Higgs mass, renormalization group and naturalness in (quantum) cosmology

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Fundaments of Gravity, Munich, 2010

Model:

inflaton

non-minimal
curvature coupling

$$\boldsymbol{L}(g_{\mu\nu},\Phi)=\frac{1}{2}\left(M_P^2+\xi|\Phi|^2\right)R-\frac{1}{2}|\nabla\Phi|^2-\frac{\lambda}{4}(|\Phi|^2-v^2)^2$$

inflaton-graviton
sector of SM

$$\varphi^2\equiv|\Phi|^2=\Phi^\dagger\Phi$$

EW scale

Non-minimal coupling constant $\xi\gg 1$

GUT theory boson as an inflaton: $\frac{\lambda\varphi^4}{4},\ \ \frac{\Delta T}{T}\sim\sqrt{\lambda},\ \ \lambda\sim 10^{-13},$ now ruled out by WMAP

Non-minimal
curvature coupling $\frac{1}{2}\xi\varphi^2 R\ \ \rightarrow\ \ \frac{\Delta T}{T}\sim\frac{\sqrt{\lambda}}{\xi},\ \ \xi\gg 1$

B. Spokoiny (1984),
D.Salopek, J.Bond & J. Bardeen (1989),
R. Fakir& W. Unruh (1990),
A.Barvinsky & A. Kamenshchik (1994, 1998)

F.Bezrukov & M.Shaposhnikov
Phys.Lett. 659B (2008) 703:

Transcending the idea of non-minimal inflation to the Standard Model ground: Higgs boson as an inflaton – no new physics between TeV and inflation

A.O.B, A.Kamenshchik, C.Kiefer,
A.Starobinsky and C.Steinwachs (2008-2009):

Radiative corrections are enhanced by a large ξ and can be probed by current and future CMB observations and LHC experiments. *On account of RG running* with the Higgs mass in the range

$$136 \text{ GeV} < M_H < 185 \text{ GeV}$$

the SM Higgs can drive inflation with the observable CMB spectral index n_s ¸ 0.94 and a very low T/S ratio r' 0.0004.

A.O.B, A.Kamenshchik, C.Kiefer,
and C.Steinwachs (Phys. Rev. D81 (2010) 043530, arXiv:0911.1408):

This model can also generate initial conditions for the inflationary background upon which WMAP compatible CMB perturbations propagate.
These initial conditions are realized in the form of a sharp probability peak in the distribution function of inflaton for the *TUNNELING* cosmological wavefunction (we suggest the *PATH INTEGRAL* formulation of this state).

Plan

One-loop approximation and RG improvement

Inflation and CMB parameters

CMB bounds on Higgs mass

Naturalness of gradient and curvature expansion

Quantum cosmology origin of SM Higgs inflation

Tunneling cosmological state from microcanonical path integral

Problems and prospects

One-loop approximation

Effective Planck mass: $M_P^2 \to M_{\rm eff}^2(\varphi) = M_P^2 + \xi\varphi^2 \gg M_P^2$

Higgs effect due to big slowly varying inflaton: $\varphi \neq 0 \to m(\varphi) \sim \varphi$

1/m gradient and curvature expansion: $\frac{R}{m^2} \sim \frac{R}{\varphi^2} \sim \frac{\lambda\varphi^4}{3M_{\rm eff}^2\varphi^2} \simeq \frac{\lambda}{\xi} \ll 1$

suppression of graviton and Higgs loops due to $\frac{1}{M_{\rm eff}^2(\varphi)} \sim \frac{1}{M_P^2 + \xi\varphi^2} \ll \frac{1}{M_P^2}$

Gradient and curvature expansion:

$$S[g_{\mu\nu},\varphi] = \int d^4x\, g^{1/2} \left(-V(\varphi) + U(\varphi)\, R(g_{\mu\nu}) - \frac{1}{2} G(\varphi)\, (\nabla\varphi)^2\right)$$

Coefficient functions:

$$\begin{cases} V(\varphi) = \frac{\lambda}{4}(\varphi^2 - v^2)^2 + \frac{\lambda\varphi^4}{128\pi^2}\left(\boldsymbol{A}\ln\frac{\varphi^2}{\mu^2} - \boldsymbol{B}\right), \\ U(\varphi) = \frac{1}{2}(M_P^2 + \xi\varphi^2) + \frac{\varphi^2}{32\pi^2}\left(3\xi\lambda\ln\frac{\varphi^2}{\mu^2} - \boldsymbol{D}\right) \end{cases}$$

$$\xi \gg 1$$

Anomalous scaling behavior constant A

Overall Coleman-Weinberg potential:

$$\sum_{\text{particles}} \frac{\pm m^4(\varphi)}{64\pi^2} \ln \frac{m^2(\varphi)}{\mu^2} = \frac{\lambda \boldsymbol{A}}{128\pi^2} \varphi^4 \ln \frac{\varphi^2}{\mu^2} + \ldots$$

Masses in terms of SU(2),U(1) and top-quark Yukawa constants

$$m_W^2 = \frac{1}{4} g^2 \varphi^2, \quad m_Z^2 = \frac{1}{4} (g^2 + g'^2) \varphi^2, \quad m_t^2 = \frac{1}{2} y_t^2 \varphi^2,$$

$$m_H^2 = 3\lambda\varphi^2, \quad m_G^2 = \lambda\varphi^2$$

Higgs

Goldstone modes

Inflation

Range of the field at the inflation stage $\varphi^2 \gg M_P^2/\xi \gg v^2$

Smallness parameters $\frac{M_P^2}{\xi\,\varphi^2} \ll 1, \quad \frac{\boldsymbol{A_I}}{32\pi^2} \ll 1 \quad \to \quad \varepsilon, \eta \ll 1$

