Maximally Supersymmetric Solutions in Supergravity

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Introduction

- Supersymmetric solutions in supergravity are well studied, many are classified.

- They are believed to uplift to solutions of the full string theory and are related to string solitons, i.e. non-perturbative string theory.
- Moreover anti-de Sitter solutions feature prominently in AdS/CFT correspondence.

- Here: **Maximally supersymmetric solutions**.

  1. Classification of all background space-times for all gauged and deformed supergravity theories in $D \geq 3$ space-time dimensions.

  2. AdS solutions and their moduli spaces.

Outline / Results

A) Set the stage to discuss solutions in a generic framework.

B) Solutions *without* fluxes:

- Only two possible cases:

$$Mink_D \qquad \text{and} \qquad AdS_D$$

(only gauged / deformed SUGRAs)

C) Solutions *with* (non-trivial) fluxes:

- Exist only for a small class of theories.
- Solutions coincide with those of the corresponding ungauged theories.
- For these theories all solutions are known and classified.

$\rightarrow$ Exhaustive list of solutions.

D) AdS$_D$ solutions

- Characterize gaugings.
- Moduli spaces for theories with coset scalar field space.

Maximally supersymmetric backgrounds

- Classical solutions / backgrounds for which

$$\langle \delta_\epsilon B \rangle \;=\; \langle \delta_\epsilon F \rangle \;= 0\,.$$

SUSY variations of
bosonic / fermionic fields

- Here: bosonic solutions, i.e. $\langle F \rangle = 0$

$$\Rightarrow \quad \langle \delta_\epsilon B \rangle \sim \langle F \rangle = 0\,.$$

$\Rightarrow$ only remaining condition: $\langle \delta_\epsilon F \rangle = 0$

- indep. supersymmetry paramters ϵ $\quad \leftrightarrow \quad$ preserved supercharges

Supersymmetry variations

- gravitini:

$$\delta\psi^i_\mu \;=\; D_\mu\epsilon^i \quad + \quad (\mathcal{F}_{0\mu})^i_{\;j}\,\epsilon^j \quad + \quad A^i_{0\,j}\,\gamma_\mu\epsilon^j$$

fluxes from the
gravity multiplet

gaugings /
deformations

covariant derivative: $D_\mu\epsilon^i = \nabla_\mu\epsilon^i + (\mathcal{Q}_\mu)^i_{\;j}\,\epsilon^j$

- spin-1/2 fermions from the gravity multiplet ("dilatini"):

$$\delta\chi^a \;=\; (\mathcal{F}_1)^a_{\;i}\,\epsilon^i \quad + \quad A^a_{1\,i}\,\epsilon^i$$

- spin-1/2 fermions from other multiplets ("gaugini", "hyperini", ...):

$$\delta\lambda^s \;=\; (\mathcal{F}_2)^s_{\;i}\,\epsilon^i \quad + \quad A^s_{2\,i}\,\epsilon^i$$

fluxes from
other multiplets

Solutions without fluxes (1)

- Easest case: all fluxes vanish:

$$\mathcal{F}_{0\mu} = \mathcal{F}_1 = \mathcal{F}_2 = 0$$

- Supersymmetric solutions of gauged supergravity without background fluxes need to satisfy the Killing spinor equation:

$$\delta\psi^i_\mu = \nabla_\mu \epsilon^i + A^i_{0\,j}\gamma_\mu \epsilon^j = 0$$

- Integrability condition:

$$\left(\frac{1}{4} R_{\mu\nu}{}^{\alpha\beta}\delta^i_k + 2 A^i_{0\,j} A^j_{0\,k}\delta^\alpha_\mu \delta^\beta_\nu\right)\gamma_{\alpha\beta}\epsilon^k = 0$$

$\Rightarrow$ Unbroken supersymmetry (without fluxes):
$\mathbf{Mink_D}$ and $\mathbf{AdS_D}$ are the only possible solutions.

