1/2-maximal consistent truncations of EFT and the K3 / Heterotic duality

Emanuel Malek

Arnold Sommerfeld Centre for Theoretical Physics,
Ludwig-Maximilian-University Munich.

Geometry and Physics, Schloss Ringberg, 24th November 2016

EM arXiv:1612.01692, arXiv:1612.01990

Introduction to EFT

- $E_{d(d)}$ invariant extension of supergravities.
- Similar to generalised geometry $TM \oplus T^*M$, $TM \oplus \Lambda^2 T^*M$, ...
- ...but with extra coordinates (in representation of $E_{d(d)}$).
- $E_{d(d)}(\mathbb{Z})$ is toroidal U-duality group. Does it play the same role here?
- No! $E_{d(d)}$ is linear symmetry group.
- DoFs of 11-dimensional supergravity organise into $E_{d(d)}$ reps, c.f. generalised geometry.
- $\mathrm{GL}(11) \longrightarrow \mathrm{GL}(7) \times \mathrm{GL}(4) \longrightarrow \mathrm{GL}(7) \times \mathrm{SL}(5) \times \mathbb{R}^+$
- g_{ij}, C_{ijk}, C_{ijklmn}, $\ldots \longrightarrow M_{ab}(x, Y) \in \mathrm{SL}(5)/\mathrm{USp}(4)$
- $g_{\mu i}$, $C_{\mu ij}$, $C_{\mu ijklm}$, $\ldots \longrightarrow A_\mu{}^{ab}(x, Y)$
- In general, two-forms, etc. appear too.
- 10 "extended coordinates" Y^{ab}. Why extra coordinates?
- Extra coordinates when on CY?

Consistent truncations

- EFT useful for consistent truncations to maximal gSUGRA, e.g.
  Lee, Strickland-Constable, Waldram 1401.3360; Godazgar^2, Krüger Nicolai, Pilch arXiv:1410.5090; Hohm, Samtleben arXiv:1410.8145; EM, Samtleben arXiv:1510.03433
- Makes sense: $E_{d(d)} \to$ scalar coset.
- What about partial SUSY-breaking consistent truncations?
- e.g. $\mathcal{N} = 2$ in 4-d?
- Start easy: $\mathrm{SL}(5)$ EFT for $\mathcal{N} = 2$ in 7-d with n vector multiplets, e.g. M-theory on K3.

Action plan

- Understand $\mathcal{N} = 2$ in $\mathrm{SL}(5)$ EFT.
- Express $\mathrm{SL}(5)$ EFT in terms of $\mathcal{N} = 2$ variables.
- Perform $\mathcal{N} = 2$ consistent truncation.
- Deformation to Heterotic DFT.
- K3 / Heterotic duality.

$\mathcal{N} = 2$ in 7 dimensions

- Fermions in $\mathrm{SL}(5)$ EFT $\in \mathrm{USp}(4)$ reps: $i = 1, \ldots, 4$, Ω_{ij} symplectic
- $\mathcal{N} = 2 \Rightarrow$ two well-defined spinors $\theta^{\dot{\alpha},i}$ (symplectic Majorana)

$\mathcal{N} = 2$ spinors

$$\left(\theta^{\dot{\alpha},i}\right)^* = \Omega_{ij}\epsilon_{\dot{\alpha}\dot{\beta}}\theta^{\dot{\beta},j} ,$$

$$\theta^{\dot{\alpha}\,i}\theta^{\dot{\beta}\,j}\Omega_{ij} = \kappa\epsilon^{\dot{\alpha}\dot{\beta}} .$$

- $\dot{\alpha} = 1, 2$ label $\mathrm{SU}(2)_R$.
- κ is density of weight $\frac{1}{5}$
- Generalised tangent bundle has $\mathrm{SU}(2)$-structure. Coimbra, Strickland-Constable, Waldram arXiv:1411.5721

Spinor bilinears

- Form spinor bilinears in $\mathrm{SL}(5) \times \mathbb{R}^+$ reps: $a = 1, \ldots, 5$