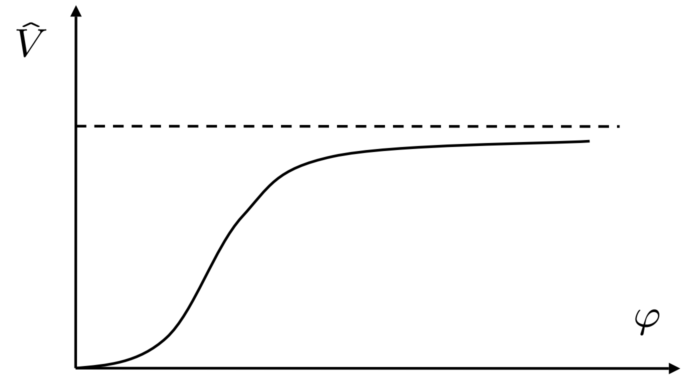
slow roll smallness parameters

Transition to Einstein frame --
Einstein frame potential:

$$(g_{\mu\nu}, \varphi) \to (\hat{g}_{\mu\nu}, \hat{\varphi})$$

$$U \to \hat{U} \equiv \frac{M_P^2}{2}, \; G \to \hat{G} \equiv 1, \; V \to \hat{V} :$$

$$\hat{V} = \left(\frac{M_P^2}{2}\right)^2 \frac{V(\varphi)}{U^2(\varphi)} \simeq \frac{\lambda M_P^4}{4\,\xi^2}\left(1 - \frac{2M_P^2}{\xi\varphi^2} + \frac{\boldsymbol{A_I}}{16\pi^2}\ln\frac{\varphi}{\mu}\right)$$



Inflationary anomalous scaling

$$\boldsymbol{A_I} = \boldsymbol{A} - 12\lambda = \frac{3}{8\lambda}\left(2g^4 + \left(g^2 + g'^2\right)^2 - 16y_t^4\right) - 6\lambda$$

e-folding # $N(\varphi) = \int\limits_{\varphi}^{\varphi_{\rm end}} d\varphi' \frac{H(\varphi')}{\dot\varphi'} \simeq \frac{48\pi^2}{\boldsymbol{A_I}} \ln\left(1 - \frac{\varphi^2}{\varphi_0^2}\right)$

end of inflation ($\varphi_{\rm end}$)

horizon crossing – formation of perturbation of wavelength related to *N*: $N(k) \simeq \ln(T_0/k)$

$$\varphi_0^2 = -\frac{64\pi^2 M_P^2}{\xi \boldsymbol{A_I}}$$

Quantum scale of inflation from quantum cosmology of the tunneling state (A.B. & A.Kamenshchik, Phys.Lett. B332 (1994) 270)

⇓

$$\frac{\varphi^2}{\varphi_0^2} = 1 - e^x, \quad x \equiv \frac{N \boldsymbol{A_I}}{48\pi^2}$$

vs tree-level result $N \simeq \frac{3}{4}\frac{\xi\varphi^2}{M_P^2}$

CMB parameters and bounds

CMB power spectrum:

Amplitude

$$\zeta^2(k) = \frac{N^2(k)}{72\pi^2}\frac{\lambda}{\xi^2}\left(\frac{e^x-1}{x\,e^x}\right)^2 \simeq 2.5\times10^{-9}$$

WMAP normalization at

$k \simeq (500\,\mathrm{Mpc})^{-1}$

$N \simeq 60$

$$\frac{\lambda}{\xi^2} \sim 0.5\times10^{-9}\left(\frac{x\,e^x}{e^x-1}\right)^2$$

quantum factor

**B. Spokoiny (1984),
D.Salopek, J.Bond & J. Bardeen (1989),
R. Fakir& W. Unruh (1990),
A.Barvinsky & A. Kamenshchik (1994, 1998),
F.Bezrukov & M.Shaposhnikov (2008)**

Determination of the quantum factor from the spectral index (and tensor to scalar ratio):

spectral index

$$n_s = 1 - \frac{2}{N}\frac{x}{e^x - 1}$$

T/S ratio

$$r = \frac{12}{N^2}\left(\frac{xe^x}{e^x - 1}\right)^2$$

$$0.934 < n_s < 0.988$$

WMAP+BAO+SN at 2σ

⇓

$$-1.57 < x < 1.79$$

$$-12.4 < A_I < 14.1$$

$$0.0006 < r < 0.015$$

Very small!

Standard Model bounds

$$m(v) = \left\{ M_Z,\ M_{W_\pm},\ M_t,\ \text{lighter masses} \right\}, \quad M_H^2 = 2\lambda v^2$$

$$\boldsymbol{A} \simeq \frac{12}{M_H^2 v^2}\left(M_Z^4 + 2M_W^4 - 4M_t^4\right) + \frac{3M_H^2}{v^2}$$

$$M_Z = 91\ \text{GeV},\ M_W = 80\ \text{GeV},\ M_t = 171\ \text{GeV},\ v = 247\ \text{GeV},$$

$$115\,\text{GeV} < m_H < 180\,\text{GeV}$$

Particle Data Group, W.-M.Yao et al (2006)

$$-48 < \boldsymbol{A_I} < -20$$

vs CMB window

$$-13.7 < \boldsymbol{A} < 16.4$$

If Higgs mass could be raised up to » 230 GeV then the SM Higgs boson could have served as the inflaton for a scenario with n_s» 0.93 and T/S» 0.0006

M_H IS TOO BIG!

