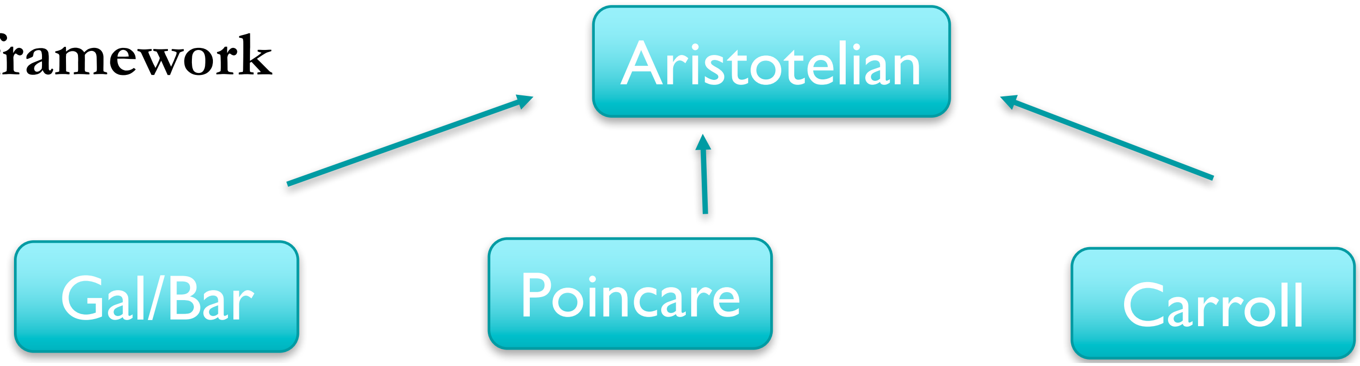
particle number conservation (if extra local U(1))

mass flux= momentum flux (local boosts)

symmetric spatial stress (local rotations)

& diffeo and (possibly) scale inv. Ward identies

Classification framework



Symmetry	Galilean/Bargmann	Poincaré	Carroll
geometry	Newton–Cartan $\tau_\mu, h^{\mu\nu}, m_\mu$	pseudo-Riemannian. $g_{\mu\nu}$	Carrollian $v^\mu, h_{\mu\nu}$
causal structure	non-rel.	Minkowski	ultra-rel.
response	energy current momentum current symmetric stress	symmetric EM tensor	energy density momentum current symmetric stress
boost Ward-identity	momentum flux = mass flux	momentum current =energy current	energy flux = 0
scaling	$\forall z$	$z = 1$	$\forall z$
dynamical geometry	HL-gravity (w. $U(1)$) CS on non-rel. algebra + dyn. non-rel. sources	GR + dyn. rel. sources	ultra-relativistic gravity + dyn. ultra-rel. sources
holographic realization	EMD-model, ... HL/CS gravity	AdS/CFT	flat space

NC/TTNC gravity

TNC geometry is a natural geometrical framework underlying HL gravity

- NC quantities combine into: $g_{\mu\nu} = -\tau_\mu \tau_\nu + \hat{h}_{\mu\nu}$
- ADM parametrization of metric used in HL gravity: Horava (0812,0901)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + \gamma_{ij} \left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

relation:

$$\tau_\mu \sim \text{lapse} \quad , \quad \hat{h}_{\mu\nu} \sim \text{spatial metric} \quad , \quad m_\mu \sim \text{shift} + \text{Newtonian potential}$$

some features:

- khronon field of BPS appears naturally $\tau_\mu = \psi \partial_\mu \tau$ Blas,Pujolas,Sibiryakov(2010)

  NC (no torsion): $N = N(t)$ projectable HL gravity

  TTNC: $N = N(t, x)$ non-projectable HL gravity
- U(1) extension of HMT emerges naturally as Bargmann U(1)
- new perspective (via chi field) on nature of U(1) symmetry

Horava,Melby-Thompson(2010)

Effective actions reproduce HL

- covariant building blocks:

- extrinsic curvature: $\hat{h}_{\nu\rho}\nabla_\mu \hat{v}^\rho = -K_{\mu\nu}$ spatial curvature $\check{R}_{\mu\nu\sigma}{}^{\rho}$.

- covariant derivative, torsion vector a_μ , inverse spatial metric $h^{\mu\nu}$

- tangent space invariant integration measure $e = \det(\tau_\mu, e^a_\nu)$

-> construct all terms that are relevant or marginal (up to dilatation weight d+z)

- in 2+1 dimensions for $1 < z \leq 2$

$$\boxed{S = \int d^3x e \left[C\left(h^{\mu\rho}h^{\nu\sigma}K_{\mu\nu}K_{\rho\sigma} - \lambda\left(h^{\mu\nu}K_{\mu\nu}\right)^2\right) - \mathcal{V}\right]}$$

kinetic terms (2^{nd} order)

potential:

$$-\mathcal{V} = 2\Lambda + c_1 h^{\mu\nu}a_\mu a_\nu + c_2\mathcal{R} + \delta_{z,2}\left[c_{10}\left(h^{\mu\nu}a_\mu a_\nu\right)^2 + c_{11}h^{\mu\rho}a_\mu a_\rho \nabla_\nu\left(h^{\nu\sigma}a_\sigma\right)\right.$$
$$\left. + c_{12}\nabla_\nu\left(h^{\mu\rho}a_\rho\right)\nabla_\mu\left(h^{\nu\sigma}a_\sigma\right) + c_{13}\mathcal{R}^2 + c_{14}\mathcal{R}\nabla_\mu\left(h^{\mu\nu}a_\nu\right) + c_{15}\mathcal{R}h^{\mu\nu}a_\mu a_\nu\right]$$

action also obtained by other method: [Afshar, Bergshoeff,Mehra,Parekh,Rollier]1512

3D HL theories

3D Einstein gravity = CS gauge theory
-> insights into classical and quantum properties of theory
holographic dualities with 2D CFTs
black holes

can 3D non-relativistic gravity theories be reformulated as CS ?

