Quantum Gravity with Anisotropic Scaling

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Workshop on Fundaments of Gravity

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based on:

arXiv:0811.2217, arXiv:0812.4287, arXiv:0901.3775,
arXiv:0902.3657,
arXiv:0909.3841 (w/ Charles Melby-Thompson),
arXiv:1003.0009 (w/ Cenke Xu)

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and work in progress,
with Charles Melby-Thompson, Kevin Grosvenor and Patrick Zulkowski.

Wider Framework: Gravity in 21st Century

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... leading to string theory: an über-geometric theory, exhibiting a web of gauge principles, dualities, etc., with gravity only a part of a bigger picture.

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(iv) Mathematical applications (cf. Poincaré conjecture and Ricci flows);

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(String theory has been a tremendous success, but for gravity it might be "too large" and "too small".)

Central idea of Lifshitz gravity

Combine gravity with the concept of anisotropic scaling.

In a spacetime with coordinates $(t, \mathbf{x}) \equiv (t, x^i)$, $i = 1, \ldots D$, consider

$$\mathbf{x} \to b\mathbf{x},$$
$$t \to b^z t.$$

Here z is the dynamical critical exponent.

In condensed matter (and now even in string theory!), many values of z are possible; integers (1, 2, . . .), fractions, . . .

Example: Lifts of static critical systems (Euclidean QFTs) to dynamical critical phenomena.

Goal: Construct similar models with propagating gravitons.

Comparison to Asymptotic Safety

Search for a UV fixed point in gravity:

Asymptotic safety: looking for relativistic, nontrivial fixed points. [Weinberg,. . .]

Lifshitz gravity: looking for nonrelativistic, often Gaussian fixed points.

Such fixed points can be UV (leading to improved short-distance behavior of gravity), or IR (emergent in condensed matter system).

Price paid for improved UV behavior: Anisotropy between space and time (or even spatial anisotropy) at short distances.

Flow between UV and IR: from $z > 1$ to $z = 1$.

Why is this interesting?

(i) Gravity duals of field theories in AdS/CFT; in particular, candidates for duals of nonrelativistic field theories;

(ii) Gravity on worldvolumes of branes;

(iii) Mathematical applications (theory of the Ricci flow);

(iv) Emergent Gaussian IR fixed points in lattice systems of condensed matter;

(v) Phenomenology of gravity in our Universe, $3+1$ dimensions. How close can this resemble GR in IR?

(vi) Conventional gravity, in spacetimes which are asymptotically anisotropic!

Update on the status of Lifshitz gravity

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Example: Lifshitz scalar field theory

Many interesting features can be illustrated by:

$$S = \frac{1}{2} \int dt\, d^D\mathbf{x} \left\{ \dot{\phi}^2 - (\Delta\phi)^2 \right\}$$

A theory closely related to the better-known

$$W = \frac{1}{2} \int d^D\mathbf{x}\, \partial_i\phi\partial_i\phi$$

The critical dimension has shifted:

$$[\phi] = \frac{D-2}{2};$$

ϕ is dimensionless in $2+1$ dimensions.

[Lifshitz,1941]

Gravity at a Lifshitz point

Minimal starting point: fields $g_{ij}(t,\mathbf{x})$ (the spatial metric), action $S = S_K - S_V$, with the kinetic term

$$S_K = \frac{1}{\kappa^2}\int dt\, d^D\mathbf{x}\sqrt{g}\,\dot{g}_{ij}G^{ijk\ell}\dot{g}_{k\ell}$$

where $G^{ijk\ell} = g^{ik}g^{j\ell} - \lambda g^{ij}g^{k\ell}$ is the De Witt metric, and the "potential term"

$$S_V = \frac{1}{4\kappa^2}\int dt\, d^D\mathbf{x}\sqrt{g}\,V(R_{ijk\ell})$$

containing all terms of the appropriate dimension.
Special case, theory in "detailed balance": $V = (\delta W/\delta g_{ij})^2$.

Extending the symmetries

A good starting point, but this action is only invariant under time-independent spatial diffeomorphisms, $\tilde{x}^i = \tilde{x}^i(x^j)$, and describes dynamical propagating components g_{ij} of the spatial metric.