Solutions without fluxes (2)

- Spin-1/2 variations:

$$\delta\chi^a = A_{1i}^a\,\epsilon^i = 0\,, \qquad \delta\lambda^s = A_{2i}^s\,\epsilon^i = 0$$

Algebraic conditions

$${A_0}^2 = -\frac{\Lambda}{2(D-1)(D-2)}\,\mathbb{1}\ , \qquad A_1 = A_2 = 0$$

- Compare with the potential:

$$V = -c_0\mathrm{tr}(A_0^\dagger A_0) + c_1\mathrm{tr}(A_1^\dagger A_1) + c_2\mathrm{tr}(A_2^\dagger A_2)\,,$$

- $\mathbf{Mink_D}$ ($\Lambda = 0$) solutions exist for all theories.
- For $D \leq 7$ *most* gauged theories admit $\mathbf{AdS_D}$ ($\Lambda < 0$) solutions. (See second half of this talk.)

[de Alwis,Louis,McAllister,Triendl,Westphal][Louis,Triendl][Louis,SL][Louis,Triendl,Zagermann][Louis,Muranaka]

Solutions with non-trivial flux (1)

- For more "interesting" solutions: Allow for non-vanishing flux.
- Firstly: spin-1/2 variations:

$$\delta\chi^a = (\mathcal{F}_1)^a_i\,\epsilon^i + A^a_{1i}\,\epsilon^i = 0\,,\quad \delta\lambda^s = (\mathcal{F}_2)^s_i\,\epsilon^i + A^s_{2i}\,\epsilon^i = 0$$

- Unbroken supersymmetry:

$$\mathcal{F}_1 = \mathcal{F}_2 = A_1 = A_2 = 0 \qquad \Rightarrow \qquad \mathcal{F}_{0\mu} = 0$$

- Generically no non-trivial fluxes possible!

Two exceptions

1. theories without χ's in the gravitational multiplet.
2. chiral theories with selfdual fluxes.

Solutions with non-trivial flux (2)

- Secondly: gravitino variations:

$$\delta\psi^i_\mu = \nabla_\mu \epsilon^i + (\mathcal{Q}_\mu)^i_j + (\mathcal{F}_{0\mu})^i_j\,\epsilon^j + A^i_{0\,j}\gamma_\mu \epsilon^j = 0$$

- Integrability condition:

$$\left(\tfrac{1}{4} R_{\mu\nu\rho\sigma}\gamma^{\rho\sigma}\delta^i_j - (\mathcal{H}_{\mu\nu})^i_j + \ldots\right)\epsilon^j = 0\,,$$

  where $\mathcal{H}_{\mu\nu}$ is the field strength corresponding to $\mathcal{Q}_\mu$.

- Unbroken supersymmetry:

$$\mathcal{H}_{\mu\nu} = 0 \qquad \Rightarrow \qquad \mathcal{Q}_\mu = A_0 = 0$$

  (See also [Hristov,Looyestijn,Vandoren] [Gauntlett,Gutowski] [Akyol,Papadopoulos])

- The supersymmetry variations take the same form as in the ungauged / undeformed case.

Solutions with non-trivial flux (3)

SUGRAs with non supersymmetry breaking flux:

dimension	supersymmetry	q	possible flux	classification
$D = 11$	$N = 1$	32	$F^{(4)}$	[Figueroa-O'Farrill,Papadopoulos]
$D = 10$	IIB	32	$F^{(5)}_{+}$	[Figueroa-O'Farrill,Papadopoulos]
$D = 6$	$N = (2, 0)$	16	$5 \times F^{(3)}_{+}$	[Chamseddine,Figueroa-O'Farrill,Sabra]
$D = 6$	$N = (1, 0)$	8	$F^{(3)}_{+}$	[Gutowski,Martelli,Reall]
$D = 5$	$N = 2$	8	$F^{(2)}$	[Gauntlett,Gutowski,Hull,Pakis,Reall]
$D = 4$	$N = 2$	8	$F^{(2)}$	[Tod]

The maximally supersymmetric solutions are classified:

- $AdS_p \times S^{(D-p)}$ and $AdS_{(D-p)} \times S^p$, for p-form flux.
- Hpp-wave as Penrose-limit of $AdS \times S$ solutions.