$\mathrm{SU}(2) \subset \mathrm{SL}(5) \times \mathbb{R}^+$ structure

Generalised $\mathrm{SU}(2)$-structure $\Longleftrightarrow$ globally well-defined

$$\kappa \in \mathbf{1}\,, \qquad A^a \in \bar{\mathbf{5}}\,, \qquad A_a \in \mathbf{5}\,, \qquad B_{u,ab} \in \mathbf{10}\,, \quad u = 1,2,3\,,$$

satisfying

$$B_{u,ab}A^b = 0\,, \qquad A^a A_a = \frac{\kappa^2}{2}\,, \qquad B_{u,ab}B_{v,cd}\epsilon^{abcde} = 4\sqrt{2}\delta_{uv}\kappa A^e\,.$$

- Compatibility conditions follow from definition as spinors.
- These implicitly define a generalised metric!
- Generalised vector

$$V_u{}^{ab} = \frac{1}{\kappa}\epsilon^{abcde}B_{u,cd}A_e\,, \qquad V_u{}^{ab}B^v{}_{ab} = 4\sqrt{2}\delta_u{}^v\kappa^2\,.$$

Express in terms of $\mathcal{N} = 2$ variables

- $(\kappa, A^a, A_a, B_{u,ab})$ instead of generalised metric $\mathcal{M}$.

$\mathrm{SU}(2)_S$ connection $\tilde{\nabla}$

$$\tilde{\nabla}_{ab}\kappa = \tilde{\nabla}_{ab}A_c = \tilde{\nabla}_{ab}A^c = \tilde{\nabla}_{ab}B_{u,cd} = 0\,.$$

$\tilde{\nabla}$ always exists but may not be torsion-free.

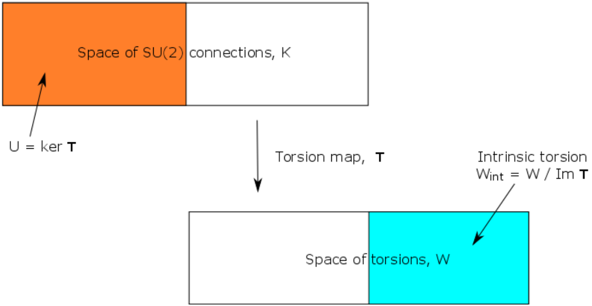
- Intrinsic torsion corresponds to fluxes.
- Scalar potential, SUSY variations, ... in terms of intrinsic torsion.
- Truncation: intrinsic torsion $\longrightarrow$ embedding tensor.

Intrinsic torsion

- Torsion is the tensor part of the connection.
- Recall: EFT has generalised Lie derivative.

$$\mathcal{L}^{\nabla}_{\Lambda} V^a - \mathcal{L}_{\Lambda} V^a = \frac{1}{2} \tau_{bc,d}{}^{a} V^d \Lambda^{bc} \, .$$

- Torsion: $\tau_{ab,c}{}^{d}$.
- Torsion depends on choice of connection.



- Independent of connection: Intrinsic torsion $\in W_{int} = W / \mathrm{Im}\, T$.

Intrinsic torsion

- Rep theory:

$$W_{int} = 2 \cdot (\mathbf{1},\mathbf{1}) \oplus 2 \cdot (\mathbf{1},\mathbf{3}) \oplus (\mathbf{3},\mathbf{1}) \oplus (\mathbf{3},\mathbf{3}) \oplus 3 \cdot (\mathbf{2},\mathbf{2}) \oplus (\mathbf{2},\mathbf{4}) \ .$$

- Intrinsic torsion: independent of $\mathrm{SU}(2)$ connection $\Rightarrow$ can find expressions without using connection.
- Look for tensors involving one derivative, e.g.

$$S = \kappa^{-1} A^a \partial_{ab} A^b = \kappa^{-1} \left(A^a \tilde{\nabla}_{ab} A^b + \frac{1}{2} A^a A^b \tau_{ca,b}{}^c \right) = \kappa^{-1} A^a A^b \tau_{ca,b}{}^c \,.$$

- Use the three generalised vectors $V_u{}^{ab}$.

$$W_{int} = {\color{red}2 \cdot (\mathbf{1},\mathbf{1})} \oplus 2 \cdot (\mathbf{1},\mathbf{3}) \oplus (\mathbf{3},\mathbf{1}) \oplus (\mathbf{3},\mathbf{3}) \oplus 3 \cdot (\mathbf{2},\mathbf{2}) \oplus (\mathbf{2},\mathbf{4}) \ .$$

