# CS–WZW CORRESPONDENCE IN OSFT


Martin Schnabl

Institute of Physics, Prague

Academy of Sciences of the Czech Republic



Fundaments of Gravity, April 12, 2010

# Open String Field Theory as Fundaments of Gravity ?

- Closed string field theory (Zwiebach 1990) — presumably THE fundamental theory of gravity — is given by a technically ingenious construction, but it is hard to work with, See i.e. N. Moeller, and also it may be viewed as too perturbative and `effective'.
- Open string field theory (Witten 1986, Berkovits 1995) might be more fundamental Sen, see also I. Sachs and M. Baumgartl. Related to holography (Maldacena) and old 1960's ideas of Sacharov.

# First look at OSFT

Open string field theory uses the following data

$$\mathcal{H}_{BCFT}, \quad *, \quad Q_B, \quad \langle . \rangle .$$

Let all the string degrees of freedom be assembled in

$$|\Psi\rangle = \sum_i \phi_i(X)|i\rangle .$$

Witten (1986) proposed the following action

$$S = -\frac{1}{g_o^2}\left[\frac{1}{2}\langle\,\Psi * Q_B\Psi\,\rangle + \frac{1}{3}\langle\,\Psi * \Psi * \Psi\,\rangle\right],$$

# First look at OSFT

This action has a huge gauge symmetry

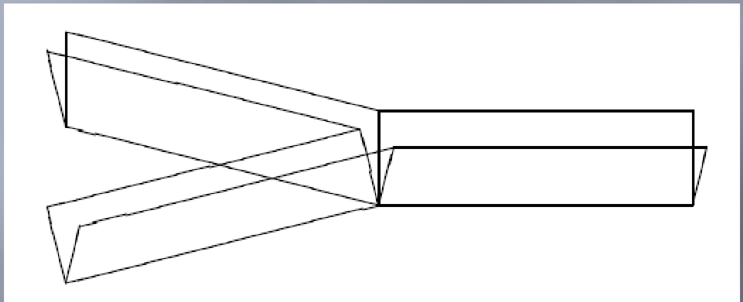
$$\delta\Psi = Q_B\Lambda + \Psi * \Lambda - \Lambda * \Psi,$$

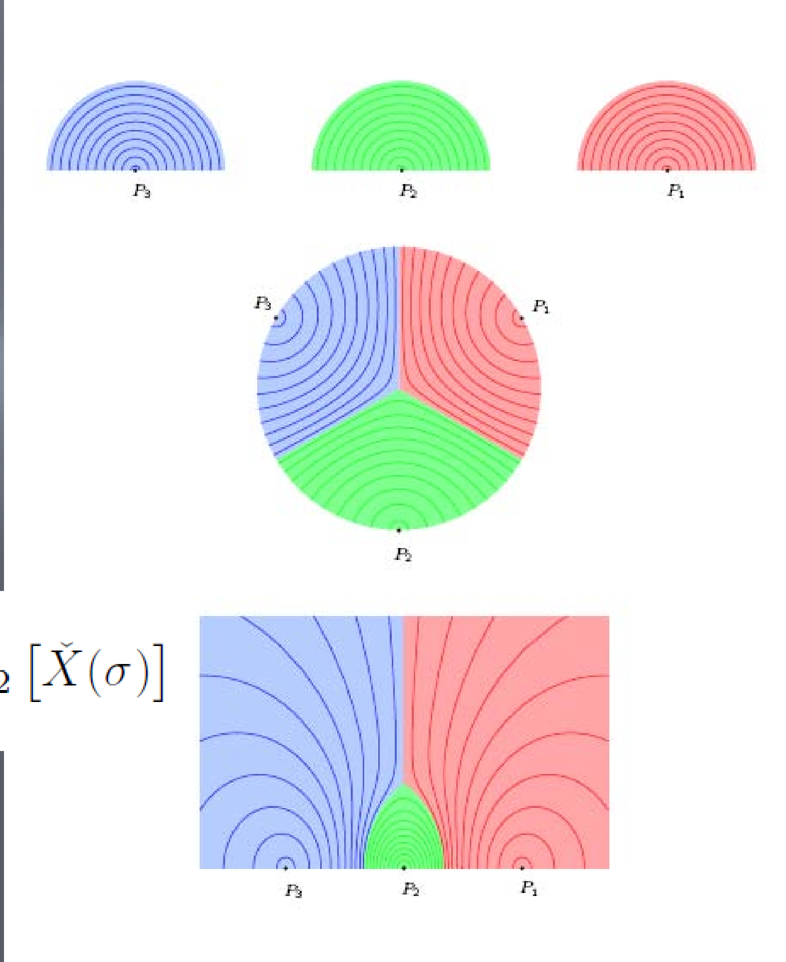
provide that the start product is associative, $Q_B$ acts as graded derivation and < . > has properties of integration.