  [Penrose;Gueven;Blau,Figueroa-O'Farrill,Hull,Papadopoulos]
- Exceptional solutions in $D = 5$. [Gauntlett,Gutowski,Hull,Pakis,Reall]

All maximally supersymmetric solutions

(with non-trival flux)

dim.	SUSY	q	$AdS \times S$	H*pp*-wave	others
$D = 11$	$\mathcal{N} = 1$	32	$\mathrm{AdS}_4 \times S^7$ $\mathrm{AdS}_7 \times S^4$	KG_{11}	-
$D = 10$	IIB	32	$\mathrm{AdS}_5 \times S^5$	KG_{10}	-
$D = 6$	$\mathcal{N} = (2, 0)$ $\mathcal{N} = (1, 0)$	16 8	$\mathrm{AdS}_3 \times S^3$	KG_6	-
$D = 5$	$\mathcal{N} = 2$	8	$\mathrm{AdS}_2 \times S^3$ $\mathrm{AdS}_3 \times S^2$	KG_5	Gödel-like, NH-BMPV*
$D = 4$	$\mathcal{N} = 2$	8	$\mathrm{AdS}_2 \times S^2$	KG_4	-

* = near-horizon limit of the BMPV [Breckenridge,Myers,Peet,Vafa] black hole

AdS solutions

- So far: classification of background *space-time geometries*.
- Now: focus on *algebraic conditions* and *target space geometry*.

- Most promising: $\mathbf{AdS_D}$ (here: $D \geq 4$):
  - Conditions $A_0^2 \sim \mathbb{1}$ and $A_1 = A_2 = 0$ require gauging.
  - May have non-trivial moduli spaces.

Motivation: AdS/CFT correspondence

AdS_D solution with q supercharges	$\leftrightarrow$	SCFT in $(D-1)$ dim. with $q/2$ supercharges
gauge symmetry	$\leftrightarrow$	global symmetry
moduli space	$\leftrightarrow$	conformal manifold

The gauged R-symmetry

- The conditions $A_0^2 \sim \mathbb{1}$ and $A_1 = A_2 = 0$ require $\mathcal{Q}_\mu \neq 0$.

$\Rightarrow$ The R-symmetry group H_R must be gauged by

$$H_R^g \subseteq H_R \,,$$

where H_R^g needs to be generated by the vector fields from the gravity multiplet, i.e. the *"graphiphotons"*.

- H_R^g is uniquely determined to be the maximal subgroup of H_R, s.t.

  a) A_0 is H_R^g invariant, i.e.
  $$[\mathfrak{h}_R^g, A_0] = 0$$

  b) The decomposition of the H_R-representation r_{GP} of the graviphotons w.r.t. H_R^g contains the adjoint representation of H_R^g, i.e.
  $$\mathrm{r}_{GP} \to \mathrm{ad}_{H_R^g} \oplus \ldots$$

Coset spaces

- From now on: Focus on theories where the scalar field space is a coset:
$$\mathcal{M}_{scal} = \frac{G}{H}\,, \qquad H = H_R \times H_{mat}$$
- Gauge group: $G^g \subseteq G$, where
$$G^g = G^g_R \times G^g_{mat}\,,$$
s.t. H^g_R is the maximal compact subgroup of G^g_R.
- Lie algebra of G:
$$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{k}$$
Notice: $[\mathfrak{h}, \mathfrak{k}] \subseteq \mathfrak{k}$, i.e. $\mathfrak{k}$ transformes in some $\mathfrak{h}$ representation.
- Decompose $\mathfrak{k}$ into $\mathfrak{h}^g_R$ irreps:
$$\mathfrak{k} = \bigoplus_i \mathfrak{k}_i\,, \qquad [\mathfrak{h}^g_R, \mathfrak{k}_i] \subseteq \mathfrak{k}_i$$