Singlets

$$T = \frac{1}{12\kappa^2} \epsilon_{uvw} V^{u,ab} \mathcal{L}_{V^v} B^w{}_{ab} \,,$$
$$S = \kappa^{-1} A^a \partial_{ab} A^b \,.$$

Intrinsic torsion - triplets

$$W_{int} = 2 \cdot (\mathbf{1}, \mathbf{1}) \oplus 2 \cdot (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{3}) \oplus 3 \cdot (\mathbf{2}, \mathbf{2}) \oplus (\mathbf{2}, \mathbf{4}) \,.$$

$(\mathbf{1}, \mathbf{3})$

$$T_u = -2\kappa A^a \mathcal{L}_{V_u} \left(A_a \kappa^{-4} \right) \,,$$
$$S_u = 2\kappa^{-6} \mathcal{L}_{V_u} \kappa^5 \,.$$

$(\mathbf{3}, \mathbf{1})$

$$T_{ab} = \frac{1}{12\kappa} \left(\delta_{ab}{}^{cd} - \frac{1}{2\sqrt{2}\kappa^2} B^v{}_{ab} V_v{}^{cd} + \frac{4}{\kappa^2} A_{[a} A^{[c} \delta_{b]}{}^{d]} \right) \mathcal{L}_{V_u} B^u{}_{cd} \,.$$

$(\mathbf{3}, \mathbf{3})$

$$T^u{}_{ab} = \frac{1}{12\kappa} \epsilon^{uvw} \left(\delta_{ab}{}^{cd} - \frac{1}{2\sqrt{2}\kappa^2} B^v{}_{ab} V_v{}^{cd} + \frac{4}{\kappa^2} A_{[a} A^{[c} \delta_{b]}{}^{d]} \right) \mathcal{L}_{V_u} B_{w,ab} \,.$$

Intrinsic torsion - doublets

$$W_{int} = 2 \cdot (\mathbf{1},\mathbf{1}) \oplus 2 \cdot (\mathbf{1},\mathbf{3}) \oplus (\mathbf{3},\mathbf{1}) \oplus (\mathbf{3},\mathbf{3}) \oplus 3 \cdot (\mathbf{2},\mathbf{2}) \oplus (\mathbf{2},\mathbf{4}) \ .$$

$(\mathbf{2},\mathbf{2})$

$$S_a = \kappa^{-2} \left(\partial_{ab} \left(A^b \kappa^2 \right) - 2 A_a A^b \partial_{bc} A^c \right) \ ,$$

$$T_a = \frac{1}{12\kappa^3} \epsilon^{uvw} B_{u,ab} V_v{}^{bc} \mathcal{L}_{V_w} A_c \ ,$$

$$U_a = \kappa^{-2} B_{u,ab} \mathcal{L}_{V^u} A^b \ .$$

$(\mathbf{2},\mathbf{4})$

$$T^u{}_a = \kappa^{-2} \left(\delta_a{}^b \delta^u{}_v + \frac{\sqrt{2}}{3\kappa} B^u{}_{ac} V_v{}^{cb} \right) \epsilon^{vwx} B_{w,bc} \mathcal{L}_{V_x} A^c \ .$$

Scalar potential

- Scalar potential = part of EFT action involving only ∂_{ab} derivatives.
- Rewrite the scalar potential in terms of the intrinsic $\mathrm{SU}(2)$ torsion.
- Intrinsic torsion involves one $\partial \Rightarrow$ build ∂^2 terms using (intrinsic torsion)^2.
- Build metric on the $(\mathbf{2}, \mathbf{2})$

$$M_{ab} = \kappa^{-3} \epsilon_{uvw} B^u{}_{ac} B^v{}_{bd} V^{w,cd} \,,$$

and on $(\mathbf{1}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{1})$

$$N_{[ab],[cd]} = \frac{1}{\kappa} \epsilon_{abcde} A^e \,.$$

- The "scalar potential" of $\mathrm{SL}(5)$ EFT can be written as

$$V\epsilon^i = \frac{1}{2}\nabla_{jk}\nabla^{ki}\epsilon^j - \frac{1}{2}\nabla_{jk}\nabla^{jk}\epsilon^i + \frac{3}{2}\nabla^{ik}\nabla_{jk}\epsilon^j \,.$$

Coimbra, Strickland-Constable, Waldram, arXiv:1112.3989.