Note that there is a gauge symmetry for gauge symmetry so one expects infinite tower of ghosts – indeed they can be naturally incorporated by lifting the ghost number restriction on the string field.

# Witten's star product

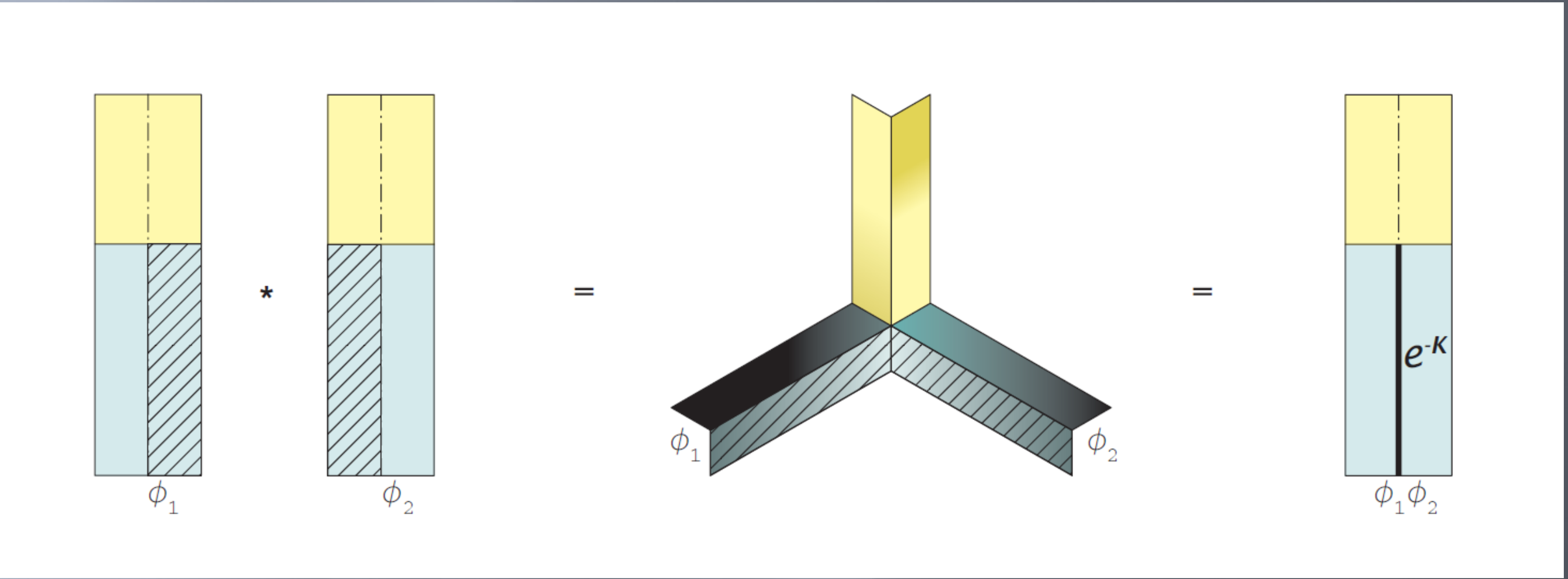
Defined by gluing three strings:






$$(\Psi_1 \star \Psi_2)\,[X(\sigma)] = \int [\mathcal{D}X_{\text{overlap}}]\,\Psi_1[\hat{X}(\sigma)]\Psi_2\,[\check{X}(\sigma)]$$