RG improvement

Big logarithms $\frac{\boldsymbol{A}_I}{64\pi^2} \ln \frac{|\varphi^2|}{v^2} \sim 2 = O(1)$

$$\boldsymbol{A}_I = \frac{3}{8\lambda}\Big(2g^4 + \big(g^2 + g'^2\big)^2 - 16y_t^4\Big) - 6\lambda$$

⇩

$$\boldsymbol{A}_I(t) = \frac{3}{8\lambda(t)}\Big(2g^4(t) + \big(g^2(t) + g'^2(t)\big)^2 - 16y_t^4(t)\Big) - 6\lambda \Big|_{t=t_{\text{end}}}$$

$$t_{\text{end}} = \ln \frac{M_P}{M_t} + \frac{1}{2} \ln \frac{4}{3\xi_{\text{end}}}$$

RG equations for running couplings

running scale:

$$t = \ln(\varphi/M_t)$$

Running coupling constants:

$$V(\varphi) = \frac{\lambda(t)}{4} Z^4(t)\,\varphi^4,$$
$$U(\varphi) = \frac{1}{2}\Big(M_P^2 + \xi(t)\,Z^2(t)\,\varphi^2\Big),$$
$$G(\varphi) = Z^2(t)$$

RG equations:

$$\frac{dg_i}{dt} = \beta_{g_i}, \quad g_i = (\lambda, \xi, g, g', g_s, y_t),$$
$$\frac{dZ}{dt} = \gamma Z$$

Beta functions:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left(18s^2\lambda^2 + \lambda \boldsymbol{A}(t)\right) - 4\gamma\lambda$$

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(-\frac{9}{4}g^2 - \frac{17}{12}g'^2 - 8g_s^2 + \frac{9}{2}sy_t^2\right)$$

$$\frac{dg}{dt} = -\frac{20-s}{6}\frac{g^3}{16\pi^2}$$

$$\frac{dg'}{dt} = \frac{40+s}{6}\frac{g'^3}{16\pi^2}$$

$$\frac{dg_s}{dt} = -\frac{7g_s^3}{16\pi^2}$$

$$\frac{d\xi}{dt} = (6\xi+1)\left(\frac{(1+s^2)\lambda}{16\pi^2} - \frac{\gamma}{3}\right)$$

$$\gamma = \frac{1}{16\pi^2}\left(\frac{9g^2}{4} + \frac{3g'^2}{4} - 3y_t^2\right)$$

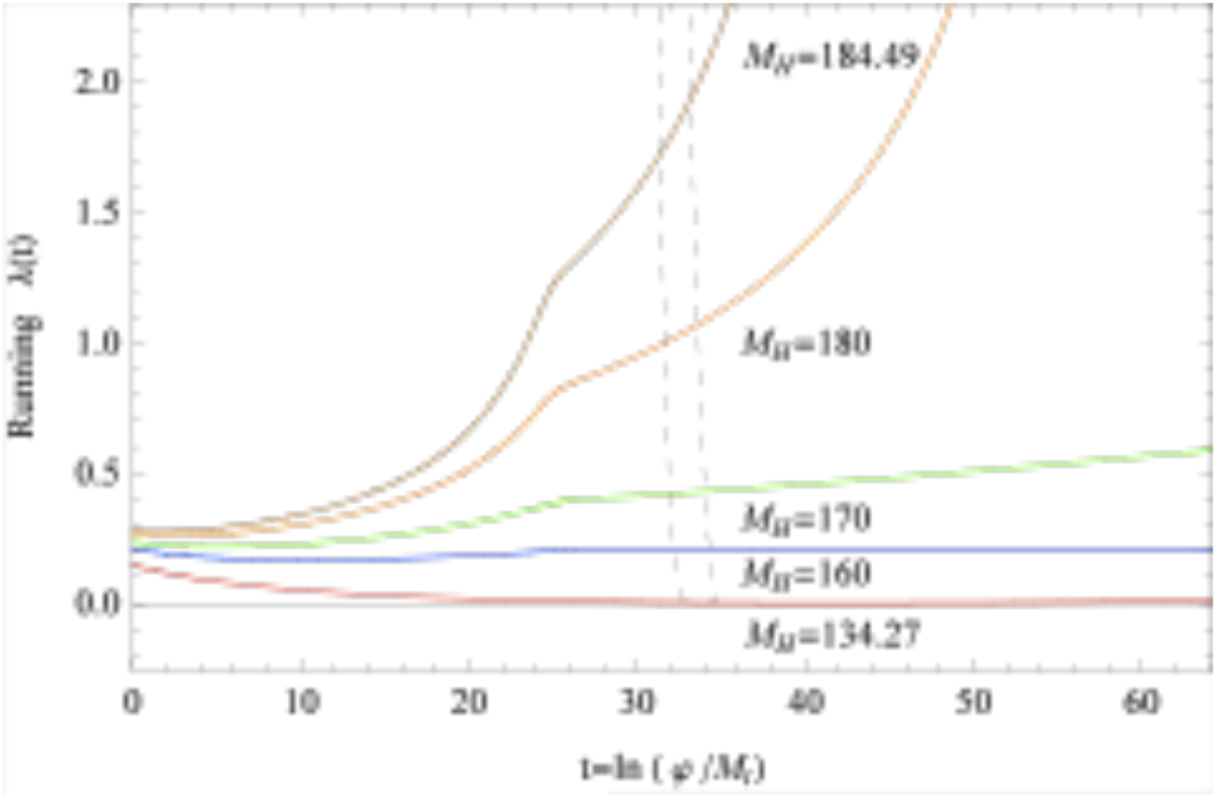
Suppression of Higgs loops due to gravitons:

$$s = \left.\frac{M_P^2 + \xi\varphi^2}{M_P^2 + (6\xi+1)\xi\varphi^2}\right|_{\varphi = M_{\rm top}\, e^t}$$

$s \simeq 1, \quad \varphi \ll \frac{M_P}{\xi}$ **EW scale**

$s \sim \frac{1}{\xi}, \quad \varphi \sim \frac{M_P}{\sqrt{\xi}}$ **inflation scale**

