

# Towards the 1-loop effective action of type IIB orientifolds

Michael Haack, (LMU Munich)  
Schloss Ringberg, November 21, 2016

1407.0027 (with Marcus Berg, Jin U Kang and Stefan Sjors)  
1511.03957 (with Jin U Kang)  
+ work in progress (with Jin U Kang)

# Overview

- Motivation
- Computational setup
- Results

# Motivation

(Perturbative) quantum corrections to effective action can be important

- if “zero effect” at tree level  
(e.g. no-scale structure of potential)
- for “precision phenomenology”  
(e.g. structure of soft SUSY breaking terms)
- increase possibilities for model building and moduli stabilization, cf. LVS

# $\mathcal{N} = 1, d = 4$ Supergravity

$$\frac{\mathcal{L}_{\text{bos}}}{(-G)^{1/2}} = \frac{1}{2\kappa^2} R - K_{,\bar{I}J} D_\mu \bar{\Phi}^{\bar{I}} D^\mu \Phi^J - \frac{1}{4} \text{Re}(f_{ab}(\Phi)) F_{\mu\nu}^a F^{b\mu\nu} \\ - \frac{1}{8} \text{Im}(f_{ab}(\Phi)) \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^b - V(\Phi, \bar{\Phi})$$

**with**  $V(\Phi, \bar{\Phi}) = e^K (G^{\bar{I}J} D_{\bar{I}} \bar{W} D_J W - 3|W|^2) + \text{Re}(f_{ab}) \mathcal{D}^a \mathcal{D}^b$   
 $D_J W \equiv \partial_{\phi^J} W + \partial_{\phi^J} K W$

- Superpotential  $W$
- Gauge kinetic function  $f_{ab}$
- Kähler potential  $K$

# Quantum Corrections

- Superpotential  $W = W^{\text{tree}} + W^{\text{non-pert}}$
- Gauge kinetic function  $f = f^{\text{tree}} + f^{1\text{-loop}} + f^{\text{non-pert}}$
- Kähler potential  $K = K^{\text{tree}} + \sum_{n=1}^{\infty} K^{n\text{-loop}} + K^{\text{non-pert}}$

# Computational setup

- Focus on type I theory and  $K$  for moduli
- Methods:
  - ★ Truncation of type II results
  - ★ Duality (heterotic theory / F-theory)
  - ★ Scattering amplitudes in type I ← focus of this talk

- String amplitudes give corrections in string frame
- Need 1-loop correction to:
  - (i) scalar metric in string frame
  - (ii) Einstein-Hilbert term
  - (iii) definition of field variables ( $\tau = \tau^{(0)} + \delta\tau$ )
- Reason: in string frame we have

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left[ (e^{-2\phi_4} + \delta E) \frac{1}{2} R + (\tilde{G}^{(0)} + \tilde{G}^{(1)}) \partial_\mu \tau^{(0)} \partial^\mu \tau^{(0)} + \dots \right]$$

(ii)  $\implies$  **Weylrescaling:**

$$g_{\mu\nu}^{(E)} = \underbrace{(e^{-2\Phi_4} + \delta E)}_{\equiv \Omega^{-2}} g_{\mu\nu}^{(S)}$$

$\implies$  ●  $\tilde{G}^{(0)}$  is multiplied by  $\Omega^2$

$$\begin{aligned} \bullet \quad R^{(S)} &= \Omega^{-2} \left( R^{(E)} - 2(D-1)g^{(E)\mu\nu} \nabla_\mu \partial_\nu \ln \Omega \right. \\ &\quad \left. - (D-2)(D-1)g^{(E)\mu\nu} \underbrace{\partial_\mu \ln \Omega \partial_\nu \ln \Omega}_{= \frac{1}{\Omega} \partial_\tau \Omega \partial_\mu \tau + \dots} \right) \end{aligned}$$

$\delta E$  depends on  $\tau$



(iii)  $\implies$

$$\begin{aligned} & \tilde{G}^{(0)}(\tau - \delta\tau) \partial_\mu \tau^{(0)} \partial^\mu \tau^{(0)} \\ &= \tilde{G}^{(0)}(\tau) \partial_\mu \tau \partial^\mu \tau \\ & \quad - \partial_\tau \tilde{G}^{(0)}(\tau) \delta\tau \partial_\mu \tau \partial^\mu \tau - 2\tilde{G}^{(0)}(\tau) (\partial_{\tau^{(0)}} \delta\tau) \partial_\mu \tau \partial^\mu \tau \end{aligned}$$

# Some generalities of the amplitude calculations

- Aim: read off scalar metric from scalar 2-pt fct.