It used to be a very complicated definition…




- The elements of string field star algebra are states in the BCFT, they can be identified with a piece of a worldsheet.
- By performing the path integral on the glued surface in two steps, one sees that in fact:

$$|\phi_1\rangle * |\phi_2\rangle = |\phi_1 e^{-K} \phi_2\rangle.$$

# Witten's star product as operator multiplication

We have just seen that the star product obeys

$$|\phi_1\rangle * |\phi_2\rangle = |\phi_1 e^{-K} \phi_2\rangle.$$

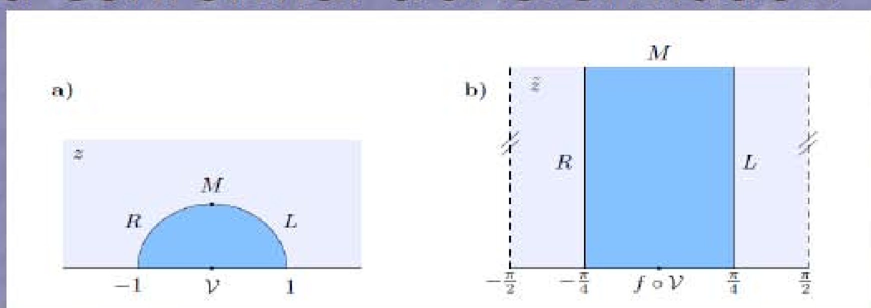
And therefore states $\hat{\phi} = e^{K/2} \phi e^{K/2}$ obey

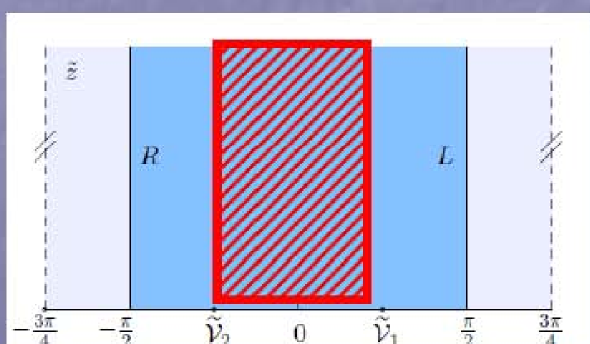
$$|\hat{\phi}_1\rangle * |\hat{\phi}_2\rangle = |\widehat{\phi_1 \phi_2}\rangle$$

The star product and operator multiplication are thus isomorphic!

# In case you wonder what $e^{-K}$ is

To find out what $e^{-K}$ stands for, one can perform a conformal transformation




$$e^{-K} = e^{-\frac{\pi}{2}\int_{-M}^{M} T_{\tilde{z}\tilde{z}}d\tilde{z}}$$

Just like

$$e^{-tL_0} = e^{-t\oint T_{ww}dw}$$

# Simple subsector of the star algebra

- The star algebra is formed by vertex operators and the operator *K*. The simplest subalgebra relevant for tachyon condensation is therefore spanned by *K* and *c*. Let us be more generous and add an operator *B* such that *QB*=*K*.
- The building elements thus obey

$$c^2 = 0, \quad B^2 = 0, \quad \{c, B\} = 1$$
$$[K, B] = 0, \quad [K, c] = \partial c$$

- The derivative *Q* acts as $Q_B K = 0, \quad Q_B B = K, \quad Q_B c = cKc.$

# Classical solutions

This new understanding lets us construct solutions to OSFT equations of motion $Q_B\Psi + \Psi * \Psi = 0$ easily.