EW vacuum instability threshould: $M_H^{\text{inst}} \simeq 134.27$ GeV



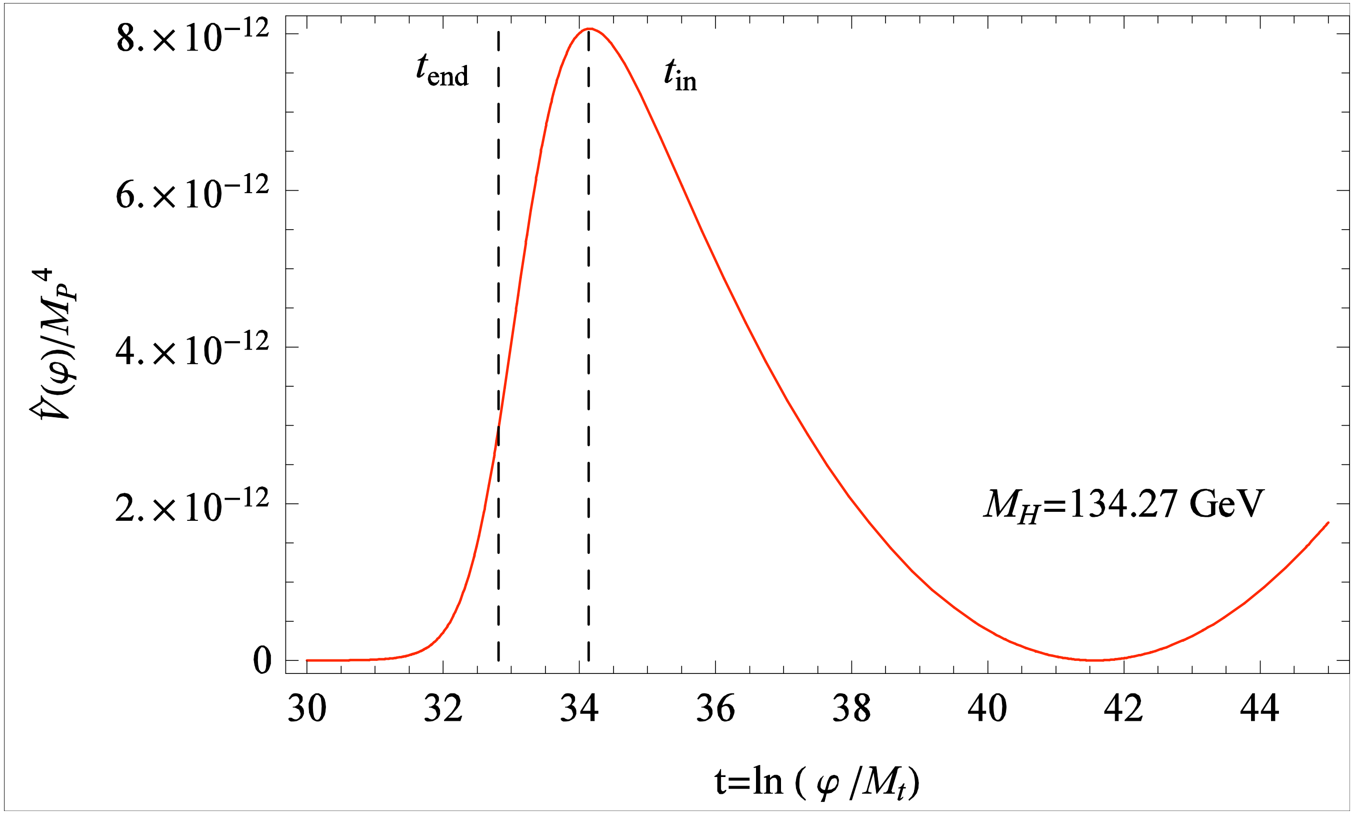
Plots of running $\lambda(t)$. Inflationary domain for a $N = 60$ CMB perturbation is marked by dashed lines.

Analogue of asymptotic freedom:

$$\beta_\lambda = \frac{1}{16\pi^2}\left(18s^2\lambda^2 + 6\lambda^2 + ...\right) - 4\gamma\lambda$$

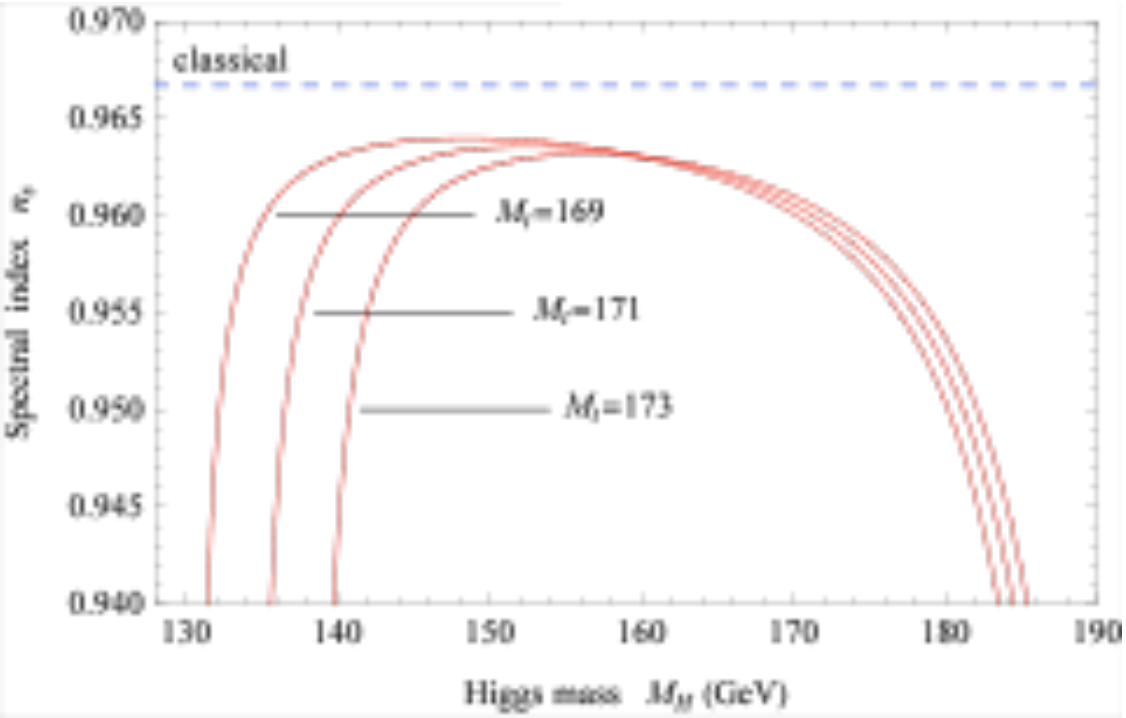
$s \to 0$

Effective potential at inflation scale (Einstein frame)



The effective potential for the instability threshold $M_{\rm H}^{\rm inst} = 134.27$ GeV. A false vacuum occurs at the instability scale $t_{\rm inst} \simeq 41.6$, $\varphi \sim 80 M_{\rm P}$. An inflationary domain for a $N = 60$ CMB perturbation is marked by dashed lines.

