Brane Induced Gravity Revisited

Fawad Hassan
Stockholm University, Sweden

(S.F. Hassan, Stefan Hofmann and Mikael von Strauss, to appear)

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Outline of the talk

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Motivation

The cosmological constant problem: Why is the observed value much smaller than that generically expected from QFT?

Resolutions:

- Modify QFT (not so easy)
- Modify gravity: make it less sensitive to Λ through IR modifications

Degravitation

An explicit realization: Brane Induced Gravity (BIG)

Outline of the talk

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Action: basic (*DGP, Dvali-Gabadadze*)

$$S = -A \int d^d x \sqrt{G} R^{(d)} - B \int d^4 \sigma \sqrt{g} R^{(4)} + S_m^{brane}$$

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Ardalan, Arfaei, Garousi and Ghodsi
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Kiritsis, Tetradis and Tomaras

Effective field theory: *DGP; Dvali, Gabadadze, Hou Sefusatti*

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Other variants: other terms, cascading setup (not considered)

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  Λ curves bulk $\Rightarrow$ flat brane classical soln. (*R. Gregory* $\cdots$)

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  Hint: Flat background + tachyon $\sim$ curved background
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  Implication: wrong background$\Rightarrow$ tachyon, ghost

Outline of the talk

Setup: Background and Fluctuations

Flat brane in flat background (easiest)

$$x_{||}^{\mu}(\sigma) = \sigma^{\mu} \quad , \quad x^{i}(\sigma) = y_0^{i}$$

$$G_{MN} = \eta_{MN} \quad , \quad g_{\mu\nu} = \eta_{\mu\nu}$$

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Gauge invariant variables:

$$H_{MN} = H^{\perp}_{MN} + \frac{1}{d}\eta_{MN}\, S$$

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But the gauge dependent ones do not drop out!

New gauge invariant variables:

$$F_\mu = f_\mu + \langle A_\mu \rangle - a_\mu + \frac{1}{2}\partial_\mu(\langle \Phi \rangle - \phi)\,,$$

$\sim$ Stückelberg fields . *Spontaneously broken realization of gauge symmetry ?*

$$F^i = y^i + \langle A^i \rangle + \frac{1}{2}\langle \partial^i \Phi \rangle\,.$$

Setup: Thick Branes and “blurred” Quantities

$$G(x_{||} - x'_{||}, x_\perp - x'_\perp) = -\int d^4k \int d^nq \frac{e^{ik(x_{||}-x'_{||})+iq(x_\perp - x'_\perp)}}{k^2 + q^2 - i\epsilon}$$

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Then, restriction to brane gives

$$\langle S\rangle(x_{||}) = \int d^nx_\perp P(x_\perp-y_0)S(x_{||},x_\perp-y_0)$$

and

$$\langle G\rangle(x_{||}-x'_{||}) = \int d^nx_\perp d^nx'_\perp\, P(x_\perp-y_0)\,G(x_{||}-x'_{||},x_\perp-x'_\perp)\,P(x'_\perp-y_0)$$

Bulk propagator restricted to brane

A very useful quantity:

$$\langle \widetilde{G} \rangle(k) = -\int d^n q \, \frac{[\widetilde{P}(q)]^2}{k^2 + q^2 - i\epsilon}$$

Brane width $\omega \Rightarrow$

$$\langle \widetilde{G} \rangle(k) = \frac{1}{\omega^{n-2}} \, \Sigma_n^{-1}(\omega^2 k^2),$$

Example: Gaussian

$$P(x_\perp) = \frac{1}{(\omega\sqrt{2\pi})^n} e^{-(x_\perp/2\omega)^2}, \qquad \widetilde{P}(q) = \frac{1}{(2\pi)^n} e^{-q^2\omega^2/2}$$

(*Antoniadis, Minasian, Vanhove*)

Bulk-brane relations

$$h_{\mu\nu} = \langle H_{\mu\nu} \rangle + \partial_\mu f_\nu + \partial_\nu f_\mu$$

gives,

$$\langle H^\perp \rangle_{\mu\nu} = h^\perp_{\mu\nu} - \partial_\mu F_\nu - \partial_\nu F_\mu - \eta_{\mu\nu} \left(\frac{1}{d} \langle S \rangle - \frac{1}{4} s \right)$$

Outline of the talk

Solutions for the Basic Model

For a brane source $T_{\mu\nu}$,

$$\widetilde{s}(k) = -\frac{2}{3B}\frac{1}{k^2 + \frac{A}{B}\frac{d-2}{2(d-5)}\langle\widetilde{G}\rangle^{-1}}\widetilde{T}$$

$$\widetilde{h}^{\perp}_{\mu\nu} = \frac{1}{B}\frac{1}{k^2 - \frac{A}{B}\langle\widetilde{G}\rangle^{-1}}\left(\widetilde{T}_{\mu\nu} - \frac{1}{3}(\eta_{\mu\nu} - \frac{k^\mu k^\nu}{k^2})\widetilde{T}\right)$$