- 2-pt fct. = 0 on-shell

- Trick: use  $p_1 + p_2 \neq 0 \iff \delta \equiv p_1 \cdot p_2 \neq 0$

in intermediate steps

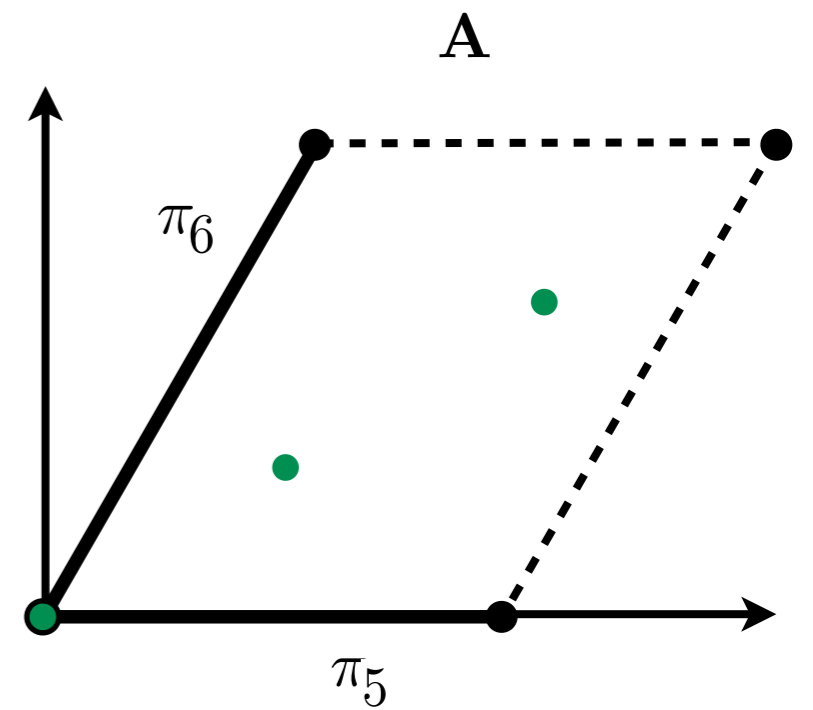
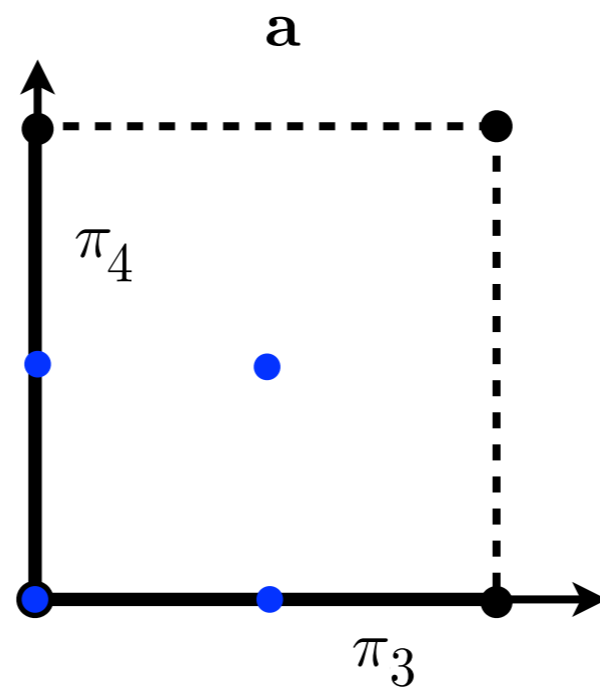
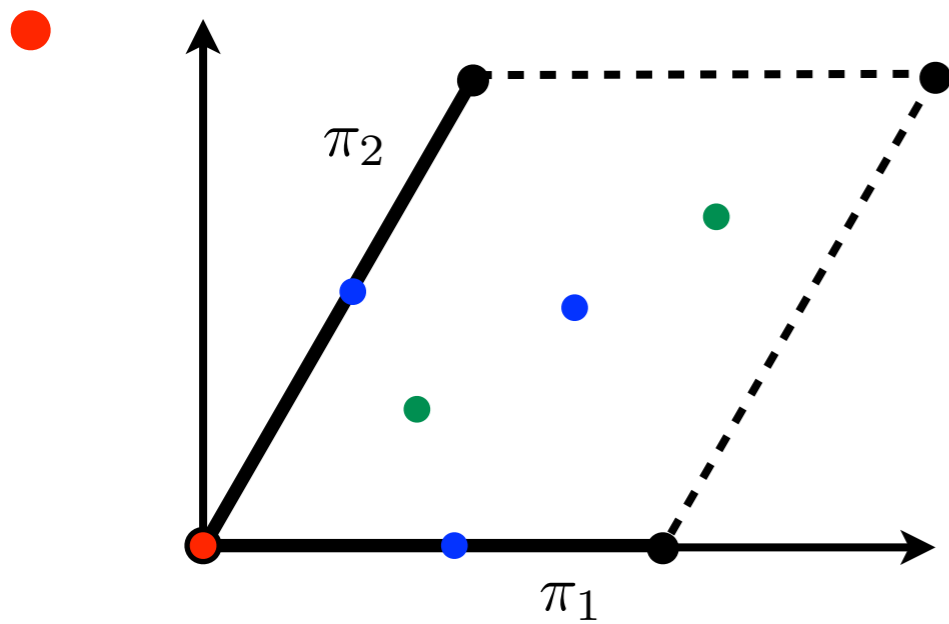
[Atick, Dixon, Sen; Minahan; Antoniadis, Bachas, Fabre, Partouche, Taylor; Antoniadis, Kiritsis, Rizos; cf. also Kiritsis, Kounnas, ...]

