

# On Mirror Symmetry for Calabi-Yau Fourfolds with Three-Form Cohomology

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Workshop on Geometry and Physics



Max-Planck-Institut für Physik  
(Werner-Heisenberg-Institut)



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F-theory effective action on elliptically fibered  $CY_4$ :

$\mathcal{N} = 1$  supergravity in  $(3 + 1)$  dimensions

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For gauge-theory on **seven-branes**  $\Rightarrow$  not relevant

For **bulk** gauge-theory:  $\Rightarrow$  Emerge from **non-trivial three-forms** of  $CY_4$

Goal: Study three-forms of  $CY_4 \Rightarrow$  Consider **toy-example: IIA on  $CY_4$ !**

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- ⑤ Summary and Outlook

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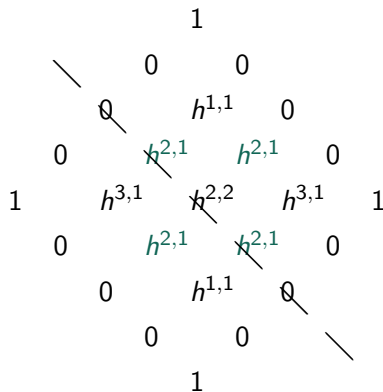
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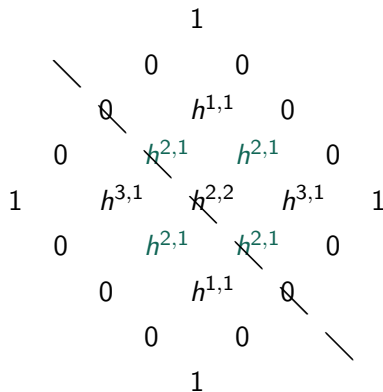
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$h^{1,1}$  complexified Kähler moduli:

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Dilaton expansion:

$$e^{-2\phi_{IIA}^{(10)}(x)} = \frac{1}{\mathcal{V}(x)} e^{-2\phi_{IIA}^{(2)}(x)}, \quad \mathcal{V} = \frac{1}{4!} \int_{Y_4} J^4$$

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Goal of our work:  
calculate  $f$ !

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with kinetic potential  $K = \log \mathcal{V} - \log \int_{Y_4} \Omega \wedge \bar{\Omega} + e^{2\phi_{IIA}^{(2)}} \mathcal{S}$

$$\mathcal{S} = \text{Re}(N)_I \left( \int_{Y_4} \Psi^I \wedge *\bar{\Psi}^m \right) \text{Re}(N)_m \quad \Rightarrow \quad \text{Im}(N)_I \text{ axionic!}$$

$\Rightarrow$  Interactions between chiral and twisted-chiral multiplets!

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- ⇒ Representation change via Legendre transformation of kinetic potential

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New: Three-form moduli are self-mirror, but change representations!

- ⇒ Representation change via Legendre transformation of kinetic potential
- ⇒ Explicit dependence of three-form metric

$$H^{lk} = \int_{Y_4} \Psi^l \wedge * \bar{\Psi}^k = -\frac{1}{2} \text{Re}(f)^{lm} C_{Am}^k v^A = (H_{\text{mirror}}^{-1})_{lk}$$

on Kähler moduli  $v^A$  in both pictures known

- ⇒ Gauge coupling  $f(z)$  linear in complex structure moduli  $z$ !

# Summary

Introducing the ansatz

$$\Psi^I = \frac{1}{2} \operatorname{Re}(f)^{lm} (\alpha_m - i \bar{f}_{mk} \beta^k) \in H^{1,2}(Y_4)$$

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Using type IIA supergravity and  $\mathcal{N} = (2, 2)$  supersymmetry, we obtain a very simple mirror map. This enables us to calculate  $f_{lm}(z)$  at large complex structure to be

$$f_{lm}(z) = z^K \hat{C}_{KI}^m$$

$$\hat{C}_{KI}^m = \int_{\hat{Y}_4} \hat{\omega}_K \wedge \hat{\alpha}_I \wedge \hat{\beta}^m$$

(Hatted objects are defined on the mirror fourfold  $\hat{Y}_4$ )



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 $\Rightarrow$  three-forms induce  $U(1)^{h^{2,1}}$ -gauge theory in the effective supergravity action

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and we calculated  $f_{\Lambda\Sigma}(z)$  and  $K^F$  at large complex structure!

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- Continued in [\[Corvilain,Grimm,Regalado '16\]](#).

Thank you for your attention!

Questions?

# Outlook - Future Work

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