- Use $\mathcal{N}=2$ spinors $\epsilon^i = \theta^i{}_{\dot\alpha}\epsilon^{\dot\alpha}$ and integrate by parts.

$$V = \nabla_{jk}\theta^{\dot\alpha}{}_i\nabla^{ik}\theta^j{}_{\dot\alpha} - \frac{1}{2}\nabla_{jk}\theta^{\dot\alpha}{}_i\nabla^{jk}\theta^i{}_{\dot\alpha} + \frac{3}{2}\nabla^{ik}\theta^{\dot\alpha}{}_i\nabla_{jk}\theta^j{}_{\dot\alpha} \,.$$

- Explicit calculation shows

$$V = \frac{1}{2}S^2 - \frac{1}{8}T^2 - \frac{1}{\sqrt{2}}ST - \frac{3}{64}T_uT^u + \frac{1}{32}T_uS^u - \frac{3}{64}S_uS^u - \sqrt{2}T_{ab}T^{ab} - \frac{9}{2\sqrt{2}}T^u{}_{ab}T_u{}^{ab} - \frac{1}{6\sqrt{2}}S_aS^a - \frac{1}{3}S_aT^a + \frac{1}{6}U_aS^a \,.$$

Consistent partially SUSY-breaking truncation

- Want to define consistent truncation to 7-d $\mathcal{N}=2$ gSUGRA.
- $\Rightarrow$ truncate on generalised $\mathrm{SU}(2)$ manifold.
- $\Rightarrow$ expand A^a, A_a, $B_{u,ab}$ in *finite* basis of tensors depending on Y^{ab}.
- $\mathcal{N}=2$ theory $\Rightarrow$ no doublets of $\mathrm{SU}(2)_S$.
- For $(\kappa, A^a, A_a, B_{u,ab})$ under $\mathrm{SL}(5) \to \mathrm{SU}(2)_S \times \mathrm{SU}(2)_R$

$$\mathbf{5} \to (\mathbf{1},\mathbf{1}) \oplus \cancel{(\mathbf{2},\mathbf{2})}\,, \qquad \mathbf{10} \to (\mathbf{1},\mathbf{3}) \oplus (\mathbf{3},\mathbf{1}) \oplus \cancel{(\mathbf{2},\mathbf{2})}\,.$$

Truncation basis

Finite basis of truncation: $(\rho,\ n^a,\ n_a,\ \omega_{M,ab})$

$$\omega_{M,ab} = (\omega_{I,ab}, \omega_{A,ab})\,, \qquad \omega_{M,ab} n^b = 0\,, \qquad M = 1,\ldots,n+3\,,$$

3 sections of $(\mathbf{1},\mathbf{3})$ and n sections of $(\mathbf{3},\mathbf{1})$ bundle of $\mathrm{SU}(2)_S \times \mathrm{SU}(2)_R$.

$$n^a n_a = 1\,, \qquad \epsilon^{abcde}\,\omega_{M,ab}\,\omega_{N,cd} = 4\eta_{MN}\, n^e\,, \qquad \eta_{MN} \text{ is } \mathrm{O}(3,n) \text{ metric.}$$

Truncation Ansatz

Scalar Truncation Ansatz

$$\kappa(x, Y) = k(x)\,\rho(Y)\,,$$
$$A^a(x, Y) = \frac{1}{\sqrt{2}} a(x)\,k(x)\,\rho(Y)\,n^a(Y)\,,$$
$$A_a(x, Y) = \frac{1}{\sqrt{2}} \frac{1}{a(x)} k(x)\,\rho(Y)\,n_a(Y)\,,$$
$$B_{u,ab}(x, Y) = \sqrt{a(x)}\,k(x)\,b_u{}^M(x)\,\rho(Y)\,\omega_{M,ab}(Y)\,.$$

- Coefficients depend on external 7-d coordinates x^μ and give 7-d gSUGRA scalars.
- Compatibility condition

$$B_{u,ab} B_{v,cd} \epsilon^{abcde} = 4\sqrt{2}\kappa A^e \Rightarrow b_u{}^M b_v{}^N \eta_{MN} = \delta_{uv}\,.$$

- Identify $b_u{}^M$ related by $\mathrm{SU}(2)_R$ symmetry.