- $\langle \Phi_i \Phi_j \rangle = \delta G_{ij} + \mathcal{O}(\delta^2)$

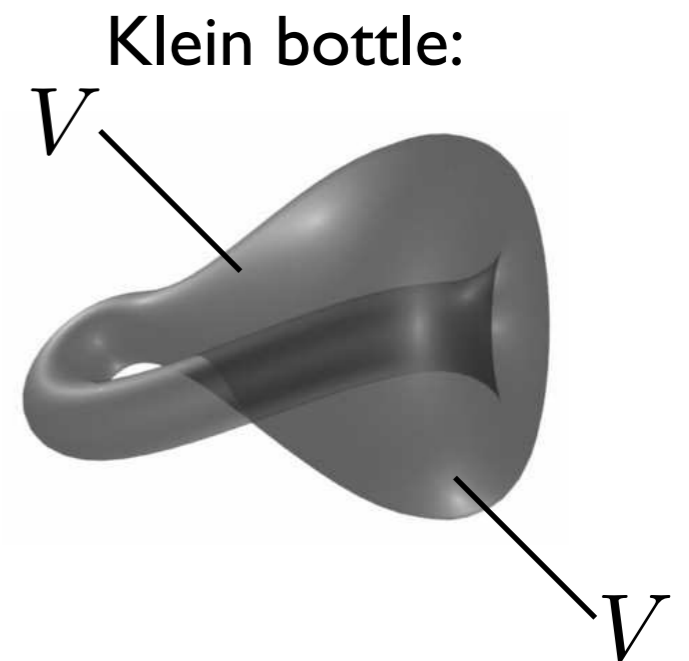
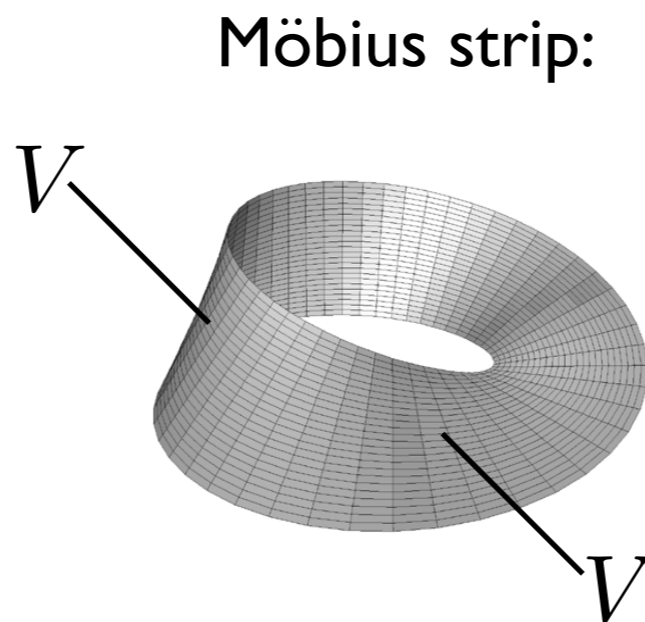
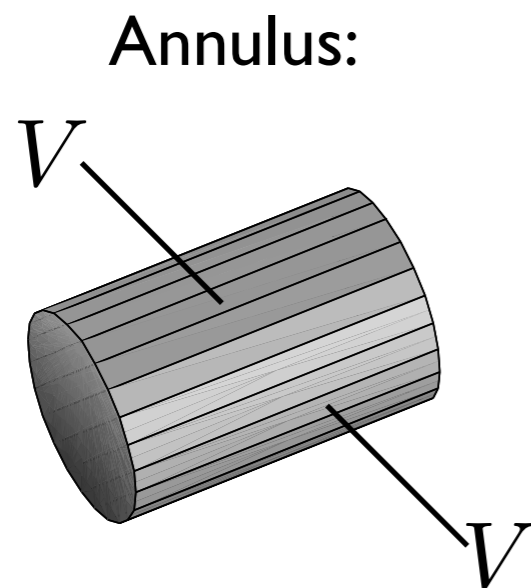
- Similarly for gravitons:  $\langle hh \rangle \sim \delta E p_2^\mu \epsilon_{1\mu\nu} \eta^{\nu\lambda} \epsilon_{2\lambda\rho} p_1^\rho$

$$T^6 / \mathbb{Z}'_6$$

- $\Theta Z^1 = e^{2\pi i v_1} Z^1$      $\Theta Z^2 = e^{2\pi i v_2} Z^2$      $\Theta Z^3 = e^{2\pi i v_3} Z^3$   
 $(v_1, v_2, v_3) = \left( \frac{1}{6}, -\frac{1}{2}, \frac{1}{3} \right)$



- Resulting 4D effective action has  $\mathcal{N} = 1$
- Model contains D9- and D5-branes (wrapped around 3rd torus)
- In addition to torus, need to calculate:



All of them have Euler number  $\chi = 2 - 2h - b - c = 0$

- E.g. annulus:

$$\mathcal{A} \sim \int_0^\infty \frac{dt}{t} \text{Tr}_{open} \left( \left[ \frac{1}{6} \sum_{k=0}^5 \Theta^k \right] q^{(p^2+m^2)/2} VV \right)$$

$$= \frac{1}{6} \sum_{k=0}^5 \int_0^\infty \frac{dt}{t} \text{Tr}_{open} \left( \Theta^k q^{(p^2+m^2)/2} VV \right)$$

projection operator  
on orbifold invariant states

$e^{-\pi t}$

vertex operator

- $\mathcal{N} = 1$  contribution, if strings twisted along all 3 tori

# Results: Scalar kinetic term

- Concretely considered:  $\tau = \text{Im}(T_3)$  with  $\tau^{(0)} \sim e^{-\Phi} \mathcal{V}_3$ 

10D dilaton

volume of 3rd torus measured with string frame metric

- Prior results: sphere, torus and  $\mathcal{N} = 2$ -sectors:

$$\tilde{G}^{(0)} = -\frac{e^{-2\Phi} \mathcal{V}}{4(\tau^{(0)})^2} \left( 1 + \frac{\zeta(3)\chi}{\mathcal{V}} \right)$$

