On Mirror Symmetry for Calabi-Yau Fourfolds with Three-Form Cohomology

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Workshop on Geometry and Physics

Motivation

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F-theory effective action on elliptically fibered CY_4:

$\mathcal{N} = 1$ supergravity in $(3+1)$ dimensions

$$S^{(4)} = \int_{\mathcal{M}_{3,1}} \frac{1}{2} R^{(4)} * 1 - K^F_{I\bar{J}} \mathcal{D} M^I \wedge * \mathcal{D} \bar{M}^J$$
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For bulk gauge-theory: $\Rightarrow$ Emerge from non-trivial three-forms of CY_4

Goal: Study three-forms of CY_4 $\Rightarrow$ Consider toy-example: IIA on CY_4!

Outline

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5. Summary and Outlook

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with $R^{(10)}$ Ricci-scalar, $H_3 = dB_2$

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with $F_2 = dC_1$, $\tilde{F}_4 = F_4 - C_1 \wedge H_3$

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$h^{1,1}$ complexified Kähler moduli:

$$t^A = b^A + iv^A$$

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Dilaton expansion: $e^{-2\phi_{IIA}^{(10)}(x)} = \frac{1}{\mathcal{V}(x)} e^{-2\phi_{IIA}^{(2)}(x)}, \quad \mathcal{V} = \frac{1}{4!} \int_{Y_4} J^4$

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Suggested by [Grimm '10]:
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Goal of our work:
calculate f!

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with kinetic potential $K = \log \mathcal{V} - \log \int_{Y_4} \Omega \wedge \bar{\Omega} + e^{2\phi_{IIA}^{(2)}} \mathcal{S}$

$$\mathcal{S} = Re(N)_l \Big(\int_{Y_4} \Psi^l \wedge * \bar{\Psi}^m \Big) Re(N)_m \quad \Rightarrow \quad Im(N)_l \text{ axionic!}$$

$\Rightarrow$ Interactions between chiral and twisted-chiral multiplets!

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⇒ Explicit dependence of three-form metric

$$H^{lk} = \int_{Y_4} \Psi^l \wedge * \overline{\Psi}^k = -\frac{1}{2} Re(f)^{lm} C^k_{Am} v^A = (H^{-1}_{\text{mirror}})_{lk}$$

on Kähler moduli v^A in both pictures known

⇒ Gauge coupling $f(z)$ linear in complex structure moduli z!

Summary

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we can include the massless-modes arising from non-trivial massless three-form modes into dimensional reductions on Calabi-Yau fourfolds.

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we can include the massless-modes arising from non-trivial massless three-form modes into dimensional reductions on Calabi-Yau fourfolds.

Using type IIA supergravity and $\mathcal{N} = (2,2)$ supersymmetry, we obtain a very simple mirror map. This enables us to calculate $f_{lm}(z)$ at large complex structure to be

$$f_{lm}(z) = z^K \hat{C}^m_{Kl} \qquad \hat{C}^m_{Kl} = \int_{\hat{Y}_4} \hat{\omega}_K \wedge \hat{\alpha}_l \wedge \hat{\beta}^m$$

(Hatted objects are defined on the mirror fourfold $\hat{Y}_4$)

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and we calculated $f_{\Lambda\Sigma}(z)$ and K^F at large complex structure!

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- Continued in [Corvilain,Grimm,Regalado '16].

Thank you for your attention! Questions?

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- Include three-form moduli in topological string/CFT framework