Spectral index vs M_H



$$0.94 < n_s(k_0) < 0.99$$

WMAP+BAO+SN
2σ CL

$$135.6 \text{ GeV} \lesssim M_H \lesssim 184.5 \text{ GeV}$$

Naturalness of gradient and curvature expansion

Energy cutoff for flat (empty) space scattering amplitudes

$$\Lambda = \frac{4\pi M_P}{\xi} \ll \frac{M_P}{\xi}$$

C. P. Burgess, H. M. Lee and M. Trott, arXiv: 0902.4465; 1002.2730 [hep-ph]

J. L. F. Barbon and J. R. Espinosa, arXiv: 0903.0355 [hep-ph]

M.Hertzberg, arXiv:1002.2995[hep-ph]

Background field method with effective Planck mass:

$$M_P \to \sqrt{M_P^2 + \xi\varphi^2} > \sqrt{\xi}\varphi \quad \Longrightarrow \quad \Lambda \to \Lambda(\varphi) = \frac{4\pi\varphi}{\sqrt{\xi}}$$

running cutoff

$$\frac{R}{\Lambda^2} \sim \frac{\lambda}{16\pi^2}$$ curvature expansion

$$\frac{\partial}{\Lambda} \sim \frac{1}{\Lambda}\frac{\dot\varphi}{\varphi} \simeq \frac{\sqrt{\lambda}}{48\pi}\sqrt{2\hat\varepsilon}$$ gradient expansion

slow roll smallness parameter

} perturbation theory range of SM $\quad \dfrac{\lambda}{16\pi^2} \ll 1$

⇓ justification of truncation

$$S = \int d^4x\, g^{1/2} \left(-V + U\, R(g_{\mu\nu}) - \frac{1}{2} G\, (\nabla\varphi)^2 + ...\right)$$

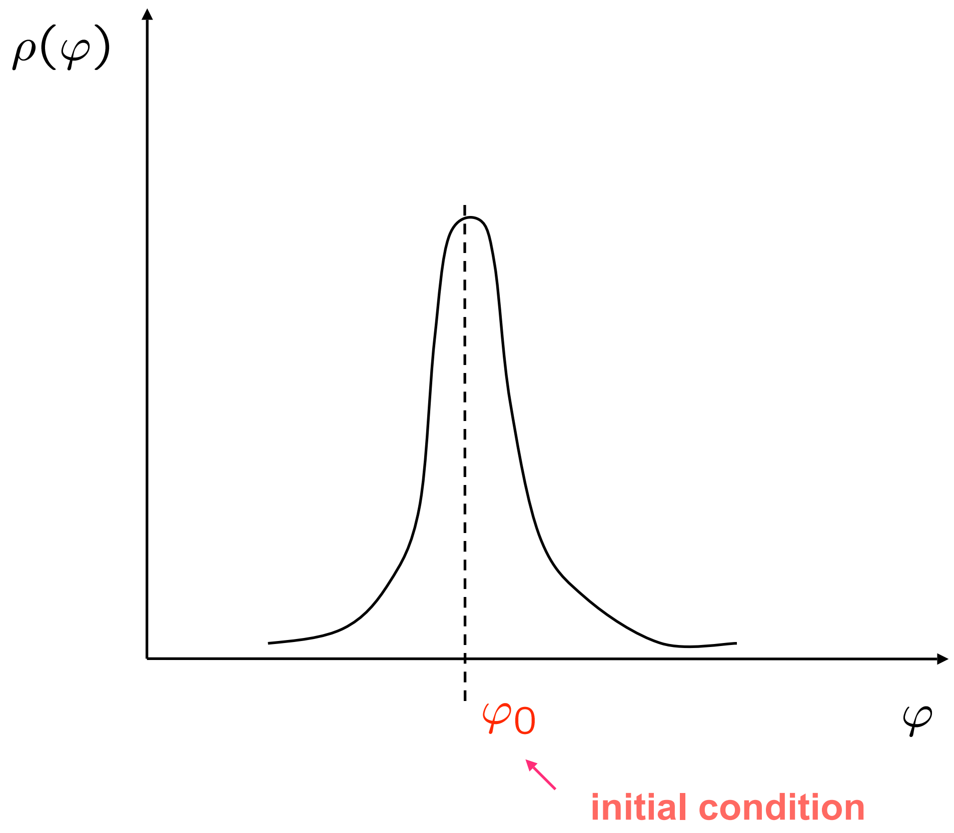
$$U, V, G = \sum_n c_n \left(\frac{\varphi}{\Lambda}\right)^n, \qquad 1 \not\gg \frac{\varphi}{\Lambda} \simeq \text{const}$$

preserves flatness of potential

Tunneling cosmological state: quantum origin of SM Higgs inflation

Generation of quantum initial conditions for inflation:

$$|\Psi\rangle = \Psi(\varphi, \Phi(\mathbf{x})) \;\to\; \rho(\varphi) = \int d[\,\Phi(\mathbf{x})\,]\,\Big|\Psi(\varphi, \Phi(\mathbf{x}))\Big|^2$$