Euler number (48 for  $\mathbb{Z}'_6$ )

overall string frame volume

$$\tilde{G}^{(1)} \sim \frac{\chi}{(\tau^{(0)})^2} + a_1 \frac{e^{-\Phi}}{(\tau^{(0)})^3} E_2(U_3) + a_2 \frac{\mathcal{V}_2}{(\tau^{(0)})^2} E_2(-1/U_2)$$

complex structure of 3rd torus

$$E_2(U) \equiv \sum_{(m,n) \neq (0,0)} \frac{(\text{Im}(U))^2}{|m + nU|^2}$$

- New result:  $\mathcal{N} = 1$  sectors [Berg, M.H., Kang, Sjörs]
- Usual lore:  $\mathcal{N} = 1$  sectors less interesting, because they do not lead to moduli dependent results
- Moduli dependence in  $\mathcal{N} = 1$  sectors via:
  - ★ Normalization of vertex operators
  - ★ Weyl rescaling to Einstein frame
- Expect further moduli dependence in  $\mathcal{N} = 1$  in presence of world volume fluxes or for branes at angles [Berg, M.H., Kang]

# Gauge coupling thresholds

- $$\frac{1}{g_a^2(\mu)} = \frac{1}{g_{a,\text{string}}^2} + \frac{b_a}{16\pi^2} \ln \left( \frac{M_s^2}{\mu^2} \right) + \Delta_a$$

- **Branes at angles:** [Lüst, Stieberger;  
Akerblom, Blumenhagen, Lüst, Schmidt-Sommerfeld]

$$\Delta_a \sim \sum_b \ln \left( \frac{\Gamma(\varphi_{ab}^1) \Gamma(\varphi_{ab}^2) \Gamma(1 + \varphi_{ab}^3)}{\Gamma(1 - \varphi_{ab}^1) \Gamma(1 - \varphi_{ab}^2) \Gamma(-\varphi_{ab}^3)} \right)$$

- $\varphi_{ab} = \varphi_a - \varphi_b$  depend on torus complex structure



- For  $\tau = \text{Im}(T_3)$  in  $\mathbb{Z}'_6$  from  $\mathcal{A}, \mathcal{M}, \mathcal{K}$ : [Berg, M.H., Kang, Sjörs]

$$\tilde{G}_{(\mathcal{N}=1)}^{(1)} = \frac{5}{2^9 \pi^3} \overbrace{\text{Cl}_2\left(\frac{\pi}{3}\right)}^{\approx 3.197 \cdot 10^{-4}} \frac{1}{(\tau^{(0)})^2}$$

2nd Clausen function  $\text{Cl}_2(\varphi) = \sum_{k=1}^{\infty} \frac{\sin(k\varphi)}{k^2}$

- This is a correction to the usual torus contribution

$$\sim \frac{\chi}{(\tau^{(0)})^2}$$

# Results: EH-term

- Prior results:

- ★ Type II: [Antoniadis, Ferrara, Minasian, Narain]

$$S^{(II)} = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left[ e^{-2\Phi_4} + \chi(\zeta(3)) e^{-2\Phi_4} \pm \frac{\pi^2}{6} \right] \frac{R}{2} + \dots$$

IIB  
IIA

- ★ Type I (on  $K_3 \times T^2$ ): [Antoniadis, Bachas, Fabre, Partouche, Taylor]

$$S^{(I)} = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left[ e^{-2\Phi_4} + a \underbrace{\frac{1}{\mathcal{V}_{T^2}} E_2(U)}_{\text{from } \mathcal{A}, \mathcal{M}, \mathcal{K}} \right] \frac{R}{2} + \dots$$

★ Heterotic string: No 1-loop contribution, i.e.

$$S^{(het)} = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left[ e^{-2\Phi_4} \left( 1 + \frac{\chi \zeta(3)}{\nu} \right) \right] \frac{R}{2} + \dots$$

[Antoniadis, Gava, Narain; Kiritsis, Kounnas]

This can be understood via

- World sheet calculation: integrand of torus & higher loop graviton 2-point function is total derivative

[Kiritsis, Kounnas, Petropoulos, Rizos]

- $10D$   $R^4$ -terms

- **Type II:** [Gross, Witten; Green, Schwarz; Grisar, van de Ven, Zanon]

$$\underbrace{(\zeta(3)e^{-2\Phi} + \frac{\pi^2}{6})t_8t_8R^4}_{\text{leads to correction to 4D kinetic terms of scalars}} - \underbrace{(\zeta(3)e^{-2\Phi} \pm \frac{\pi^2}{6})\epsilon_{10}\epsilon_{10}R^4}_{\text{leads to correction to 4D EH-term}}$$

IIB  
 IIA

leads to correction to 4D  
kinetic terms of scalars

leads to correction  
to 4D EH-term

- **Heterotic:** [Cai, Nunez; Gross, Sloan; Sakai, Tani; Abe, Kubota, Sakai]

$$(\zeta(3)e^{-2\Phi} + \frac{\pi^2}{6})t_8t_8R^4 - \zeta(3)e^{-2\Phi}\epsilon_{10}\epsilon_{10}R^4$$

- How is this compatible with heterotic / type I duality?

[Tseytlin; Green, Rudra]

$$g_{\mu\nu}^{(I)} = e^{-\Phi_{het}} g_{\mu\nu}^{(het)} \quad , \quad \Phi_I = -\Phi_{het}$$

- How is this compatible with heterotic / type I duality?

