Heterotic T-fects, 6D SCFTs, and F-Theory

joint work with A. Font, I. García-Etxebarria, D. Lüst and S. Massai:
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Christoph Mayrhofer

Arnold Sommerfeld Center, LMU München

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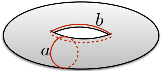
Motivation

- To understand landscape of string vacua need to go away from the lamppost and look at non-geometric string compactifications too;
- Because (most probably) amount of such vacua is much larger than geometric ones;
- Step in this direction is understanding of following 6d heterotic vacua and dualities among them;

Heterotic String Theory on T^2 I

- From compactification of het. string on T^2 obtain following moduli in 8d:
  - complexified Kähler modulus: $\rho = \int_{T^2} B + \omega \wedge \bar{\omega}$ with $\omega(G)$;
  - complex structure modulus: $\tau = \frac{\int_b \omega}{\int_a \omega}$;



  - Wilson line moduli: $\beta^i = \int_a A^i + i \int_b A^i$;
- Moduli space of het. torus compactification is (Narain space):

$$O(2) \times O(2+n_{WL}) \backslash O(2,2+n_{WL}) / O(2,2+n_{WL},\mathbb{Z})\,;$$

[Narain '86]

Main case of interest: $n_{WL} = 1$ (and $n_{WL} = 0$);

Heterotic String Theory on T^2 II

- For $n_{WL} = 1$, above Narain space can be mapped to Siegel upper half plan of genus two

$$\mathbb{H}_2 = \left\{ \Omega = \begin{pmatrix} \tau & \beta \\ \beta & \rho \end{pmatrix} \middle| \Im(\det(\Omega)) > 0 \wedge \Im(\rho) > 0 \right\}$$

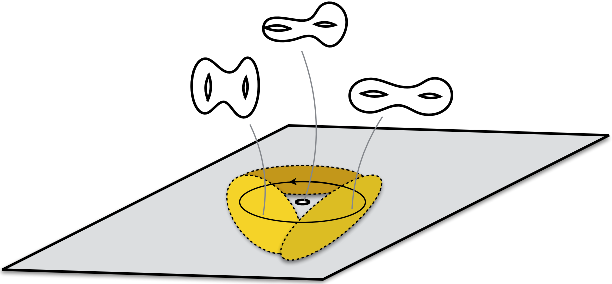
quotient by $Sp(4,\mathbb{Z})$-action $\Omega \to (A\Omega + B)(C\Omega + D)^{-1}$ with

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(4,\mathbb{Z})\,;$$

- Should note, map is not bijective, only on $O(2) \times O(3) \backslash O(2,3)/SO^{+}(2,3,\mathbb{Z})$; [Vinberg '13, Malmendier&Morrison '14]
- Above moduli fields are entries of Ω;
- $\mathbb{H}_2/Sp(4,\mathbb{Z})$ is (complex structure) moduli space of genus two curves, i.e. $\Omega_{ij} = \int_{b_i} \omega_j / \int_{a_i} \omega_j$ with a_i, b_i the cycles of $\mathcal{C}_{g=2}$ and ω_i the holomorphic one-forms;

Genus Two Fibration I

- Interested in vacua with non-trivial moduli fields background; Let torus compactification vary (adiabatically) along two real dimensions;
- Allow for stringy (patching) dualities, i.e. identifications under $Sp(4, \mathbb{Z})$ action;
- Like in F-theory, geometrify information of varyring fields in terms of fibration, i.e. genus two fibration;



- That way end up with non-geometric compactification; Allow for identifications with inverse of metric, or even total mixing of three moduli τ, ρ and β;

Genus Two Fibration II

- To fulfil (susy) EOM, fibration has to be holomorphic; Hence, degenerates at (complex) co-dim one loci;
- Degeneration points are location of quotient singularities, non-pert. objects like NS5 branes, or more generally T-fects;
- All degenerations of genus two curves are classified;
  [Ogg '66, Namikawa&Ueno '73]
- Natural question: can we find identification/interpretation of physical objects at all these degenerations?