Main features at a glance:

(1) 4-dim limit: $\omega \to 0$, no vDVZ discontinuity.
(2) s is tachyonic
(3) s is a ghost
(4) massive, unstable 4-dim gravitons $h^{\perp}$

Will come back to these later

Outline of the talk

Extrinsic Curvature Contributions

Could the neglected terms in the action resolve the ghost/tachyon problem?

$$S_\Omega = C \int d^4\sigma \sqrt{-g}\ \left(\Omega^M_{\alpha\beta}\ \Omega_M{}^{\alpha\beta} - \Omega^{M\alpha}_\alpha\ \Omega_{M\beta}{}^\beta\right)$$

where

$$\Omega^M_{\alpha\beta} = \partial_\alpha\partial_\beta x^M - \gamma^\lambda_{\alpha\beta}\,\partial_\lambda x^M + \Gamma^M_{NK}\,\partial_\alpha x^N \partial_\beta x^K$$

It involves, $\partial_i H|_{brane}$

For a thick brane, use $\langle\partial_i H\rangle \equiv \langle\frac{\partial}{\partial x^i_\perp} H\rangle(x_{||})$.

Outcome: No change in $\widetilde{s}$ and $\widetilde{h}^\perp$ (negative and positive aspects).

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$G(x - x')$: All propagations; $\langle G \rangle (x_{||} - x'_{||})$: Brane restricted

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Optical Theorem:

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brane-to-brane propagator with a physical brane:

$$\widetilde{G}^{c}_{bb} = (k^2 - \frac{A}{B} \langle \widetilde{G} \rangle^{-1})^{-1}$$

$\Rightarrow$ unstable 4-dim gravitons since,

$$Im\, \langle \widetilde{G}_{bb} \rangle^{-1} = \frac{A}{B}\, Im\, \langle \widetilde{G} \rangle^{-1}$$

Optical Theorm in the Complete Theory

brane-to-Bulk propagator:

$$\widetilde{G}_{Bb}(k,q) = \frac{-\widetilde{P}(q)\,\langle\widetilde{G}^{-1}\rangle}{k^2+q^2}\left[\frac{1}{B}\frac{1}{k^2+(A/B)\langle\widetilde{G}\rangle^{-1}}\right]$$

varifies optical theorem with physical brane,

$$2\,Im\,\widetilde{G}_{bb}^{-1} = \sigma^{B\neq 0}_{brane\to bulk}$$

$\sigma^{B\neq 0}_{brane\to bulk}$ given by $\widetilde{G}_{bB}$ with proper amputations

Brane theory alone is not unitary

Quantitative analysis of graviton propagator

$$G_{bb} = \frac{1}{B} \frac{1}{k^2 - \frac{A}{B} \langle \widetilde{G} \rangle^{-1}} = \frac{\omega^2}{B} \, \frac{1}{\omega^2 k^2 - \frac{A \omega^n}{B} \, \Sigma(\omega^2 k^2 - i\epsilon)}$$

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Example, for even n, $m = \frac{n}{2} - 1$,

$$\Sigma^{-1}(u - i\epsilon) = N[\, u^m e^u\, E_1(u - i\epsilon) + \sum_{r=0}^{m} (-1)^r (r-1)!\, u^{m-r}\,]$$

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and $\qquad \lim_{\epsilon\to 0} E_1(u - i\epsilon) = -Ei(-u) + i\pi$

$i\epsilon$ prescription $\Rightarrow$ sign of imaginary part

$$\Sigma(u) = \Sigma_1(u) + i\Sigma_2(u)$$

Standard QFT form for unstable particles

Validity of usual approximation for unstable graviton

$$G_{bb} = \frac{\omega^2}{B} \; \frac{1}{\omega^2 k^2 - \frac{A\omega^n}{B} \left[\Sigma_1(u) + i\Sigma_2(u) \right]}$$

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Exact complex pole: difficult even in QFT

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Mass defined by

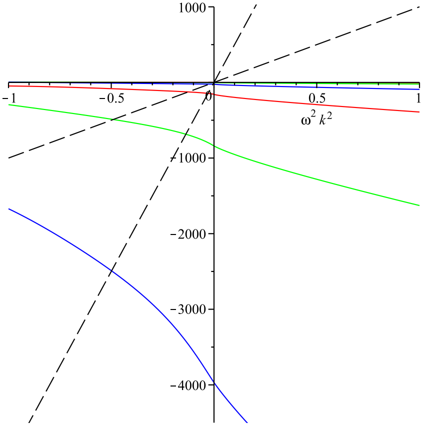
$$\omega^2 k^2 - \frac{A\omega^n}{B}\Sigma_1(\omega^2 k^2) = 0$$