[Tseytlin; Green, Rudra]

$$g_{\mu\nu}^{(I)} = e^{-\Phi_{het}} g_{\mu\nu}^{(het)} \quad , \quad \Phi_I = -\Phi_{het}$$

- Possible answer:  $\mathcal{S}^{(het)}$  in 10D contains

[Green, Rudra]

$$\sqrt{g^{(het)}} e^{-\Phi_{het}/2} J_0 E_{3/2}(e^{-\Phi_{het}}) - \sqrt{g^{(het)}} \frac{\pi^2}{6} \mathcal{I}_2$$

$$\star \quad J_0 = t_8 t_8 R^4 - \frac{1}{8} \epsilon_{10} \epsilon_{10} R^4$$

$$\star \quad \mathcal{I}_2 = -\frac{1}{8} \epsilon_{10} \epsilon_{10} R^4 + \dots$$

$$\star \quad E_{3/2} = \zeta(3) e^{-3/2 \Phi_{het}} + \frac{\pi^2}{6} e^{\Phi_{het}/2} + non-pert.$$

**S-duality invariant**

- How is this compatible with heterotic / type I duality?

[Tseytlin; Green, Rudra]

$$g_{\mu\nu}^{(I)} = e^{-\Phi_{het}} g_{\mu\nu}^{(het)} \quad , \quad \Phi_I = -\Phi_{het}$$

- Possible answer:  $\mathcal{S}^{(het)}$  in 10D contains

[Green, Rudra]

$$\underbrace{\sqrt{g^{(het)}} e^{-\Phi_{het}/2} J_0 E_{3/2}(e^{-\Phi_{het}}) - \sqrt{g^{(het)}} \frac{\pi^2}{6} \mathcal{I}_2}$$

$$\rightarrow \sqrt{g^{(I)}} e^{-\Phi_I/2} J_0 E_{3/2}(e^{-\Phi_I})$$

$$\star \quad J_0 = t_8 t_8 R^4 - \frac{1}{8} \epsilon_{10} \epsilon_{10} R^4$$

$$\star \quad \mathcal{I}_2 = -\frac{1}{8} \epsilon_{10} \epsilon_{10} R^4 + \dots$$

$$\star \quad E_{3/2} = \zeta(3) e^{-3/2 \Phi_{het}} + \frac{\pi^2}{6} e^{\Phi_{het}/2} + non-pert.$$

**S-duality invariant**

- How is this compatible with heterotic / type I duality?

[Tseytlin; Green, Rudra]

$$g_{\mu\nu}^{(I)} = e^{-\Phi_{het}} g_{\mu\nu}^{(het)} \quad , \quad \Phi_I = -\Phi_{het}$$

- Possible answer:  $\mathcal{S}^{(het)}$  in 10D contains

[Green, Rudra]

$$\underbrace{\sqrt{g^{(het)}} e^{-\Phi_{het}/2} J_0 E_{3/2}(e^{-\Phi_{het}})}_{\rightarrow \sqrt{g^{(I)}} e^{-\Phi_I/2} J_0 E_{3/2}(e^{-\Phi_I})} - \underbrace{\sqrt{g^{(het)}} \frac{\pi^2}{6} \mathcal{I}_2}_{\rightarrow \sqrt{g^{(I)}} \frac{\pi^2}{6} e^{-\Phi_I} \mathcal{I}_2}$$

$$\rightarrow \sqrt{g^{(I)}} e^{-\Phi_I/2} J_0 E_{3/2}(e^{-\Phi_I})$$

$$\rightarrow \sqrt{g^{(I)}} \frac{\pi^2}{6} e^{-\Phi_I} \mathcal{I}_2$$

$$\star \quad J_0 = t_8 t_8 R^4 - \frac{1}{8} \epsilon_{10} \epsilon_{10} R^4$$

$$\star \quad \mathcal{I}_2 = -\frac{1}{8} \epsilon_{10} \epsilon_{10} R^4 + \dots$$

$$\star \quad E_{3/2} = \zeta(3) e^{-3/2 \Phi_{het}} + \frac{\pi^2}{6} e^{\Phi_{het}/2} + non-pert.$$

**S-duality invariant**



- How is this compatible with heterotic / type I duality?

[Tseytlin; Green, Rudra]

$$g_{\mu\nu}^{(I)} = e^{-\Phi_{het}} g_{\mu\nu}^{(het)} \quad , \quad \Phi_I = -\Phi_{het}$$

- Possible answer:  $\mathcal{S}^{(het)}$  in 10D contains

[Green, Rudra]

$$\underbrace{\sqrt{g^{(het)}} e^{-\Phi_{het}/2} J_0 E_{3/2}(e^{-\Phi_{het}})}_{\rightarrow \sqrt{g^{(I)}} e^{-\Phi_I/2} J_0 E_{3/2}(e^{-\Phi_I})} - \underbrace{\sqrt{g^{(het)}} \frac{\pi^2}{6} \mathcal{I}_2}_{\rightarrow \sqrt{g^{(I)}} \frac{\pi^2}{6} e^{-\Phi_I} \mathcal{I}_2}$$

$$\rightarrow \sqrt{g^{(I)}} e^{-\Phi_I/2} J_0 E_{3/2}(e^{-\Phi_I})$$

$$\rightarrow \sqrt{g^{(I)}} \frac{\pi^2}{6} e^{-\Phi_I} \mathcal{I}_2$$

$$\star J_0 = t_8 t_8 R^4 - \frac{1}{8} \epsilon_{10} \epsilon_{10} R^4$$

Disk level! 

$$\star \mathcal{I}_2 = -\frac{1}{8} \epsilon_{10} \epsilon_{10} R^4 + \dots$$

$$\star E_{3/2} = \zeta(3) e^{-3/2 \Phi_{het}} + \frac{\pi^2}{6} e^{\Phi_{het}/2} + non-pert.$$

**S-duality invariant**

- How is this compatible with heterotic / type I duality?