Duality with F-Theory I

- Reminder: F-theory is IIB with varying axio-dilaton; Axio-dilaton is encoded in complex structure of elliptic curve/torus over every point of 10d space-time, i.e. elliptic fibration; [Vafa '96]
- Het. string on T^2 and F-Theory on elliptically fibered K3 are dual to each other; [Morrison&Vafa '96]
- Duality is best understood in large volume/stable degeneration limit; [Morrison&Vafa '96] At this point in moduli space, base $\mathbb{P}^1$ of K3 splits in two; The het. data, i.e. τ and β^i $(\rho \to i\infty)$, can be read off from intersection of two components of degenerated K3; [Friedman et al. '97]
- But for $n_{WL} = 0$ and $n_{WL} = 1$, there is even identification in terms of moduli space; [Cardoso '96, McOrist et al. '10, Malmendier&Morrison '14]

Duality with F-Theory II

- For both cases ($n_{WL} = 0, 1$) F-Theory K3 given by

$$y^2 = x^3 + (a\,u^4 + c\,u^3)\,x + (b\,u^6 + d\,u^5 + e\,u^7)\,;$$

- K3 has II^* sing. at $u = \infty$ and III^* sing. (or II^* in case of $c = 0$) at $u = 0$; Therefore, Picard number of K3 is 17 ($n_{WL} = 1$) or 18 ($n_{WL} = 0$), respectively;
- Moduli spaces agree with het. ones and can even be mapped:
  - $n_{WL} = 1$ ($e = 1$): $a = -\frac{1}{48}\psi_4(\Omega)$, $b = -\frac{1}{864}\psi_6(\Omega)$, $c = -4\chi_{10}(\Omega)$, $d = \chi_{12}(\Omega)$;
    Siegel modular forms ψ_4, ψ_6, χ_{10} and χ_{12} fix Ω uniquely;
  - $n_{WL} = 0$ ($c = 0$): $j(\tau)j(\rho) = -1728^2\frac{a^3}{27d\,e}$,
    $(j(\tau) - 1728)(j(\rho) - 1728) = 1728^2\frac{b^2}{4d\,e}$ and $\beta = 0$;

Duality with F-Theory III

- Therefore, have identification of $E_8 \times E_7$ K3 with genus two curve, and identification of $E_8 \times E_8$ K3 with two tori glued together at one point (degenerated genus two curve);
- Further, if genus two curve is given in terms of sextic, i.e.

$$y^2 = c_6 x^6 + c_5 x^5 + \dots ,$$

  then a, b, c, d of K3 are simply given by Igusa-Clebsch invariants of sectic, i.e. polynomials of coefficients c_i;
- Fortunately, all degenerations of genus two curves are in this form; Therefore, can easily map them to (singularities of) K3;
- Note, to go from K3 to representation of hyperelliptic curve is more involved;

Resolutions of F-Theory Side I

- Degeneration of hyperelliptic curve is parametrised by t, i.e. c_i vary with t, with singular curve at $t = 0$;
- From c_i's obatin a, b, c, d, which are then functions (sections) of t too;
- On K3 (fibre) have already III^* singularity at $u = 0$ which will enhance at $u = t = 0$ to non-min./beyond Kodaira type sing.;
- Need to blow up base to resolve such singularities; [Miranda '83, Grassi '93, Aspinwall&Morrison '97]
- To determine which base blow-ups must be done, take sort of toric approach; Write down f and g in terms of its (leading) monomials in u and t,

$$f = \sum_i f_i \, u^{m_i^1} \, t^{m_i^2} \,, \qquad g = \sum_i g_i \, u^{l_i^1} \, t^{l_i^2} \,,$$

  and ask for allowed 'blow-up direction' **n** such that hypersurface

$$y^2 = x^3 + f\,x + g$$

  is still CY;

Resolutions of F-Theory Side II

- Condition on vanishing first Chern class translates to:

$$(m_i^1-4)n_1+(m_i^2-4)n_2 \geq -4 \quad \text{and} \quad (l_i^1-6)n_1+(l_i^2-6)n_2 \geq -6$$