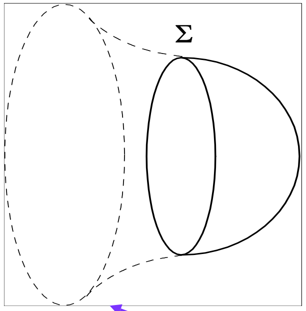


No-boundary vs tunneling wavefunctions (hyperbolic nature of the Wheeler-DeWitt equation):

$$\left|\Psi_\pm\big(\varphi,\Phi(\mathbf{x})\big)\right| = \exp\left(\mp\frac{1}{2}S_E(\varphi)\right)\left|\Psi_{\text{matter}}\big(\varphi,\Phi(\mathbf{x})\big)\right|$$

Euclidean action of quasi-de Sitter instanton

$$S_E(\varphi) \simeq -\frac{24\pi^2 M_P^4}{V(\varphi)} < 0$$



No-boundary (+): probability maximum at the mininmum of the potential

Tunneling (-): probability maximum at the maximum of the potential

$$\rho_\pm^{1-\text{loop}}(\varphi) = \int d\big[\Phi(\mathbf{x})\big]\left|\Psi_\pm\big(\varphi,\Phi(\mathbf{x})\big)\right|^2 = \exp\left(\mp S_E(\varphi) - S_E^{1-\text{loop}}(\varphi)\right)$$

contradicts renormalization theory

Tunneling state from microcanonical path integral in cosmology

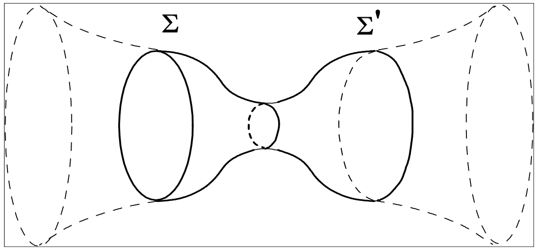
EQG density matrix

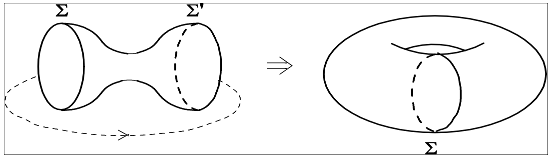
$$\rho(q,q') = e^{\Gamma} \int_{g,\phi\,|_{\Sigma,\Sigma'} = (q,q')} D[g_{\mu\nu},\phi]\, e^{-S_E[g_{\mu\nu},\phi]}$$

collective notation for configuration space coordinates

$q = (g_{ij}(\mathbf{x}), \phi(\mathbf{x}))$

D.Page (1986)

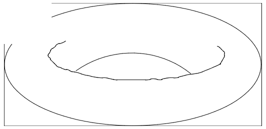




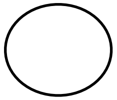
Statistical sum:

$$e^{-\Gamma} = \int_{\text{periodic}} D[g_{\mu\nu},\phi]\, e^{-S_E[g_{\mu\nu},\phi]}$$

on *S^3*£ *S^1* — **thermal**



including as a limiting case *S^4* — **vacuum**



Configuration space decomposition:

minisuperspace background

$$[\,g_{\mu\nu},\phi\,] = [\,a(\tau), N(\tau); \Phi(x)\,]$$

$$ds^2 = N^2\,d\tau^2 + a^2\,d^2\Omega^{(3)}$$ **FRW metric**

lapse (↖ N) scale factor (↖ a)

$$\Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \ldots)$$

quantum "matter" – cosmological perturbations

$$e^{-\Gamma} = \int_{\text{periodic}} D[\,a,N\,]\,e^{-S_{\text{eff}}[\,a,N\,]}$$

$$e^{-S_{\text{eff}}[\,a,N]} = \int_{\text{periodic}} D\Phi(x)\,e^{-S_E[\,a,N;\Phi(x)\,]}$$

quantum effective action of Φ on minisuperspace background

Range of integration over *N*?

The answer from physical QG in spacetime with Lorentzian signature:

Microcanonical density matrix

$$\hat{\rho} \sim \text{``}\Big(\prod_{\mu} \delta(\hat{H}_\mu)\Big)\text{''}$$

A.O.B., Phys.Rev.Lett. 99, 071301 (2007)

⇓

Wheeler-DeWitt equations

$$\hat{H}_\mu(q, \partial/i\partial q)\, \rho(q, q') = 0$$

⇓

Canonical (phase-space or ADM) path integral in Lorentzian theory:

3-metric and matter fields $q = (g_{ij}(\mathbf{x}), \phi(\mathbf{x}))$; p **-- conjugated momenta**

$$\rho(q_+, q_-) = e^{\Gamma} \int_{q(t_\pm)=q_\pm} D[\,q, p, N\,]\ \exp\left[\, i \int_{t_-}^{t_+} dt\, (p\,\dot{q} - N^\mu H_\mu)\right]$$

lapse and shift functions (N^μ)

constraints $H_\mu = H_\mu(q, p)$

Range of integration over N^μ: $-\infty < N^\mu < \infty$

Calculate this integral by "minisuperspace-quantum matter" decomposition and use semiclassical expansion and saddle points:

$$\Gamma_0 = S_{\rm eff}[\,a_0, N_0\,]$$

$$\frac{\delta S_{\rm eff}[\,a_0, N_0\,]}{\delta N_0} = 0$$

No periodic solutions of effective equations with real Lorentzian lapse *N*. Saddle points comprise Wick-rotated (Euclidean) geometry:

