Arnold Sommerfeld Center for Theoretical Physics

Interfaces and their entropy

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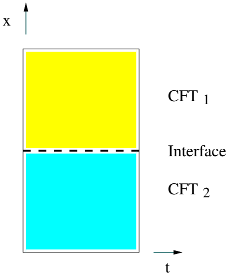
Ringberg 2016, To the memory of Ioannis Bakas

Overview

- Introduction to interfaces in 2d CFT
- $N = 2$ SUSY
- Interface entropy = Calabi-diastasis
- Further remarks

Based on work with Costas Bachas, Michael Dougals and Leonardo Rastelli (and ongoing work with Peter Mayr and Martin Vogrin)

Interfaces for conformal field theories

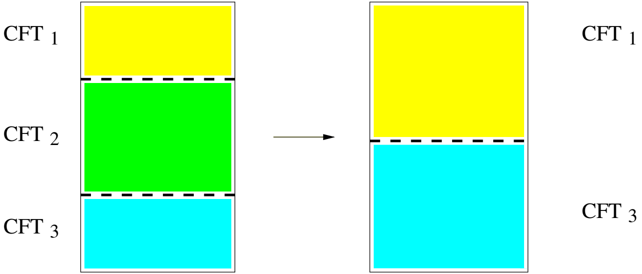


- Two conformal field theories are joined along a common interface, which is required to preserve conformal invariance

  Bachas-de Boer-Dijkgraaf-Ooguri, Petkova-Zuber

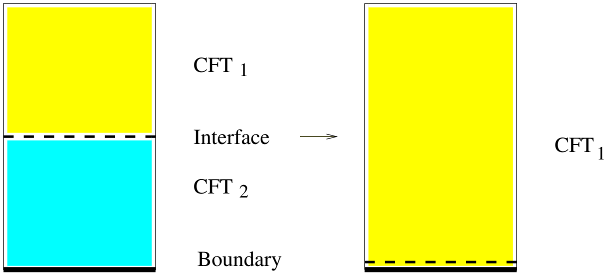
- The interface can carry additional degrees of freedom which are not inherited from the bulk.

Fusion of interfaces



- Two interfaces merge to form a new interface. In general:singular process.
- In the limit, obtain a new interface between theory CFT_1 and CFT_3.

Action on boundary conditions



- Special case: Interfaces can fuse with boundaries to form new boundaries.
- String theory: Interfaces act naturally on D-branes.

Conformal interfaces

- We want to preserve conformal invariance.
- We require that $T - \bar{T}$ is continuous across the interface.

$$T^{(1)} - \bar{T}^{(1)} = T^{(2)} - \bar{T}^{(2)}$$

- A special class are the totally reflecting interfaces, which are boundaries for the two theories. In this case the rhs and lhs of the above equation vanish separately at the boundary.
- Another special case are topological interfaces, where

$$T^{(1)} = T^{(2)}, \;\; \bar{T}^{(1)} = \bar{T}^{(2)} \text{ on the interface } .$$

- The interface can hence be deformed or moved across the world sheet in arbitrary ways – as long as it does not hit a field insertion.

Defects in $N = (2,2)$ theories, I

- Consider a theory with 4 anti-commuting supercharges with the usual anti-commutation relations

$$\{Q_\pm, \bar{Q}_\pm\} = H \pm P \ .$$

- We are interested in defects that preserve at least half of the supersymmetry. Just as for boundary conditions or orientifolds, there are two ways to do so.
- B-type defect: We demand that the combination $Q_B = Q_+ + Q_-$ is preserved everywhere. This means that along the interface the supercharges have to fulfill the “gluing conditions”

$$\begin{aligned} Q_+^{(1)} + Q_-^{(1)} &= Q_+^{(2)} + Q_-^{(2)}, \\ \bar{Q}_+^{(1)} + \bar{Q}_-^{(1)} &= \bar{Q}_+^{(2)} + \bar{Q}_-^{(2)} \ . \end{aligned}$$

where the superscripts refer to the two theories separated by the defect.