The moduli space

- Flat directions of the potential $V(\phi)$:

$$\begin{aligned}\mathfrak{f} &= \{\delta\phi \in \mathfrak{k} : V(\langle\phi\rangle + \delta\phi) = V(\langle\phi\rangle)\} \\ &= \{\delta\phi \in \mathfrak{k} : [\delta\phi, \mathfrak{g}_R^g] \subseteq \mathfrak{g}_R^g\}\end{aligned}$$

  ($\mathfrak{f}$ is the non-compact part of the normalizer $N_{\mathfrak{g}}(\mathfrak{g}_R^g)$)

- Observation:

$$\mathfrak{f} = \mathfrak{k}_R^g \oplus \mathfrak{k}_0$$

non-compact part of $\mathfrak{g}_R^g$
$\rightarrow$ Goldstone bosons

$\mathfrak{h}_R^g$ singlets in $\mathfrak{k}$, i.e.
$[\mathfrak{h}_R^g, \mathfrak{k}_0] = 0$
$\rightarrow$ span the moduli space

- The moduli $\mathfrak{k}_0$ are the non-compact part of $\mathfrak{g}_0 = \mathfrak{h}_0 \oplus \mathfrak{k}_0$ (which is the centralizer $C_{\mathfrak{g}}(\mathfrak{g}_R^g)$).

$$\Rightarrow \quad \mathcal{M}_{AdS} = \frac{G_0}{H_0}$$

All AdS$_D$ solutions for $D \geq 4$

dim.	SUSY	H_R^g	$\mathcal{M}_{AdS}$	
$D=7$	$\mathcal{N}=4$	USp($\mathcal{N}$)	•	[Pernici,Pilch,van Nieuwenhuizen] [Louis,SL,Rüter (to app.)]
	$\mathcal{N}=2$		•	[Louis,SL]
$D=6$	$\mathcal{N}=(1,1)$	SU(2) $\otimes$ SU(2)	•	[Romans]
$D=5$	$\mathcal{N}=8$	SU(4)	$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}$	[Günaydin,Romans,Warner] [Louis,SL,Rüter]
	$\mathcal{N}=6$	U($\mathcal{N}/2$)	•	[Ferrara,Porrati,Zaffaroni]
	$\mathcal{N}=4$		$\frac{\mathrm{SU}(1,p)}{\mathrm{U}(1)\times \mathrm{SU}(p)}$	[Corrado,Günaydin,Warner,Zagermann] [Louis,Triendl,Zagermann]
	$\mathcal{N}=2$		Kähler	[Louis,Muranaka]
$D=4$	$\mathcal{N}=8$	SO($\mathcal{N}$)	•	[de Wit, Nicolai][Louis,SL,Rüter]
	$\mathcal{N}=5,6$		•	[de Wit,Nicolai]
	$\mathcal{N}=4$		•	[Louis,Triendl]
	$\mathcal{N}=3$		•	
	$\mathcal{N}=2$		Kähler	[de Alwis, et al.]
	$\mathcal{N}=1$		real	[de Alwis, et al.]

$\rightarrow$ Perfect agreement with marginal deformations of SCFTs [Cordova,Dumitrescu,Intriligator]

Summary

- Systematic classification of all maximally supersymmetric supergravity backgrounds:
  - no fluxes: only flat space-time and anti-de Sitter.
  - fluxes: - generically not possible
    - in the gauged case: same solutions as for ungauged theories
    - all solutions are known and classified
- General recipe for the computation of AdS_D moduli spaces for theories with coset scalar manifold.

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Thank You!