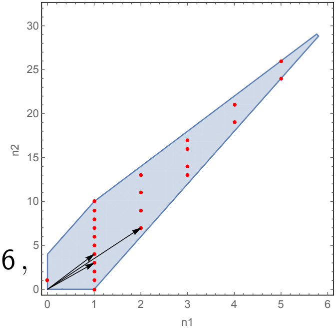
for all $\mathbf{m}^i$, $\mathbf{l}^i$ with $\mathbf{n}=(n_1,n_2)$ direction of blow-up, i.e. $t,\, u \rightarrow e^{n_1}t, e^{n_2}u$;

Ex. het. $[\mathrm{II}^* - \mathrm{I}_0]$ singularity: Solution set $\{\mathbf{n}^j\}$ to inequalities

$$(6,-1)\cdot\mathbf{n} \geq -4\,,\ (1,0)\cdot\mathbf{n} \geq -4\,,$$

for f and for g:

$$(4,-1)\cdot\mathbf{n} \geq -6\,,\ (-1,0)\cdot\mathbf{n} \geq -6\,,$$
$$(-6,1)\cdot\mathbf{n} \geq -6\,;$$



Resolutions of F-Theory Side III

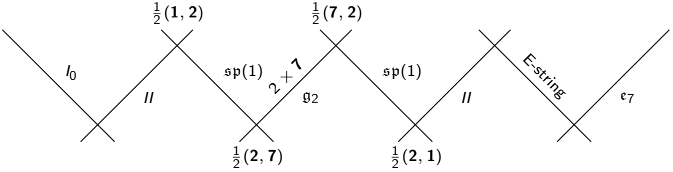
- Not all singularities which obtained from degenerations of genus two can be resolved on F-Theory side; Can give simple criterion for when can resolve in above way:

  $$\mu(a) < 4 \quad \text{or} \quad \mu(b) < 6 \quad \text{or} \quad \mu(c) < 10 \quad \text{or} \quad \mu(d) < 12\,,$$

  where μ is vanishing order at $t = 0$;
- If this is fulfilled, solution set $\{\mathbf{n}^j\}$ to above inequalities is finite;
- From $\{\mathbf{n}^j\}$ read off self-intersection numbers, because for adjacent $\mathbf{n}^j$ one has $a_j \mathbf{n}^j = \mathbf{n}^{j+1} + \mathbf{n}^{j-1}$ with a_j self-intersection number of $\mathbf{n}^j$;
- Further vanishing orders of f, g and Δ along e_i's are immediately obtained;
- To work out gauge algebras and matter representations standard techniques have to be applied; [Bershadsky '96, Katz&Vafa '96,Grassi&Morrison '12,...]

Example: III-III

- The genus two curve is given by
$$y^2 = x(x-1)(x^2+t)\left[(x-1)^2+t\right] .$$
- Note again, degeneration at $t \to 0$;
- Monodromies around t=0 are
$$\tau \to \frac{\rho}{\beta^2 - \rho\tau}\,, \quad \beta \to -\frac{\beta}{\beta^2-\rho\tau}\,, \quad \rho \to \frac{\tau}{\beta^2-\rho\tau}\,;$$
- From map, obtain following CY3 singularity (to leading orders):
$$y^2 = x^3 + \left[t^6u^3 + t^2u^4\right]x + t^4u^6 + t^6u^5 + u^7\,;$$
- Get following resolved geometry:



Dual Theories

- Having mapped all genus two degenerations to F-Theory, can search for dual theories;
- In subclass of elliptic models, obtain the following dual theories:

$\mu(I_{10})$	dual models
2	$[\mathrm{I}_0 - \mathrm{II}]_{0112}$
3	$[\mathrm{I}_0 - \mathrm{III}]_{0113}$
4	$[\mathrm{I}_0 - \mathrm{IV}]_{0224}$, $[\mathrm{II} - \mathrm{II}]_{0224}$
5	$[\mathrm{IV} - \mathrm{I}_1]_{0325}$, $[\mathrm{II} - \mathrm{III}]_{0225}$
6	$[\mathrm{I}_0 - \mathrm{I}_0^*]_{0226}$, $[\mathrm{III} - \mathrm{III}]_{0226}$, $[\mathrm{IV} - \mathrm{II}]_{0336}$
7	$[\mathrm{I}_0^* - \mathrm{I}_1]_{0227}$, $[\mathrm{IV} - \mathrm{III}]_{0337}$
8	$[\mathrm{I}_0 - \mathrm{IV}^*]_{0448}$, $[\mathrm{IV} - \mathrm{IV}]_{0448}$, $[\mathrm{I}_0^* - \mathrm{II}]_{0338}$
9	$[\mathrm{I}_0 - \mathrm{III}^*]_{0339}$, $[\mathrm{I}_0^* - \mathrm{III}]_{0339}$
10	$[\mathrm{I}_0 - \mathrm{II}^*]_{0\,5\,5\,10}$, $[\mathrm{IV}^* - \mathrm{II}]_{0\,5\,5\,10}$, $[\mathrm{I}_0^* - \mathrm{IV}]_{0\,4\,4\,10}$
11	$[\mathrm{II} - \mathrm{III}^*]_{0\,4\,4\,11}$, $[\mathrm{IV}^* - \mathrm{III}]_{0\,5\,5\,11}$

Interpretation of Dualities

- For these elliptic models can find some insight by looking at monodromy relations;
- Start from $\mathrm{III} - \mathrm{III}$ and apply following moves:

$$\begin{aligned}
[\mathrm{III} - \mathrm{III}] &= A_1 B_1 A_1 A_2 B_2 A_2 \\
&= A_1 B_1 A_1 A_1 B_1 A_1 \qquad (\rho \to \tau) \\
&= A_1 B_1 A_1 B_1 A_1 B_1 \qquad (\text{braid}) \\
&= (A_1 B_1)^3 = [\mathrm{I}_0 - \mathrm{I}_0^*]\,,
\end{aligned}$$

  where A_i, B_i are Dehn twists around a_i, b_i of genus two curve;
- Colliding of IV and II singularity gives also $[\mathrm{I}_0 - \mathrm{I}_0^*]$
- Reason for why $\rho \to \tau$ should be valid operation can be seen from duality map with $\beta = 0$:

$$j(\tau)j(\rho) = -1728^2 \frac{a^3}{27 d\, e}\,, \ (j(\tau) - 1728)(j(\rho) - 1728) = 1728^2 \frac{b^2}{4\, d\, e}\,;$$

  Can 'compensate' for ρ degeneration by enhance τ degeneration;
- Note, have duality between non-gemetric/geometric vacua, cf. [Malmendier&Morrison '14];

Comment on Compact 6d Non-Geometric Heterotic Models

- Map to F-Theory side enforces coefficients of sextic to be sections of anti-canonical bundle of $\mathbb{P}^1$;
- Therefore, discriminant (of sextic) is a polynomial of degree 20, i.e. sextic generically degenerates over 20 points (loci of T-fects);
- As example can extend $[\mathrm{III} - \mathrm{III}]$ degeneration to global fibration;
- Obtain in addition to $[\mathrm{III} - \mathrm{III}]$ a $[\mathrm{I}_2 - 0 - 0]$ singularity at one point and $[\mathrm{I}_1 - 0 - 0]$ singularities at 12 further points;

Going 'backwards' I

- So far started always from het. side and mapped to F-theory;
- Furthermore, set of theories obtained that way looks very limited;
  - E.g. didn't see 'bare' I_1 degeneration on het. side, i.e. $(\tau, \rho) \to (\tau + 1, \rho - 1)$;
- Therefore, (at least) for $E_8 \times E_8$ invert map from above:

$$y^2 = x^3 + au^4v^4xz^4 + (bu^5v^7 + cu^6v^6 + du^7v^5)z^6$$

$$\Rightarrow \qquad \begin{array}{l} j(\tau) = -8\frac{4\,a^3+27(c^2-4bd)-\sqrt{\gimel}}{bd} \\ j(\rho) = -8\frac{4\,a^3+27(c^2-4bd)+\sqrt{\gimel}}{bd} \end{array}$$

with $\gimel = 12^3a^3bd + \left[4a^3 + 27(c^2 - 4bd)\right]^2$;

- Fibration of τ and ρ now given in terms of F-theory coefficients;