[Tseytlin; Green, Rudra]

$$g_{\mu\nu}^{(I)} = e^{-\Phi_{het}} g_{\mu\nu}^{(het)} \quad , \quad \Phi_I = -\Phi_{het}$$

- Possible answer:  $\mathcal{S}^{(het)}$  in 10D contains

[Green, Rudra]

$$\underbrace{\sqrt{g^{(het)}} e^{-\Phi_{het}/2} J_0 E_{3/2}(e^{-\Phi_{het}})}_{\rightarrow \sqrt{g^{(I)}} e^{-\Phi_I/2} J_0 E_{3/2}(e^{-\Phi_I})} - \underbrace{\sqrt{g^{(het)}} \frac{\pi^2}{6} \mathcal{I}_2}_{\rightarrow \sqrt{g^{(I)}} \frac{\pi^2}{6} e^{-\Phi_I} \mathcal{I}_2}$$

$$\rightarrow \sqrt{g^{(I)}} e^{-\Phi_I/2} J_0 E_{3/2}(e^{-\Phi_I})$$

$$\rightarrow \sqrt{g^{(I)}} \frac{\pi^2}{6} e^{-\Phi_I} \mathcal{I}_2$$

$$\star J_0 = t_8 t_8 R^4 - \frac{1}{8} \epsilon_{10} \epsilon_{10} R^4$$

$$\star \mathcal{I}_2 = -\frac{1}{8} \epsilon_{10} \epsilon_{10} R^4 + \dots$$

$$\star E_{3/2} = \zeta(3) e^{-3/2 \Phi_{het}} + \frac{\pi^2}{6} e^{\Phi_{het}/2} + non-pert.$$

Disk level!

Disk level correction  
to 4D EH-term?

S-duality invariant

- New results for  $\mathbb{Z}'_6$ : [M.H., Kang]

from  $\mathcal{N} = 1$ -sectors of  $\mathcal{A}, \mathcal{M}, \mathcal{K}$

$$\delta E = \frac{\chi}{(2\pi)^3} \left( 2\zeta(3)e^{-2\Phi} + \frac{\pi^2}{3} \right) + \frac{5}{64\pi^2} \text{Cl}_2 \left( \frac{\pi}{3} \right) - \frac{5}{256\pi^2} \left[ \frac{64\pi^2\alpha'}{\mathcal{V}_3} E_2(U_3) - \frac{12\pi^2\alpha'}{\mathcal{V}_2} E_2(U_2) - \frac{3\mathcal{V}_2}{4\pi^2\alpha'} E_2(-1/U_2) \right]$$

from  $\mathcal{N} = 2$ -sectors of  $\mathcal{A}, \mathcal{M}, \mathcal{K}$

- Follows closely a calculation by [Epple]
- Generalization to  $\mathbb{Z}_3$  [M.H., Kang] and  $\mathbb{Z}_6, \mathbb{Z}_7, \mathbb{Z}_{12}$  [with Tailin Li, unpublished]

# Results: Field redefinitions

- Examples:

- ★ Type I (on  $K_3 \times T^2$ ):  $\tau = \tau^{(0)} + a^2 + \frac{E_2(U)}{e^{-\Phi}\mathcal{V}}$

[Antoniadis, Bachas, Fabre,  
Partouche, Taylor]

open string scalars (cf. KKLMNT)

- ★ Type II on CY:  $e^{-2\tilde{\Phi}_4} = e^{-2\Phi_4} \left( 1 + a_1 \frac{\chi}{\mathcal{V}} + \dots \right)$

[Antoniadis, Minasian,  
Theisen, Vanhove]

$$\tilde{\mathcal{V}} = \mathcal{V} \left( 1 + a_2 \chi e^{2\Phi_4} + \dots \right)$$

- Kählerness of metric and shift symmetry in  $c$ , i.e.

$$G_{\tau_i \tau_j} = G_{\tau_i \tau_j}(\tau), \quad G_{c_i c_j} = G_{c_i c_j}(\tau), \quad K = K(\tau),$$

fix the field variables  $T = c + i\tau$  completely!

- $K_{T\bar{T}} \partial_\mu T \partial^\mu \bar{T} = \frac{1}{4} K_{\tau\tau} (\partial_\mu c \partial^\mu c + \partial_\mu \tau \partial^\mu \tau)$  implies:

$$\star \quad G_{c_i \tau_j} = 0$$

$$\star \quad G_{\tau_i \tau_j} = \frac{1}{4} \frac{\partial^2 K}{\partial \tau_i \partial \tau_j} = G_{c_i c_j}$$

$$\star \quad \partial_{\tau_k} G_{c_i c_j} = \partial_{\tau_i} G_{c_j c_k} = \partial_{\tau_j} G_{c_k c_i}$$

- $\delta c_i = 0$

- E.g. (for  $\mathbb{Z}'_6$ )

$\mathcal{T}, \mathcal{A}, \mathcal{M}, \mathcal{K}$

$\mathcal{N} = 1$

$$\begin{aligned}
 \delta\tau_3 = a_1 & \sqrt{\frac{T_3^{(0)} - \bar{T}_3^{(0)}}{(S^{(0)} - \bar{S}^{(0)})(T_1^{(0)} - \bar{T}_1^{(0)})(T_2^{(0)} - \bar{T}_2^{(0)})}} \\
 e^{-\Phi} \mathcal{V} & + a_2 \frac{(T_3^{(0)} - \bar{T}_3^{(0)})E_2(U_2)}{(S^{(0)} - \bar{S}^{(0)})(T_2^{(0)} - \bar{T}_2^{(0)})} \\
 & + a_3 \frac{E_2(-1/U_2)}{(T_1^{(0)} - \bar{T}_1^{(0)})} + a_4 \frac{E_2(U_3)}{(S^{(0)} - \bar{S}^{(0)})}
 \end{aligned}$$