$$t = \tau, \quad N_{\rm Lorentzian} = -iN_{\rm Euclidean}$$

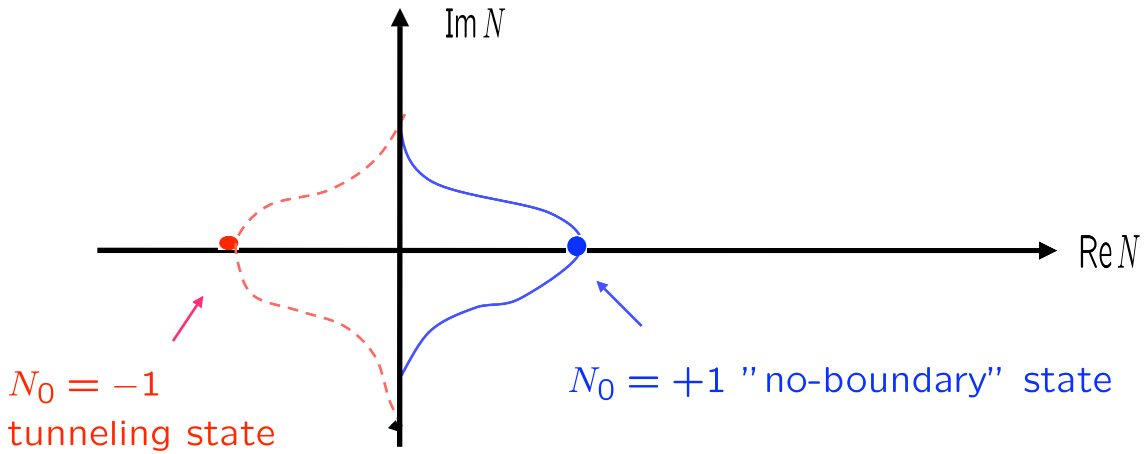
⇓

Euclidean lapse

Lorentzian path integral = EQG path integral with the imaginary lapse integration contour:

$$e^{-\Gamma} = \int\limits_{N \in [-i\infty,\, i\infty]} D[\,a, N\,]\; e^{-S_{\rm eff}[\,a,\,N\,]}$$

conformal "rotation"



Deformation of the original contour of integration

$$-i\infty < N < i\infty$$

into the complex plane to pass through the saddle point

$$N_0 = \pm 1$$

gauge (diffeomorphism) inequivalent!

$$\Gamma_\pm = S_{\rm eff}[\,a_0(\tau), \pm 1\,]$$

$$\exp\Big(-\,\Gamma_{\rm no-boundary/tunnel}\Big) = e^{-\Gamma_\pm}$$

Recipe for the *TUNNELING* state: calculate and *renormalize* effective action in Euclid for *N>0* and analytically continue to *N=-1*

Heavy massive quantum fields – local expansion:

$$S_{\rm eff}[g_{\mu\nu}] = \int d^4x\, g^{1/2} \left(M_{\rm P}^2 \Lambda - \frac{M_{\rm P}^2}{2} R(g_{\mu\nu}) + ... \right)$$

Effective Planck mass and cosmological constant

$$S_{\rm eff}[\,a, N\,] = 6\pi M_{\rm P}^2 \int d\tau\, N(-aa'^2 - a + H^2 a^3)$$

S^4 (vacuum) instanton:

$$a_0(\tau) = \frac{1}{H} \sin(H\tau), \quad H = \sqrt{\frac{\Lambda}{3}}$$

$\Rightarrow$

Analytic continuation – Lorentzian signature dS geometry:

$$\tau = \pi/2H + it$$

$$a_L(t) = \frac{1}{H} \cosh(Ht)$$

$$\Gamma_\pm = \mp \frac{8\pi^2 M_{\rm P}^2}{H^2}$$

Probability distribution on the ensemble of dS universes:

$$\rho_{\rm tunnel}(\Lambda) = \exp\left(-\frac{24\pi^2 M_{\rm P}^2}{\Lambda} \right), \quad \Lambda > 0$$

$$\rho_{\rm tunnel}(\Lambda) = 0, \quad \Lambda < 0$$

$$S_{\text{eff}}[g_{\mu\nu},\varphi] = \int d^4x\, g^{1/2} \left(-V(\varphi) + U(\varphi)\, R(g_{\mu\nu}) - \frac{1}{2} G(\varphi)\, (\nabla\varphi)^2 \right)$$

Transition to the Einstein frame

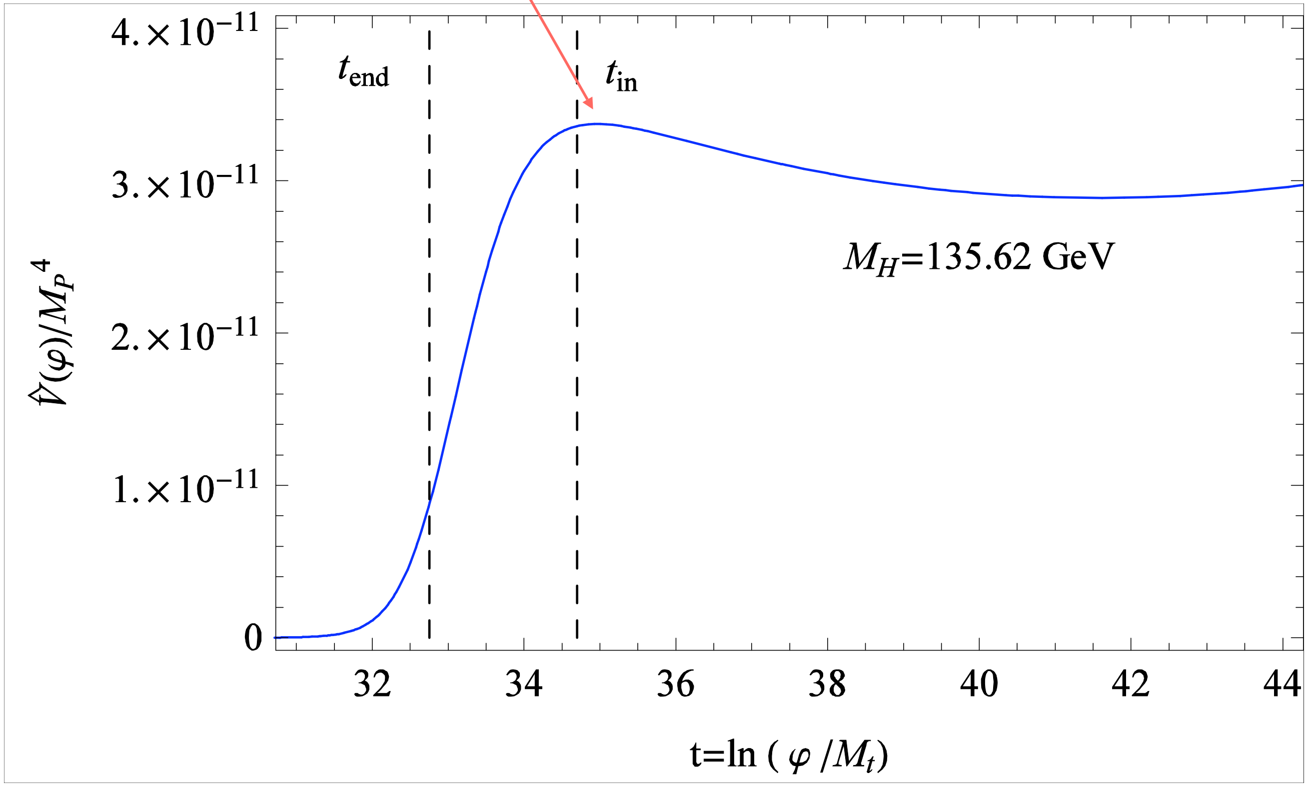
$$\rho_{\text{tunnel}}(\varphi) = \exp\left(-\frac{24\pi^2 M_{\text{P}}^4}{\hat{V}(\varphi)} \right)$$