Scalar coset space

- Recall: $a(x)$, $k(x)$, ${b_u}^M(x)$ with ${b_u}^M {b_v}^N \eta_{MN} = \delta_{uv}$, identify by $\mathrm{SU}(2)_R$.

Scalar degrees of freedom

${b_u}^M(x)$ has $3n + 9 - 6 - 3 = 3n$ degrees of freedom.

$$ {b_u}^M b^{u,N} = \frac{1}{2}\left(\eta^{MN} - \mathcal{H}^{MN}\right) \in \frac{\mathrm{O}(3,n)}{\mathrm{O}(3)\times\mathrm{O}(n)} \,. $$

$a(x)$: dilaton, $k(x)$: $|g_7|$.

$$ \Rightarrow \mathcal{M}_{scalar} = \frac{\mathrm{O}(3,n)}{\mathrm{O}(3)\times\mathrm{O}(n)} \times \mathbb{R}^+ \times \mathbb{R}^+ \,. $$

Truncation Ansatz for other fields

- $7-d$ vector fields

$$A_\mu{}^{ab}(x,Y) \to A_\mu{}^M(x)\,\rho(Y)\,\omega_{M,ab}(Y)\,, \tag{1}$$

$\Rightarrow$ $n+3$ vector fields of 1/2-max 7-d gSUGRA coupled to n vector multiplets.

- 2-form

$$\mathcal{B}_{\mu\nu,a}(x,Y) = \mathcal{B}_{\mu\nu}(x)\,\rho^2(Y)\,n_a(Y)\,, \tag{2}$$

- 3-form

$$\mathcal{C}_{\mu\nu\rho}{}^a(x,Y) = \mathcal{C}_{\mu\nu\rho}(x)\,\rho^3(Y)\,n^a(Y)\,. \tag{3}$$

Consistent truncation

- Conditions for consistent truncation involve generalised Lie derivative.
- Use generalised vectors: ${\omega_M}^{ab} = \epsilon^{abcde}\omega_{M,cd}n_e$.
- $\mathcal{N} = 2 \Rightarrow$ no doublets! Intrinsic torsion: $3 \cdot (\mathbf{2}, \mathbf{2}) \oplus (\mathbf{4}, \mathbf{2}) = 0$.

Doublet conditions for consistent truncation

$$n^a \mathcal{L}_{\omega_M} \omega_{N,ab} = 0\,,$$
$$\partial_{ab}\left(n^b\rho^3\right) = \rho^3 n_a n^b \partial_{bc} n^c\,.$$

Closure condition for consistent truncation

$\omega_{M,ab}$ must form closed set under generalised Lie derivative:

$$\mathcal{L}_{\omega_M} {\omega^N}_{ab} = \frac{1}{4}\left(\mathcal{L}_{\omega_M} {\omega^N}_{cd}\right) {\omega_P}^{cd} {\omega^P}_{ab}\,.$$

Embedding tensor

Embedding tensor

$\mathcal{L}_{\omega_M}\omega_N{}^{ab}\omega_{P,ab}$ has only three irreps:

$$\mathcal{L}_{\omega_{[M}}\omega_N{}^{ab}\omega_{P],ab} = f_{MNP}\,,$$
$$\mathcal{L}_{\omega_M}\omega_{(N}{}^{ab}\omega_{P),ab} \propto n^a\mathcal{L}_{\omega_M}n_a\eta_{NP} = f_M\eta_{NP}\,,$$
$$\mathcal{L}_{\omega_{(M}}\omega_{N),ab}\omega_P{}^{ab} \propto \mathcal{L}_{\omega_P}\rho\eta_{MN} = \xi_P\eta_{MN}\,.$$

Additionally, define

$$\Theta = n^a\partial_{ab}n^b\,.$$

These are the $\mathrm{O}(3,n)$ embedding tensor and singlet deformation parameter Θ.