$\mathcal{A}, \mathcal{M}, \mathcal{K}$   
 $\mathcal{N} = 2$

- E.g. (for  $\mathbb{Z}'_6$ )

$\mathcal{T}, \mathcal{A}, \mathcal{M}, \mathcal{K}$   
 $\mathcal{N} = 1$

$$\begin{aligned}
 \delta\tau_3 = a_1 & \sqrt{\frac{T_3^{(0)} - \bar{T}_3^{(0)}}{(S^{(0)} - \bar{S}^{(0)})(T_1^{(0)} - \bar{T}_1^{(0)})(T_2^{(0)} - \bar{T}_2^{(0)})}} \\
 e^{-\Phi} \mathcal{V} & + a_2 \frac{(T_3^{(0)} - \bar{T}_3^{(0)})E_2(U_2)}{(S^{(0)} - \bar{S}^{(0)})(T_2^{(0)} - \bar{T}_2^{(0)})} \\
 & + a_3 \frac{E_2(-1/U_2)}{(T_1^{(0)} - \bar{T}_1^{(0)})} + a_4 \frac{E_2(U_3)}{(S^{(0)} - \bar{S}^{(0)})}
 \end{aligned}$$

$\mathcal{A}, \mathcal{M}, \mathcal{K}$   
 $\mathcal{N} = 2$

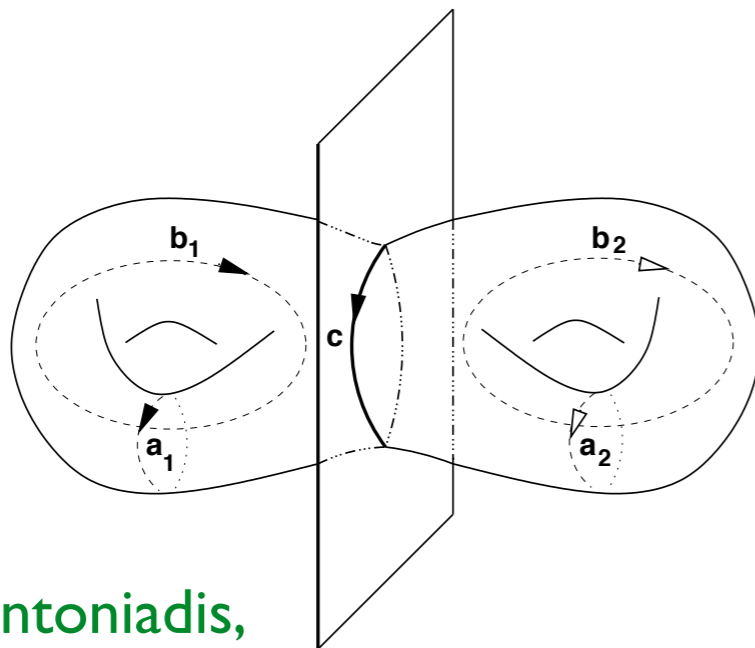
analog of correction by

[Antoniadis, Bachas, Fabre,  
 Partouche, Taylor]

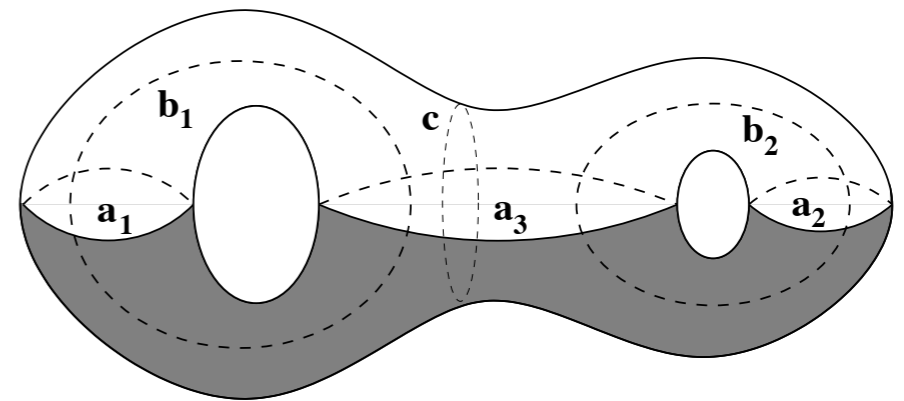
# Check of field redefinition of $T_3$ from holomorphic gauge kinetic function of D5-branes (wrapped around 3rd torus)

- At disk level:  $f_{D5} = T_3^{(0)} \sim e^{-\Phi}$
- 1-loop correction to  $T_3^{(0)}$  appears at  $\chi = 2 - 2h - b - c = -1$ :

E.g.



[from: Antoniadis, Taylor]



[from: Antoniadis, Narain, Taylor]

+ 3 more diagrams

$(b = 1, c = 2), (b = 2, c = 1), (h = 1, c = 1)$



Preliminary conjecture for Kähler potential of  
 untwisted closed moduli for  $T^6/\mathbb{Z}'_6$  (up to 1-loop):

$$\begin{aligned}
 K = & -\ln(S - \bar{S}) - \ln[(T_1 - \bar{T}_1)(T_2 - \bar{T}_2)(T_3 - \bar{T}_3)] - \ln(U_2 - \bar{U}_2) \\
 & + c_1 \chi \zeta(3) \sqrt{\frac{(T_1 - \bar{T}_1)(T_2 - \bar{T}_2)(T_3 - \bar{T}_3)}{(S - \bar{S})^3}} \\
 & + c_2 \frac{E_2(U_2)}{(T_2 - \bar{T}_2)(S - \bar{S})} + c_3 \frac{E_2(-1/U_2)}{(T_1 - \bar{T}_1)(T_3 - \bar{T}_3)} + c_4 \frac{E_2(U_3)}{(T_3 - \bar{T}_3)(S - \bar{S})} \\
 & + c_5 \frac{1}{\sqrt{(T_1 - \bar{T}_1)(T_2 - \bar{T}_2)(T_3 - \bar{T}_3)(S - \bar{S})}}
 \end{aligned}$$