It does not take much trying to find the simplest solution is $\Psi = \alpha c - cK$

$$Q\Psi = \alpha(cKc) - (cKc)K$$

$$\Psi * \Psi = \alpha^2 c^2 - \alpha c^2 K - \alpha cKc + (cK)(cK)$$

More general solutions are of the form

$$\Psi = Fc\frac{KB}{1-F^2}cF,$$

Here *F*=*F(K)* is arbitrary

Schnabl 2005, Okawa, Erler 2006

# Classes of solutions

- The space of all such solutions has not been completely classified although we are close quite close (Erler).
- Let us restrict our attention to different choices of *F(K)* only.
- Let us call a state *geometric* if *F(K)* is of the form

$$F(K) = \int_0^\infty d\alpha\, f(\alpha) e^{-\alpha K}$$

where *f(α)* is a tempered distribution

# Classes of solutions

- Therefore *F(K)* must be holomorphic for *Re(K)>0* and bounded by a polynomial there.
- Since formally $\Psi = (1 - FBcF)Q(1 - FBcF)^{-1},$

and $(1 - FBcF)^{-1} = 1 + \frac{F}{1 - F^2} BcF$

the state is trivial if $F/(1 - F^2)$ is well defined

# Classes of solutions

- There is another useful criterion. One can look at the cohomology of the theory around a given solution. It is given by an operator

$$Q_\Psi = Q_B + \{\Psi, \cdot\}_*.$$

- The cohomology is formally trivialized by an operator $A = \frac{1 - F^2}{K} B,$ which obeys $\{Q_\Psi, A\} = 1.$

# Classes of solutions

- Therefore in this in this class of solutions, the trivial ones are those for which $F^2(0) \neq 1$.
- Tachyon vacuum solutions are those for which $F^2(0) = 1$ but the zero of $1\text{-}F^2$ is first order
- When the order of zero of $1\text{-}F^2$ at $K=0$ is of higher order the solution is not quite well defined, but it has been conjectured (Ellwood, M.S.) to correspond to multibrane solutions.

# Examples

$$F(K) = a \quad (const.)$$

… trivial solution

$$F(K) = \sqrt{1 - \beta K}$$

…. ‘tachyon vacuum’ only *c* and *K* turned on

$$F(K) = e^{-K}$$

…. M.S. ‘05

$$F(K) = \frac{1}{\sqrt{1+K}}$$

… Erler, M.S. ’09 – the simplest solution so far

# Towards the super OSFT

- There has been much progress on many fronts in our field, we are finding new solutions (new classical backgrounds), study cohomology, couplings to closed strings etc.
- One of the outstanding problems is to find a generalization to open superstrings. One of the most popular actions is due to Berkovits:

$$S = \frac{1}{2g^2}\Big\langle \left(e^{-\Phi} Q e^{\Phi}\right)\left(e^{-\Phi}\eta_0 e^{\Phi}\right) - \int_0^1 dt\,\left(e^{-t\Phi}\partial_t e^{t\Phi}\right)\left(e^{-t\Phi} Q e^{t\Phi}\right)\left(e^{-t\Phi}\eta_0 e^{t\Phi}\right)\Big\rangle\,,$$

since it is non-polynomial, it has been very hard to work with.

# CS-WZW correspondence

- In our hopefully-soon-to-appear work with P.A. Grassi we borrowed some classical results from the CS-WZW correspondence and rewrote the action as

$$S = -\frac{1}{g^2}\left[\frac{1}{2}\langle\langle\, \mathbb{A} * \mathbb{Q}\mathbb{A}\,\rangle\rangle + \frac{1}{3}\langle\langle\, \mathbb{A} * \mathbb{A} * \mathbb{A}\,\rangle\rangle\right].$$

- In principle, in this form the action should be amenable to straightforward quantization (work under progress).