Covariantization of the theory:

(1) Introduce ADM-like variables N (lapse) and N_i (shift), known from the space-time decomposition of the spacetime metric;

(2) Replace $\dot{g}_{ij} \to K_{ij} = \frac{1}{N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$,

$$\sqrt{g} \to N\sqrt{g}.$$

Gauge symmetries: Foliation-preserving diffeomorphisms $\mathrm{Diff}_{\mathcal{F}}(M)$,

$$\delta t = f(t),\ \delta x^i = \xi^i(t, x^j).$$

The transformation rules follow from a nonrelativistic contraction of spacetime diffeomorphisms; N and N_i are gauge fields of $\mathrm{Diff}_{\mathcal{F}}(M)$:

$$\delta N = \dot{f}(t)N + \ldots, \quad \delta N_i = \dot{\xi}_j + \ldots$$

In the minimal (="projectable") realization, N is a function of only t.

Symmetries reminiscent of the Causal Dynamical Triangulations (CDT) approach to quantum gravity on the lattice.

Simplest example: $z = 2$ gravity

The action is $S = S_K - S_V$, with

$$S_k = \frac{1}{\kappa^2} \int dt\, d^D\mathbf{x} \sqrt{g} N \, \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

and

$$S_V = \int dt\, d^D\mathbf{x} \sqrt{g} N \, \left(\alpha R_{ij} R^{ij} + \beta R^2 + \ldots \right).$$

Shift in the critical dimension, as in the Lifshitz scalar:

$$[\kappa^2] = 2 - D.$$

The minimal theory with $N(t)$ has the usual number of transverse-traceless graviton polarizations, plus an extra scalar DoF, all with the dispersion relation $\omega^2 \sim k^4$.

Two special values of λ: 1 and 1/D.

Another example: $z = 3$ gravity

The action is again $S = S_K - S_V$, with

$$S_K = \frac{1}{\kappa^2} \int dt\, d^D\mathbf{x} \sqrt{g} N \, \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

and

$$S_V = \int dt\, d^D\mathbf{x} \sqrt{g} N \, C_{ij} C^{ij}.$$

where $C^{ij} = \varepsilon^{ik\ell} \nabla_k (R^j_\ell - \frac{1}{4} R \delta^j_\ell)$ is the Cotton-York-ADM tensor. The shift of the critical dimension is

$$[\kappa^2] = 3 - D.$$

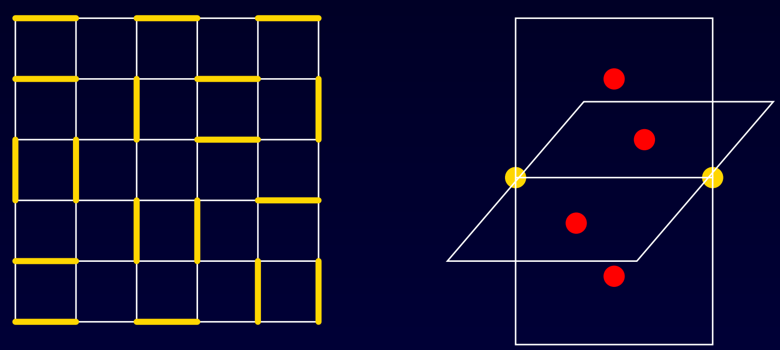
Anisotropic Weyl invariance eliminates the scalar graviton classically.

Emergent gravity at a Lifshitz point

[Cenke Xu and P.H., arXiv:1003.0009]

These models with $z = 2$ or $z = 3$ gravitons can emerge as IR fixed points on the fcc lattice. Emergent gauge invariance stabilizes new algebraic bose liquid phases.

Recall the emergence of $U(1)$ "photons" in dimer models [Fradkin,Kivelson,Rokhsar,...]:



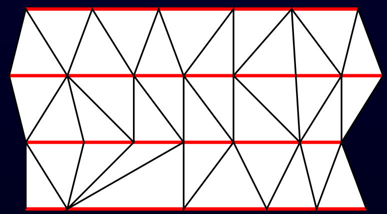
Lattice symmetries protect $z = 2$ or $z = 3$ in IR, forbid G_N.
But: interacting Abelian gravity is possible!