Defects in $N = (2,2)$ theories, II

- A-type defect: Same for $Q_A = Q_+ + \bar{Q}_-$.
- A-type and B-type defects are interchanged by mirror symmetry
- In situations with both defects and boundaries, B (A) defects preserve the same SUSY as B (A) D-branes
- There are special defects that actually preserve the full supersymmetry.

$$Q_\pm^{(1)} = Q_\pm^{(2)}, \quad \bar{Q}_\pm^{(1)} = \bar{Q}_\pm^{(2)} \quad \text{on } \mathbf{R} \, ,$$

The SUSY algebra implies that those defects also fulfill

$$H^{(1)} = H^{(2)}, \quad P^{(1)} = P^{(2)} \quad \text{on } \mathbf{R} \, .$$

They preserve translational invariance in space and time.

- We can define a second class of defects preserving the full supersymmetry. It can be obtained by twisting with the mirror automorphism.

$$\begin{aligned} Q_+^{(1)} &= Q_+^{(2)}, \quad \bar{Q}_+^{(1)} = \bar{Q}_+^{(2)} \\ Q_-^{(1)} &= \bar{Q}_-^{(2)}, \quad \bar{Q}_-^{(1)} = Q_-^{(2)} \end{aligned}$$

- In particular, mirror symmetry itself can be interpreted as a defect.

The folding trick

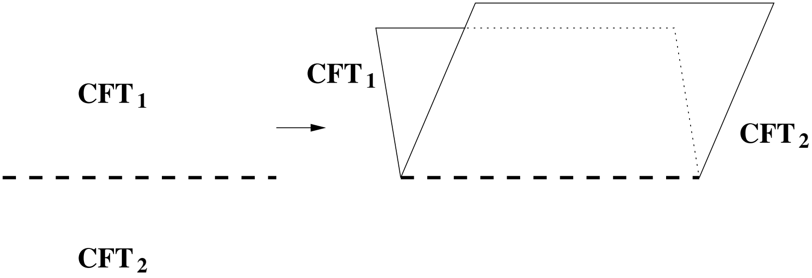


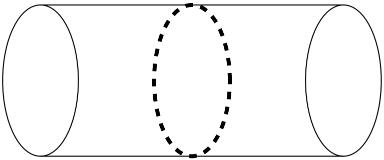
Figure: Folding trick.

- Instead of a theory on the full plane with an interface along the real line and theories CFT_1 and CFT_2 on the upper and lower half plane, one can consider the theory $\mathrm{CFT}_1 \otimes \overline{\mathrm{CFT}}_2$ on the upper half plane. Here, $\overline{\mathrm{CFT}}_2$ is obtained from CFT_2 by exchanging left and rightmovers.

Defect entropy

- Quite generally, one defines entropies

$$S = -\beta^2 \frac{\partial F}{\partial \beta}, \quad F = -\beta^{-1} \ln Z$$



- For defects, one can define an entropy by inserting a defect in the partition function. Asymptotically, this entropy becomes

$$S = \ln g, \quad g = \langle 1 \rangle = \langle 0 | D | 0 \rangle$$

- Defect operator $D = g|0\rangle\langle 0| + \ldots$
- This generalizes the boundary entropy

$$S = \ln g, g = \langle 1 \rangle_B,$$

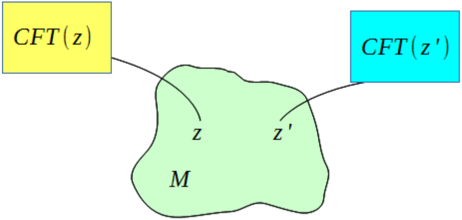
Possible interpretations/applications of g

- We can regard a boundary as a special case of a defect. In a string theory context, the g-factor corresponds to the tension of a D-brane.
- One can try to use g to define a "distance" between the two theories separated by the interface. Proposal

$$d(1,2) = \min \sqrt{\log g}$$

- The minimum is to be taken over some suitable set of interfaces between the two theories
- Appealing features
  - If the defect merely implements a symmetry, the distance is 0
  - For moduli spaces of theories, the g-distance becomes the Zamolodchikov distance for small distances.