- Section condition $\Rightarrow$ closure of algebra of generalised Lie derivative $\Rightarrow$ f_{MNP}, f_M, ξ_M, Θ satisfy quadratic constraints.

Intrinsic torsion and T-tensor

Intrinsic torsion components become the T-tensor of 1/2-max gSUGRA.

$$
\begin{aligned}
S &\sim k\, a^2 \Theta \,, \\
T &\sim \frac{k}{\sqrt{a}} \epsilon^{uvw} b_u{}^M b_v{}^N b_w{}^P f_{MNP} \,, \\
T_u &\sim \frac{k}{\sqrt{a}} b_u{}^M \left(3\xi_M - f_M\right) \,, \\
S_u &\sim \frac{k}{\sqrt{a}} b_u{}^M \xi_M \,, \\
T_{ab} &\sim k\, \omega_{M,ab} \left(\eta^{MN} + \mathcal{H}^{MN}\right) \left(\xi_N - f_N\right) \,, \\
T^u{}_{ab} &\sim k\, \omega^M{}_{ab} \left(\eta^{MN} + \mathcal{H}^{MN}\right) \epsilon^{uvw} b_v{}^P b_w{}^Q f_{NPQ} \,, \\
&\vdots \qquad \vdots
\end{aligned}
$$

Scalar potential

Scalar potential

We obtain the potential of the half-maximal gauged supergravity

$$V = \frac{k^2}{a}\left(-\frac{1}{12}f_{MNP}f_{QRS}\mathcal{H}^{MQ}\mathcal{H}^{NR}\mathcal{H}^{PS} + \frac{1}{4}f_{MNP}f_{QRS}\mathcal{H}^{MQ}\eta^{NR}\eta^{PS} - \frac{1}{6}f_{MNP}f_{QRS}\eta^{MQ}\eta^{NR}\eta^{PS}\right) + (f_M)^2 + (\xi_M)^2 + \Theta^2 + \dots\,.$$

- Similarly, for kinetic terms, topological term, SUSY variation.
- All Y-dependence appears only through embedding tensor: f_{MNP}, f_M, ξ_M, Θ.
- Constant f_{MNP}, f_M, ξ_M, $\Theta \Rightarrow$ consistent truncation.

Heterotic DFT as a deformation

- What if we allow coefficients to depend on Y^{ab} as well?

Heterotic Deformation Ansatz

$$\begin{aligned}
\kappa(x, Y) &= k(x, Y)\,\rho(Y)\,,\\
A^a(x, Y) &= \frac{1}{\sqrt{2}} a(x, Y)\,k(x, Y)\,\rho(Y)\,n^a(Y)\,,\\
A_a(x, Y) &= \frac{1}{\sqrt{2}}\frac{1}{a(x, Y)} k(x, Y)\,\rho(Y)\,n_a(Y)\,,\\
B_{u,ab}(x, Y) &= \sqrt{a(x, Y)}\,k(x, Y)\,b_u{}^M(x, Y)\,\rho(Y)\,\omega_{M,ab}(Y)\,,
\end{aligned}$$

- But no doublet derivatives $n^a\partial_{ab}k = n^a\partial_{ab}a = n^a\partial_{ab}b_u{}^M = 0$.

Heterotic doubled space

- Impose $n^a \partial_{ab} k = n^a \partial_{ab} a = n^a \partial_{ab} {b_u}^M = 0$.
- Assume $(\mathbf{1}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{1})$ can be expanded in $\omega_{M,ab}$:

$$\partial_{ab} = \frac{1}{4} \omega^M{}_{ab} \omega_M{}^{cd} \partial_{cd} \,.$$

- Define $D_M = \frac{1}{2} \omega_M{}^{ab} \partial ab$.

$$[D_M, D_N] = \frac{1}{2} f_{MN}{}^P D_P - \omega_N{}^{[ab} \partial_{ab} \omega_M{}^{cd]} \partial_{cd} \,.$$

- Impose $f_{MN}{}^P D_P = 0$ and section condition on $\omega_M{}^{ab}$.
- Can write $D_M = \partial_M \longrightarrow 3 + n$ coordinates of Heterotic DFT.
- f_{MNP} encodes gauge group of Heterotic string Graña, Marques arXiv:1201.2924
  $f_{MN}{}^P \partial_P = 0$ is required in Heterotic DFT Siegel hep-th/9305073; Kwak, Hohm arXiv:1103.2136.