Gravity on the lattice

Causal dynamical triangulations approach [Ambjørn,Jurkiewicz,Loll] to $3+1$ lattice gravity:

Naive sum over triangulations does not work (branched polymers, crumpled phases).

Modify the rules, include a preferred causal structure:



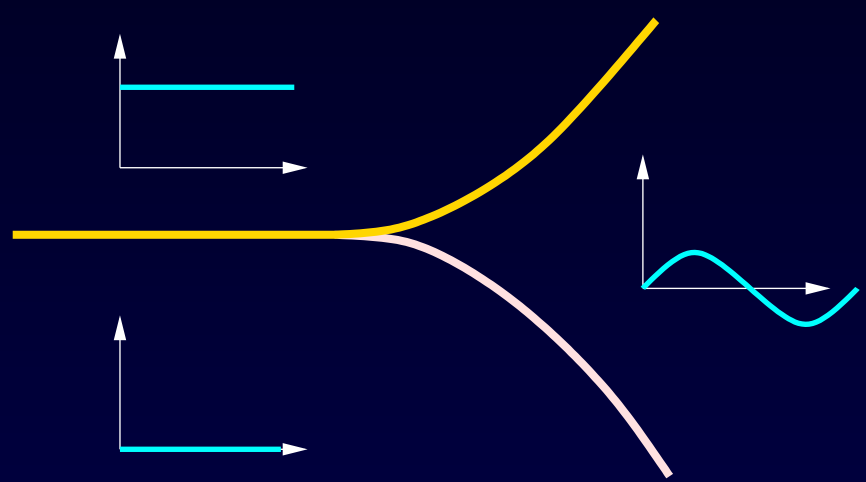
With this relevant change of the rules, a continuum limit appears to exist: The spectral dimension $d_s \approx 4$ in IR, and $d_s \approx 2$ in UV. Continuum gravity with anisotropic scaling: $d_s = 1 + D/z$. ([Benedetti,Henson,2009]: works in $2+1$ as well.)

Relevant deformations, RG flows, phases

The Lifshitz scalar can be deformed by relevant terms:

$$S = \frac{1}{2}\int dt\, d^D\mathbf{x}\, \left\{ \dot{\phi}^2 - (\Delta\phi)^2 - \mu^2 \partial_i\phi\partial_i\phi + m^4\phi^2 - \phi^4 \right\}$$

The undeformed $z = 2$ theory describes a tricritical point, connecting three phases – disordered, ordered, spatially modulated ("striped") [A. Michelson, 1976]:



RG flows in gravity: $z = 1$ in IR

Theories with $z > 1$ represent candidates for the UV description.
Under relevant deformations, the theory will flow in the IR.
Relevant terms in the potential:

$$\Delta S_V = \int dt\, d^D\mathbf{x} \sqrt{g} N \left(\ldots + \mu^2 R - 2\Lambda \right).$$

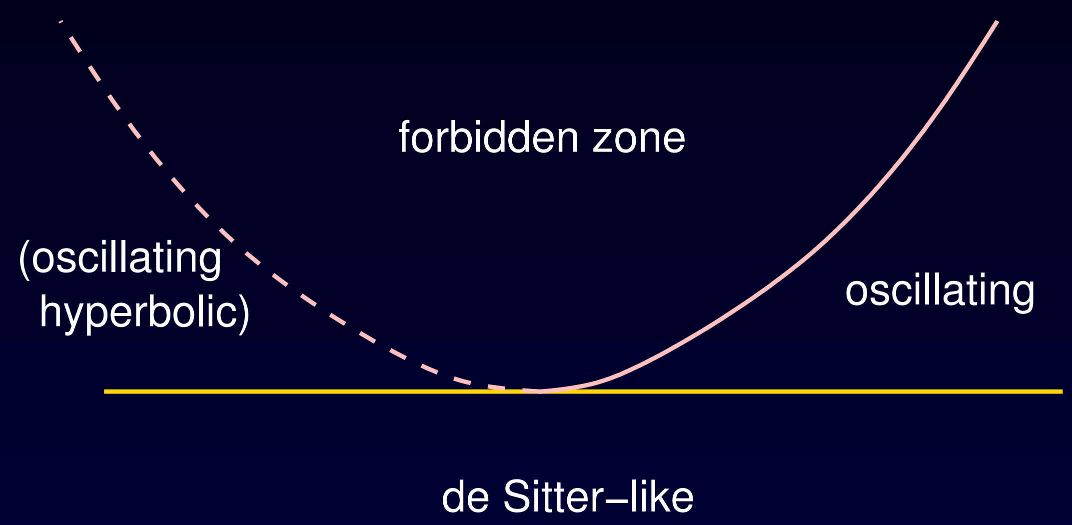
the dispersion relation changes in IR to $\omega^2 \sim k^2 + \ldots$
the IR speed of light is given by a combination of the couplings
μ^2 combines with $\kappa, \ldots$ to give an effective G_N.