Deformation interfaces



a) moduli space M of CFT's

b) deformation interface

- Example: Free boson compactified on a circle of radius R. The interface mediates between CFT(R_1) and CFT(R_2).
- Example: $N = (2,2)$ superconformal theories

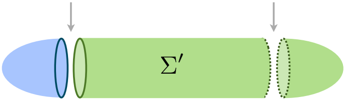
Supersymmetry preserving perturbations in N=(2,2)

- Two types of perturbations that preserve SUSY in the bulk
- (c,c) perturbations $\Delta S = \int d^2x d\theta^+ d\theta^- \Phi|_{\bar{\theta}^\pm=0}$
  Geometrically: Complex structure deformations
- (a,c) perturbations $\Delta S = \int d^2x d\theta^+ d\bar{\theta}^- \Psi|_{\bar{\theta}^+=\theta^-=0}$
  Geometrically: Kähler deformations
- In theories with boundaries, supersymmetry can be preserved if the perturbation is (c,c) [(a,c)] and the boundary is A-type [B-type]. Hori-Iqbal-Vafa
- For deformation interfaces: (c,c) [(a,c)] perturbations are described by A [B] type defects, and the behavior of D-branes under such perturbations can be described by fusion.
- Monodromy interfaces are special perturbation interfaces.

Interface entropy and Calabi diastasis

- We can now compute the g-function for deformation interfaces of $N = (2, 2)$ theories.
- Result: $2 \log g = K(t, \bar{t}) + K(t', \bar{t}') - K(t, \bar{t}') - K(t', \bar{t})$
- t, t' are the moduli on the two sides of the interface
- K is the Kähler potential
- $2 \log g$ is the Calabi diastasis function known from Kähler geometry
- Interfaces provide a "quantum" formulation of this object, relying only on supersymmetry (not a geometric interpretation).
- CFT formula for the g-factor for a defect Δ_f

$$g^2 = \frac{{}_{\mathrm{RR}}\langle\bar{0}|\,\Delta_{f,A}\,|0'\rangle_{\mathrm{RR}} \times {}_{\mathrm{RR}}\langle\bar{0}'|\,\Delta^{\dagger}_{f,A}\,|0\rangle_{\mathrm{RR}}}{{}_{\mathrm{RR}}\langle\bar{0}|0\rangle_{\mathrm{RR}} \times {}_{\mathrm{RR}}\langle\bar{0}'|0'\rangle_{\mathrm{RR}}} \quad ,$$



$$g^2 = \frac{{}_{\rm RR}\langle\bar{0}|\,\Delta_{f,A}\,|0'\rangle_{\rm RR} \times {}_{\rm RR}\langle\bar{0}'|\,\Delta^\dagger_{f,A}\,|0\rangle_{\rm RR}}{{}_{\rm RR}\langle\bar{0}|0\rangle_{\rm RR} \times {}_{\rm RR}\langle\bar{0}'|0'\rangle_{\rm RR}} \quad ,$$

- $\log\left({}_{\rm RR}\langle\bar{0}|0\rangle_{\rm RR}\right) = -K(t,\bar{t})$.
- The amplitude

$${}_{\rm RR}\langle\bar{0}|\Delta_{f,A}|0'\rangle_{\rm RR} \equiv e^{-K(t',\bar{t})}$$

defines the analytic extension of the quantum Kähler potentials to independent holomorphic and antiholomorphic moduli.

Remark: Monodromy interfaces and SUSY gauge theories

- Deformation interfaces exist also for theories in higher dimensions
- Behavior of g-function as one approaches singularities in CY target space
- Expect to find a g-function of the field theory if the gravity part decouples from the field theory part
- This can be made concrete for monodromy interfaces. They have a matrix representation M acting on the period vectors.
- $2 \log g_{FT} = -i\Pi(\Sigma + M^T \Sigma M - M^T \Sigma - \Sigma M)\Pi$
- Results for monodromies around superconformal points: $2 \log g_{FT} = nK$, n an integer computed from the monodromy matrix.

Conclusions

- Conformal interfaces provide a rich set of non-local observables in CFTs
- They introduce new structure in CFTs; defect fusion provides a multiplication law
- There are applications in many different fields
  - statistical mechanics
  - Kähler geometry