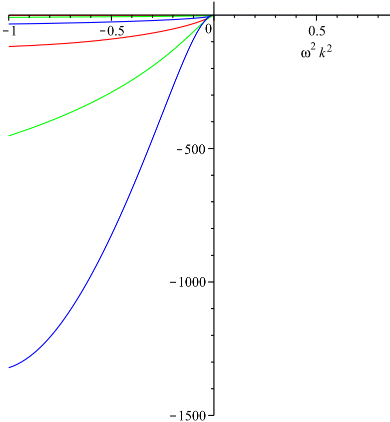
$\Sigma_2|_{pole}$: Decay width

From plots: good approximation for small mass

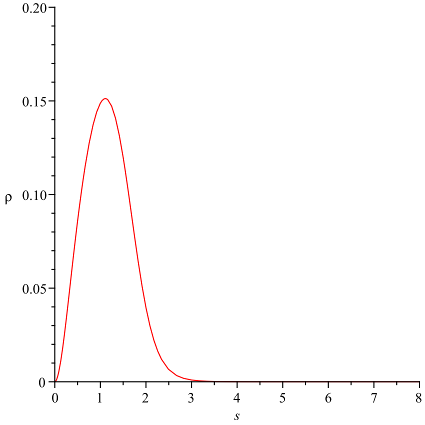
Validity of usual approximation for unstable graviton

$$(\omega^2 k^2)/(\frac{A\omega^n}{B}) + \Sigma_1(\omega^2 k^2) = 0, \qquad \Sigma_2(\omega^2 k^2)$$





A Sample Spectral Density Function



Outline of the talk

Screening of Λ

$$\Lambda \Rightarrow \widetilde{T}_{\mu\nu} = \widetilde{T}^{(m)}_{\mu\nu} + \Lambda\, \eta_{\mu\nu}\, \delta^{(4)}(k)$$

In any theory, $\widetilde{h}^{\perp}_{\Lambda\mu\nu} = 0$. But ...

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EH gravity:

$$\widetilde{s}_\Lambda \sim \Lambda\,\delta^{(4)}(k)/k^2 \quad or \quad \Box_4 s_\Lambda \sim \Lambda$$

$\Rightarrow$ Instability of flat space (dS)

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In BIG:

$$\widetilde{s}_\Lambda(k) = c\, \delta^{(4)}(k) \quad or \quad s_\Lambda(k) = c$$

$$c = -\frac{16}{3}\, \frac{d-5}{d-2}\, \frac{\Lambda}{A}\, \langle \widetilde{G} \rangle(0) > 0, \text{ finite for } n > 2$$

Flat background is stable

Screening of Λ: Discussion

- Relevant: $\langle \widetilde{G} \rangle (0) =$ *finite*, Irrelevant $Im \langle \widetilde{G} \rangle$, $\langle \widetilde{G} \rangle (k)$

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- No $d = 4 + n$ dim behaviour as $k^2 \to 0$ (not needed)
- Graviton decay $\Rightarrow$ Accelerated expansion (??)
- Contrast with massive Fierz-Pauli gravity:

$$m_s = \infty \;\Rightarrow s = 0 \quad \text{except for} \quad s_\Lambda \sim \Lambda/m_2 \;!$$

Implication for Newton Constant

$$g_{\mu\nu}(x) = (1 + c)\,\eta_{\mu\nu} + G_N\; h^{(m)}_{\mu\nu}(x) \qquad\qquad (G_N = 1/B)$$

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For canonical flat metric η_{MN},

$$g'_{\mu\nu} = \frac{g_{\mu\nu}}{(1+c)} = \eta_{\mu\nu} + \frac{G_N}{1+c}\; h^{(m)}_{\mu\nu}(x)$$

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Here, $c < 1$. But is the observed smallness of G_N related to the unobserved largeness of Λ(??)

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- Suggests that the tachyon/ghost problem is, at least partly, an artifact of the flat background approximation.

Outline of the talk

Further Issue: 4d Effective Action

4 dim Effective action $S_{eff}[h, F]$

Tensor, vector form

Stückelberg form even with general covariance (broken phase realization (?))

Further Issue: Dilaton Couplings

Bulk and Brane dilatonic couplings exist

Can be tuned to kill the ghost or make it very heavy