Sign of k^2 in dispersion relation is opposite for the scalar and the tensor modes! Can we classify the phases of gravity? Can gravity be in a modulated phase?

Modulated phases of gravity

[in progress, w/ Patrick Zulkowski and Charles Melby-Thompson]

First, classify all spatially homogeneous and isotropic phases. Take $g_{ij} = a^2(t)\gamma_{ij}(k)$, with $k = 0, \pm 1$; set $N_i = 0$. The phase diagram for $k = 1$ (at fixed R^2 terms) looks like this:

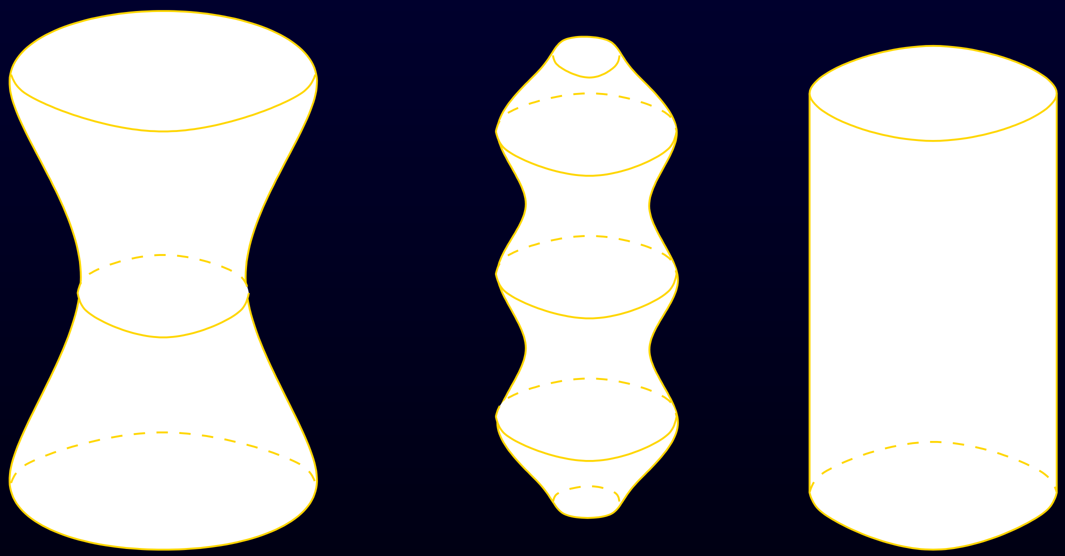


Governed by the Friedmann equation,

$$(\dot{g})^2 + R^2 + \mu^2 R - 2\Lambda = 0.$$

Spatially homogeneous isotropic phases of gravity

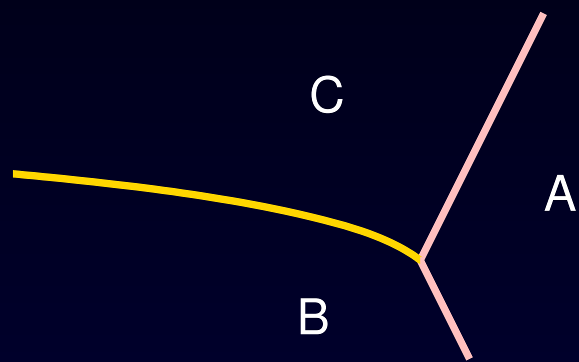
Examples of phases of gravity with $k = 1$: a de Sitter-like phase, an oscillating cosmology ($=$"temporally modulated" phase); the Einstein static universe appears at the phase transition line, where the theory satisfies detailed balance.