Going 'backwards' II

- Note: around simple (mod 2) roots of $\daleth$ obtain monodromies exchanging τ and ρ;
- But first want to look into monodromies that keep ρ and τ 'separate', i.e. $SL(2,\mathbb{Z}) \times SL(2,\mathbb{Z})$;
- Use Shioda-Inose structure/ansatz:

$$a = -3 f_\tau f_\rho,\ b = b_\tau b_\rho,\ c = -\tfrac{27}{2} g_\tau g_\rho,\ d = d_\tau d_\rho,$$
$$\Delta_\tau = 4 b_\tau d_\tau,\ \Delta_\rho = 4 b_\rho d_\rho;$$

$\Rightarrow$ $\daleth = 3^{12}\left[f_\tau^3 g_\rho^2 - f_\rho^3 g_\tau^2\right]^2$ and τ-/ρ-fibration are given by Weierstraß eqns.:

$$y^2 = x^3 + f_{\tau/\rho} x + g_{\tau/\rho}$$

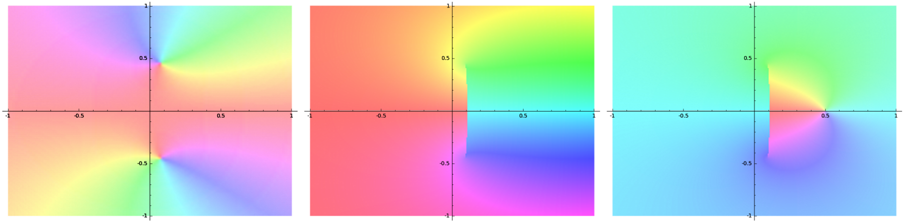
- Simple example: NS5 on top of I_1 degeneration (het. side), i.e. $(\tau,\rho) \to (\tau+1,\rho)$, leads to

$$a = 9 f_\rho\,, \quad b = \Delta_\rho t(t+4)\,, \quad c = -\tfrac{27}{2}(2+t) g_\rho\,, \quad d = \tfrac{27}{16}$$

with f_ρ, g_ρ const. and t the base coordinate; Monodromy appears when going around $t=0$;

Splitting of heterotic 'bare' I_1 degeneration I

- Now want to move NS5 away from I_1 to obtain a 'bare' I_1, i.e. deform previous setting;
- Since location of NS5s given by $b\,d = 0$, change b to $\Delta_\rho(t-\mu)(t+4)$; Therefore, moved NS5 from $t=0$ to $t=\mu$
- To see what this deformation does to het. side, look at plots for $\daleth$, $j(\tau)$, and $j(\rho)$:

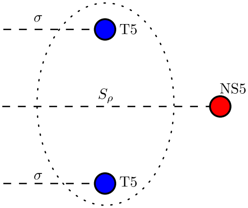


(a) $\daleth$ (b) $j(\tau)$ (c) $j(\rho)$

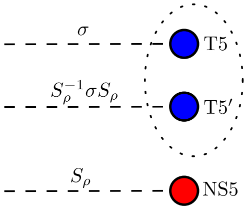
- Note: splitting into three components;

Splitting of heterotic 'bare' I_1 degeneration II

- With bit of work can find branch cut arrangement of fig. (d), where σ, S_ρ and S_τ indicate $\tau \leftrightarrow \rho$, $\rho \to \rho + 1$ and $\tau \to \tau + 1$,respectively;



(d) Symmetric presentation



(e) Decoupled presentation

- Can deform branch cuts to obtain fig. (e);

$\Rightarrow$ Two components (T5 and T5') which are conjugate to each other and NS5;

- Monodromy around T5 plus T5': $(\tau, \rho) \to (\tau + 1, \rho - 1)$;
- Hence, 'bare' I_1 of het. side splits (due to quantum corrections) into two components, T5 and T5';

Summary & Outlook

- Analysed all genus two degenerations from F-theory side;
- Identified dual models in this list;
- Interpretation for some of them;
- Compact examples;
- Inversion of duality in case of $E_8 \times E_8$;

- Extend the analysis to $SO(32)$ case;
- Understand, also in case of one non-vanishing Wilson line, map from F-theory to het. better (away from stable degeneration limit);
  - In-depth study also of this solutions;

Thank you for your attention!