Hartong,Lei,NO(1604)

CS on extended Bargmann algebra
= 3D U(1)-invariant projectable HL gravity = 3D dynamical NC gravity
(with/without cosmo constant)

CS on extended Schroedinger algebra
= novel conformal non-projectable U(1)-invariant HL gravity
= novel dynamical TTNC gravity -> CS Schroedinger gravity

CS on ext. Bargmann: Papageorgiou,Schroers (0907)
Bergshoeff,Rollier (1604): CS on ext. Bargmann from non-rel limit of Einstein gravity & 2 vectors
& extended Bargmann supergravity

Chern-Simons on non-relativistic algebras

Hartong,Lei,NO

CS Lagrangian $\mathcal{L} = \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$

need invariant bilinear form -> non-trivial requirement for non. rel algebras (non-semi simple Lie algebras)

Galilean algebra $[J\,, P_a] = \epsilon_{ab} P_b\,, \quad [J\,, G_a] = \epsilon_{ab} G_b\,, \quad [H\,, G_a] = P_a$

Bargmann algebra
(central extension = mass generator) $[P_a\,, G_b] = N \delta_{ab}$

2+1 dim special:
can further extend
(S,Y,Z central wrt Gal
but not nec. rest)

$$[G_a\,, G_b] = S\epsilon_{ab}\,, \quad [P_a\,, P_b] = Z\epsilon_{ab}$$
$$[P_a\,, G_b] = N\delta_{ab} - Y\epsilon_{ab}\,.$$

can add a SL(2,R) generated by H,D,K to this

From 1st order formulation to 3D HL/NC gravity

CS action on extended Bargmann (extra S-generator) includes a term:

$$\mathcal{L}_{\rm kin} = R^2(G) \wedge e^1 - R^1(G) \wedge e^2 + \tau \wedge \Omega^1 \wedge \Omega^2 - m \wedge d\omega + \zeta \wedge d\tau$$

write in metric form by integrating out:

zeta = Lagrange multiplier for torsionless constraint
Omega1,2 = boost connections
omega = (2dim) rotation connection

leaves as indep variables: NC variables τ_μ, e^a_μ and m_μ

-> gives U(1)-inv. 3D HL in NC covariant form [Hartong,Lei,NO]

$$S = \int d^3x e \left[C \left(h^{\mu\rho} h^{\nu\sigma} K_{\mu\nu} K_{\rho\sigma} - (h^{\mu\nu} K_{\mu\nu})^2 - \tilde{\Phi} \mathcal{R} \right) - \mathcal{V} \right]$$

Horava,Melby-Thompson(2010)
Hartong,NO(1504)

Chern-Simons Schroedinger Gravity

Hartong,Lei,NO(1604)

CS with gauge connection on "triply-extended" Schroedinger algebra

$$\begin{aligned} A \;=\; & H\tau + P_a e^a + G_a \omega^a + J\omega + Nm + Db + Kf \\ & + S\zeta + Y\alpha + Z\beta \,. \end{aligned} \tag{17}$$

(gives action with 3 distinct invariants -> 3 central charges)

-> new way of constructing conformal actions for non-projectable HL gravity (with U(1) sym.)

- useful starting point to do holography with HL (in CS form)
- theory has Lifshitz vacua ("minimal" setup to do Lif holography)
- new dualities involving novel class of 2D non-rel field theories on bdry featuring:
- novel (infnite dimensional) conformal non-relativistic algebras

[Hartong,Lei,NO (to appear)}

- analogue of Lorentz CS terms (Galilean boost, rotation anomalies ?)

Non-relativistic FT and coupling to NC/HL gravity

have tools/building blocks to covariantly couple HL/NC gravity to non-relativistic field theories:

- massless/massive spin (0),1/2,1,3/2 for no-torsion using non-relativistic contraction: [Bergshoeff,Rosseel,Zojer]1512
- spin ½ for generic NC using non-rel limit: [Fuini,Karch,Uhlemann]1510
- GED (Galilean electrodynamics) coupled to TNC [Festuccia,Hansen,Hartong,NO]1607

methods to construct non-trivial (interacting) non-rel field theories

- from scratch (use symmetries)
- take non-rel. (c-> infinity) limits
- perform null reduction of D+1 dim relativistic theories

Main points & outlook

- non-Lorentzian geometries play an important role in
  field theory, gravity and non-AdS holography
- constructed a new 3D theory of gravity w. Lifshitz vacua: **CS Schroedinger gravity**
  & novel (infinite dim) asymptotic symmetry groups: new class of holographic dualities

- landscape of non-Lorentzian field theories

  hydrodynamics, transport coeffs, flows, anomalies, 2D FTs with scaling,

- non-AdS holography for systems with non-CFT critical exponents

  models of Lif holography, finite density (charge), fluid/gravity

- **toy models of quantum gravity**

  asymptotic sym. groups of Schr. CS gravity, properties of dynamical NLG,
  probes (particle, string), extensions, connections with non-rel ST

- "black holes" in dynamical non-Lorentzian geometry

  black holes in CS Schroedinger gravity ?

- non-AdS holography and non-Lorentzian geometry

  holography in dyn. NLG bulk, BMS, holographic EE, spacetime reconstruction