Cosmology: [Kiritsis et al, Brandenberger et al, Lüst et al, many others]

Phase structure in the CDT approach

Compare to the phase diagram in the causal dynamical triangulations:
[Ambjørn et al, 1002.3298]



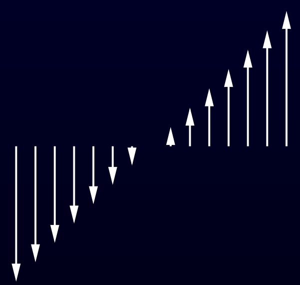
Note: $z = 2$ sufficient to explain three phases.
Possibility of a nontrivial $z \approx 2$ fixed point in $3 + 1$ dimensions?

Spatially modulated phases of gravity

Simplify, go to $2+1$ dimensions. In the forbidden zone, there is a class of solutions: Flat space with a shift flow, for example

$$g_{ij} = \delta_{ij}, \quad N_x = Cy, \quad N_y = 0.$$

Couette flow (hydrodynamics): resulting geometry is spatially inhomogeneous, but space-time homogeneous.



Another solution: $N_x = Cx,\ N_y = -Cy$.

What are these solutions? Snapshots of gravitational waves. Expectation: Spatially modulated phases of gravity exist in higher dimensions.

Applications in General Relativity and String Theory

[P.H. and Charles Melby-Thompson, arXiv:0909.3841]

The concept of anisotropic scaling in gravity is useful also in conventional relativistic GR and string theory, for understanding solutions which are asymptotically anisotropic near infinity (such as in AdS/CMT).

Penrose's notion of conformal infinity is insufficient for handling holographic renormalization in such spaces, but can be extended to the notion of anisotropic conformal infinity.

Another use: Defining boundary conditions for Euclidean path integrals in gravity with anisotropic scaling near infinity.

Anisotropic Weyl symmetry

First, local version of anisotropic scaling symmetries can be defined,

In the simplest example, for general values of z, we define

$$g_{ij} \to \exp(2\Omega(t, \mathbf{x}))g_{ij}, \quad N_i \to \exp(2\Omega(t, \mathbf{x}))N_i,$$

$$N \to \exp(z\Omega(t, \mathbf{x}))N.$$

Such anisotropic Weyl transformations form a closed symmetry group with the foliation-preserving diffeomorphisms.

Anisotropic conformal infinity

We have seen that anisotropic Weyl transformations with $z \neq 1$ are compatible with foliation-preserving diffeomorphisms,

Main point: In spacetime geometries whose asymptotic isometries are compatible with $\mathrm{Diff}_{\mathcal{F}}(M)$, anisotropic conformal transformations naturally define an anisotropic conformal infinity/boundary of spacetime.

The boundary is equipped with a natural anisotropic conformal structure.

Example: Black holes in spacetimes with anisotropic infinity (e.g. in warped AdS_3).

Ultralocal gravity

In retrospect, one example of a theory of gravity with anisotropic scaling has appeared in the literature already in the 1970's: the ultralocal theory of gravity [Isham;Teitelboim;Henneaux]

It results simply from eliminating all derivative terms from the potential, and setting

$$S_V = 2\Lambda.$$

This case can be viewed from two perspectives, either as $z = 0$ or $z = \infty$.

Remarkably, this theory is "generally covariant" – it has the same number of gauge symmetries per spacetime point as GR. The symmetry algebra is not that of GR, instead it is deformed into spatial diffeomorphisms and a local $U(1)$ symmetry.

Projectable vs. nonprojectable

Simplest attempt to relax projectability: Declare N to be a function of everything, see what happens. This approach has worked in the ultralocal theory, leading to general covariance and the closure of the constraints.

Effective field theory logic: Allow all terms in S compatible with symmetries. New terms: built out of $\nabla_i N/N$. New constraints second-class, no additional gauge invariance.

Artificially disallowing such terms: The constraint algebra appears in trouble, for $z > 1$. (Still, one apparently consistent way of quantizing this system: With detailed balance, $\mathcal{H}_\perp$ are quadratic in "a_{ij} variables." Declaring a_{ij}'s to be the first-class constraints (or a_{ij} and $a^\dagger_{ij}$ as second-class pairs) closes the algebra. This leads to quantizing the theory as topological.)