Heterotic generalised Lie derivative

Heterotic generalised Lie derivative

Take $V^{ab} = \rho(Y)\,\omega_M{}^{ab}(Y)\,V^M(x,Y), \quad \Lambda^{ab} = \rho(Y)\,\omega_M{}^{ab}(Y)\,\Lambda^M(x,Y)$

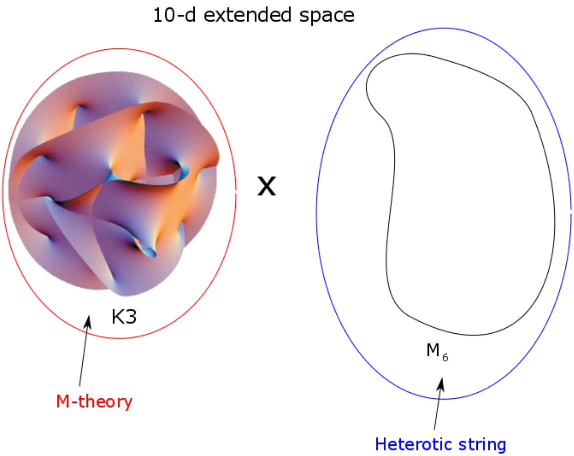
$$\omega^M{}_{ab}\mathcal{L}_\Lambda V^{ab} = \Lambda^N\partial_N V^M - V^N\partial_N\Lambda^M + \eta^{MN}\eta_{PQ}V^P\partial_N\Lambda^Q + f^M{}_{NP}V^N\Lambda^P\,.$$

We obtain the Heterotic generalised Lie derivative.

- Section condition becomes $\eta^{MN}\partial_M \otimes \partial_N = 0$.
- Restricts dependence to 3 coordinates.
- Action becomes Heterotic $\mathrm{O}(n,3)$ DFT action with 7 external directions.
- $b_u{}^M \longrightarrow$ "frame-like formulation of DFT" Siegel, hep-th/9305073; Hohm, Hull, Zwiebach arXiv:1006.4823; Kwak, Hohm arXiv:1011.4101.

M-theory / Heterotic duality

- Consider consistent truncation on K3 $\Rightarrow \mathrm{SU}(2)$-structure.
- Dependence only on Y^{i5}, $i = 1, \ldots, 4$.
- $n^5 = n_5 = 1$, $n^i = n_i = 0$.
- $\omega_{M,ij}$ are 22 harmonic forms.
- Doublet constraints $\propto d\omega_M = 0$.
- All gaugings vanish: ungauged 7-D SUGRA.
- Duality is change of section.
- Interpret Y^{i5} as section: $\Rightarrow$ four M-theory coordinates $\Rightarrow$ K3 truncation of M-theory.
- Interpret 3 out of Y^{ij} as section: $\Rightarrow$ Deformation to Heterotic DFT with gauge group $U(1)^{22} \Rightarrow T^3$ reduction of Heterotic SUGRA.
- K3 lives in the extended space for the heterotic string!


10-d extended space
K3
x
M$_6$
M-theory
Heterotic string

Conclusions and Outlook

- Studied consistent $1/2$-SUSY truncations of $\mathrm{SL}(5)$ EFT.
- These give $1/2$-maximal 7-d $\mathrm{SO}(3, n)$ gSUGRA.
- Embedding tensor is found from generalised Lie derivative.
- Full gaugings are obtained, including singlet deformation Θ.
- $\mathrm{SU}(2)$-structure in extended space gives rise to heterotic DFT: embedding tensor defines gauge group.
- K3 gauge enhancement at orbifold singularities?
- Non-geometric $\mathrm{SU}(2)$-backgrounds?
- Uplift 7-d stable deSitter gSUGRA? Contains $\Theta \neq 0$. Dibitetto, Fernández-Melgarejo, Marqués arXiv:1506.01294.
- Lower dimensions, less SUSY.