Preliminary conjecture for Kähler potential of untwisted closed moduli for  $T^6/\mathbb{Z}'_6$  (up to 1-loop):

$$K = -\ln(S - \bar{S}) - \ln[(T_1 - \bar{T}_1)(T_2 - \bar{T}_2)(T_3 - \bar{T}_3)] - \ln(U_2 - \bar{U}_2)$$

$$+c_1 \chi \zeta(3) \sqrt{\frac{(T_1 - \bar{T}_1)(T_2 - \bar{T}_2)(T_3 - \bar{T}_3)}{(S - \bar{S})^3}} \quad \leftarrow S^2$$

$$+c_2 \frac{E_2(U_2)}{(T_2 - \bar{T}_2)(S - \bar{S})} + c_3 \frac{E_2(-1/U_2)}{(T_1 - \bar{T}_1)(T_3 - \bar{T}_3)} + c_4 \frac{E_2(U_3)}{(T_3 - \bar{T}_3)(S - \bar{S})}$$

$$+c_5 \frac{1}{\sqrt{(T_1 - \bar{T}_1)(T_2 - \bar{T}_2)(T_3 - \bar{T}_3)(S - \bar{S})}}$$

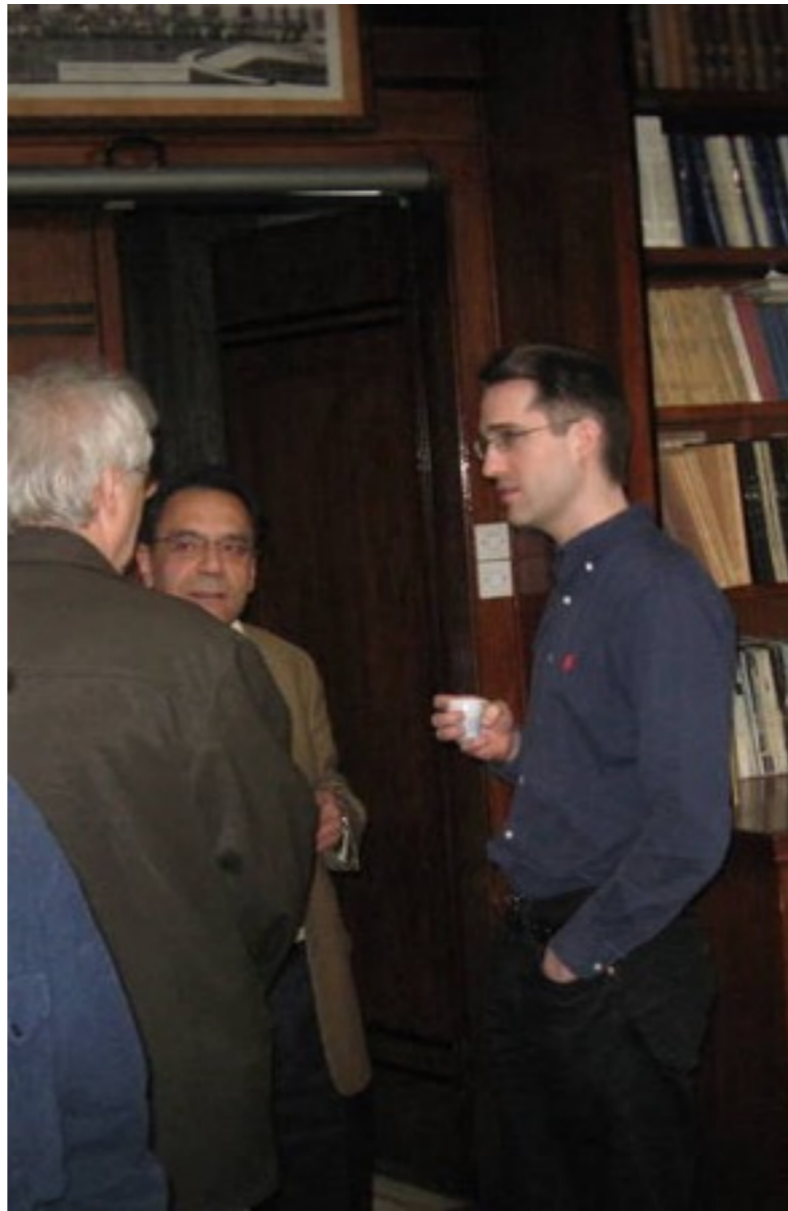
$\mathcal{A}, \mathcal{M}, \mathcal{K}$   
 $\mathcal{N} = 2$

$\mathcal{T}, \mathcal{A}, \mathcal{M}, \mathcal{K}$   
 $\mathcal{N} = 1$

# Outlook

- Work out coefficients in  $K$  and field redefinitions
- Check indirect prediction of field variables by direct string calculation (indirect arguments make many predictions for string calculations)
- Check field redefinition by  $\chi = -1$  amplitudes
- Applications to string model building?

# Thank You!



Spring School on „Strings, Cosmology  
and Particles“  
(Serbia 2009, Belgrade and Nis)