# CS-WZW correspondence

- Let us consider the following 2+1 dimensional theory with gauge group G defined on a manifold $\Sigma\times R$

$$S_{CS} = -\frac{1}{2g^2}\int_{\Sigma\times\mathbb{R}} \mathrm{Tr}(AdA + \frac{2}{3}A^3),$$

- Let $A_0=0$ on the boundary $\partial\Sigma$. Let us decompose $A=A_0+\tilde{A}$. Then

$$S = -\frac{k}{4\pi}\int_{Y}\mathrm{Tr}\left(\tilde{A}\frac{\partial}{\partial t}\tilde{A}\,\mathrm{d}t\right) + \frac{k}{2\pi}\int_{Y}\mathrm{Tr}\left[A_0(\tilde{\mathrm{d}}\tilde{A}+\tilde{A}^2)\right]$$

# CS–WZW correspondence

- After integrating out $A_0$ we get $\delta(F')$, where $F'=d'\tilde{A}+\tilde{A}\tilde{A}$. This constraint can be easily solved by $\tilde{A}=U d' U^{-1}$
- Let us use U as the integration variable instead of $\tilde{A}$

$$\int \mathrm{D}\tilde{A}\,\delta(\tilde{F}) = \int \mathrm{D}U$$

- The Jacobian of this transformation is one

$$S = kS_C^+(U) \equiv \frac{k}{4\pi}\int_{\partial Y} \mathrm{Tr}\left(U^{-1}\partial_\phi U U^{-1}\partial_t U\right) \mathrm{d}\phi\, \mathrm{d}t + \frac{k}{12\pi}\int_Y \mathrm{Tr}\left(U^{-1}\mathrm{d}U\right)^3$$

Elitzur, Moore, Schwimmer, Seiberg 1989

# CS–WZW correspondence for the super OSFT

- Let us tensor the large Hilbert space of super OSFT with space of differential forms on the unit interval parameterized by $t \in [0, 1]$,
- This makes all elements of the Hilbert space *t*-dependent, and also doubles it
- Let us assign ghost number one to *dt*, and let $\mathbb{Q} = Q + \eta_0 + d_t,$ where $d_t = dt\frac{\partial}{\partial t}.$

  Note that this operator has the correct cohomology as $H(Q + \eta_0 + d_t) \simeq H(Q + \eta_0, H(d_t))\,.$

# CS–WZW correspondence for the super OSFT

- Let us further define $\langle\langle \ldots \rangle\rangle = \int_0^1 \langle \ldots \rangle$ so that

$$\langle\langle\, \mathbb{A}_1 * \mathbb{A}_2 * \ldots * \mathbb{A}_n \,\rangle\rangle = \langle\langle\, \mathbb{A}_2 * \ldots * \mathbb{A}_n * \mathbb{A}_1 \,\rangle\rangle\,.$$

and

$$\langle\langle\, Q\mathbb{X}(t)\,\rangle\rangle = 0\,, \qquad \langle\langle\, \eta_0\mathbb{X}(t)\,\rangle\rangle = 0\,, \qquad \langle\langle\, d_t\mathbb{X}(t)\,\rangle\rangle = \langle\, \mathbb{X}(1) - \mathbb{X}(0)\,\rangle$$

# CS–WZW correspondence for the super OSFT

- The proposed action takes the form

$$S = -\frac{1}{g^2}\left[\frac{1}{2}\langle\!\langle\, \mathbb{A} * \mathbb{Q}\mathbb{A} \,\rangle\!\rangle + \frac{1}{3}\langle\!\langle\, \mathbb{A} * \mathbb{A} * \mathbb{A} \,\rangle\!\rangle\right].$$

where at the classical level the string field can be restricted to

$$\mathbb{A} = A^{(1,0,0)} + B^{(1,-1,0)} + C^{(0,0,1)},$$

# What next ?

- Complete the construction at the quantum level using the BV formalism, it is very nontrivial to solve the master equation (work in progress with Berkovits, Okawa, Torii, Zwiebach and Grassi)
- Find the tachyon vacuum solutions on non-BPS branes.
- Study brane-antibrane tachyon driven dynamics (and apply to early universe models)
- When you are done, go back and look for closed strings.