$U(1)$-Extended Diff$_{\mathcal{F}}$

Why do we want N to be the function of t and x^i? N is related to g_{00}, and that is where the Newton potential is.

Strategy: Keep the subleading, $\mathcal{O}(1/c^2)$ term in g_{00}:

$$g_{00} = -N(t)^2 + \frac{A_0(t,\mathbf{x})}{c^2} + \ldots,$$

and the subleading term α in the time reparametrizations as we take the $c \to \infty$ limit.

This α generates an extra $U(1)$ gauge symmetry,

$$\delta A_0 = \dot{\alpha}, \qquad \delta N_i = \partial_i \alpha, \qquad \delta g_{ij} = 0.$$

Requiring the invariance of the action under this $U(1)$-extended $\mathrm{Diff}_{\mathcal{F}}(M)$ symmetry implies:

(1) freezing the value of $\lambda = 1$,

(2) the presence of additional couplings between A_0 and R and N_i.

The full nonlinear theory works straightforwardly only in $2 + 1$ dimensions.

(More sophisticated constructions may work in higher spacetime dimensions – work in progress.)

More general Lifshitz scalars

A natural sequence of generalizations exists:

Split x^i into groups,

$$x^{i_1}, x^{i_2}, \dots x^{i_n},$$

with i_1 $(i_2, \dots i_n)$ taking D_1 $(D_2, \dots D_n)$ values, and $D = D_1 + D_2 + \dots D_n$.

Each k-th group has its own value of z, equal to k.

This leads to Lifshitz models with nested spatial anisotropy.

Generalizations to gravity are straightforward; in particular, a $(1+1)+D$ split is interesting: it captures features of gravity near black-hole horizons.

Theory with detailed balance

The role of the condition of detailed balance is twofold:

(1) A technical one: Reduces the number of independent couplings in the action.
In condensed matter, nongravitational examples of theories with detailed balance exhibit a simpler renormalization structure.

(2) Perhaps a more conceptual one: The condition of detailed balance arises in systems out of equilibrium, relating S to the equilibrium theory described by W.

Detailed balance can be softly broken, or eliminated altogether, in favor of the most general action of the effective field theory approach.

Application to AdS/CFT

Anisotropic gravity systems, if consistent, could provide a new class of gravity duals for CFTs, in particular those relevant for condensed matter.

Example: Start with W which has a Euclidean AdS_D solution. Then the theory with detailed balance, described by S in $D+1$ dimensions, has a static solution given by

$$N = 1, \quad N_i = 0, \quad g_{ij} = \text{Euclidean } AdS_D \text{ metric.}$$

This geometry has an $S^{D-1} \times \mathbf{R}$ boundary.

Bulk isometries = conformal symmetries of S^{D-1} plus time translations.

These are the symmetries of a quantum critical system on the boundary, already critical in the static limit.

Mathematical applications

Using detailed balance, the theory is related (in imaginary time) to the covariantized Ricci flow equation,

$$\dot{g}_{ij} = \gamma N(R_{ij} + \alpha R g_{ij}) + \nabla_i N_j + \nabla_j N_i.$$

In particular, the topological version of this theory represents a natural quantum field theory associated with the Ricci flow.

Ricci flow has been instrumental in Perelman's proof of the Poincaré conjecture.

Observables and their correlation functions should be of mathematical interest.

Entropic origin and detailed balance

Imposing detailed balance might be convenient for mathematical simplicity. However, a remarkable physics parallel exists: between gravity with detailed balance, and the Onsager-Machlup theory of non-equilibrium thermodynamics. [Onsager,Machlup 1953; Onsager 1931]

$$S=\int dt\,d^D\mathbf{x}\left(\dot{\Phi}_a M^{ab}\dot{\Phi}_b-\frac{\delta W}{\delta\Phi_a}M_{ab}\frac{\delta W}{\delta\Phi_b}\right).$$

This OM action describes the response of thermodynamic variables Φ_a to entropic forces $\delta W/\delta\Phi_a$; W itself is entropy!

Formally, gravity at a Lifshitz point with detailed balance has the same structure; mathematical formalism for understanding the possible entropic origin of gravity?

compare the heuristic ideas of [Verlinde,Jacobson,...]

Conclusions

Many interesting open questions, lots of work to be done!