$$\hat{V}(\varphi) = \left(\frac{M_{\text{P}}^2}{2} \right)^2 \frac{V(\varphi)}{U^2(\varphi)}$$

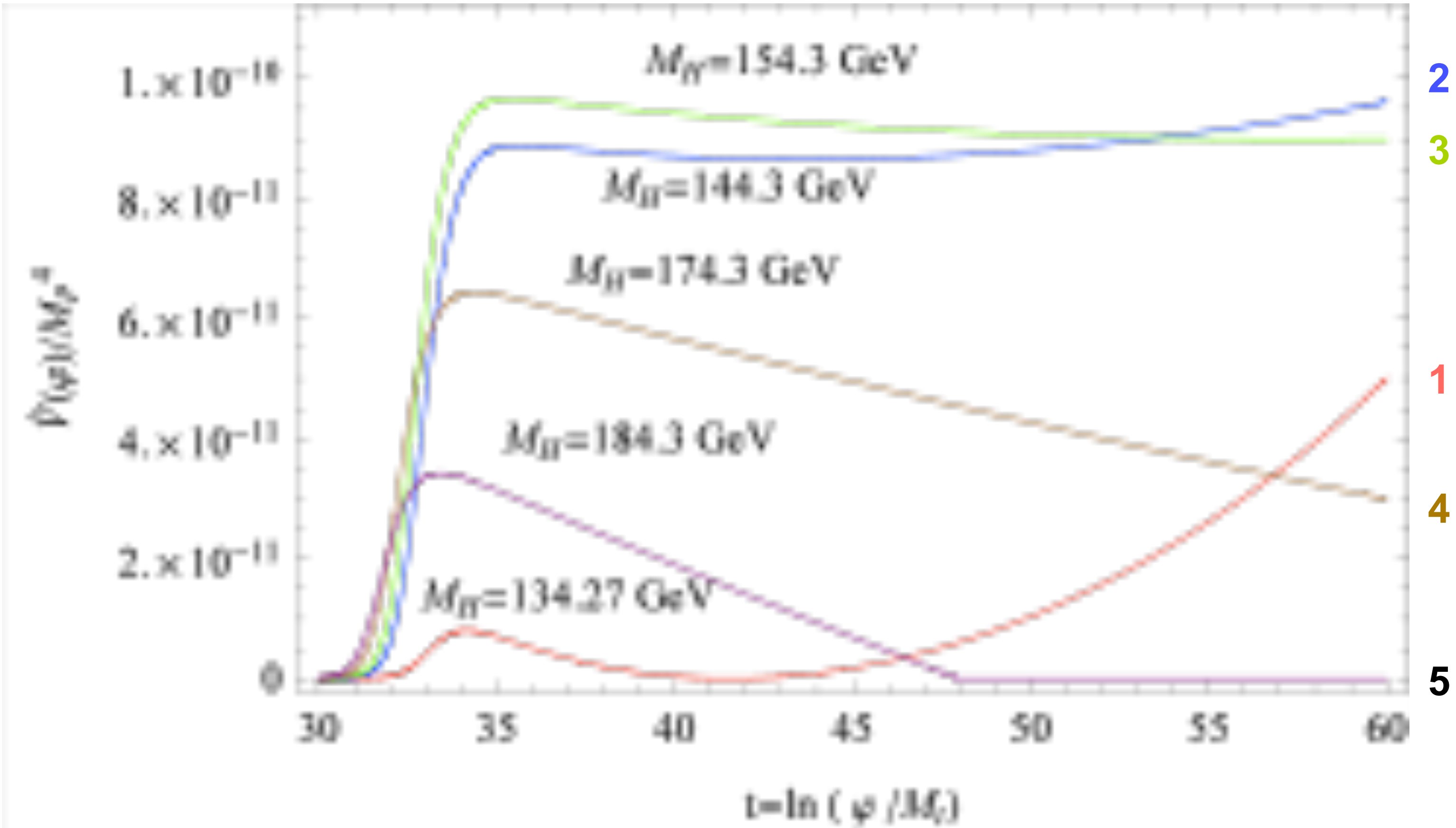
Einstein frame potential

Probability maximum at the *maximum* of the potential!

Generates a sharp probability peak in the tunneling distribution



Inflaton potential at the lowest CMB compatible value of M_{H} with a metastable vacuum at $t \simeq 42$. An inflationary domain for a $N = 60$ CMB perturbation is marked by dashed lines.



The succession $\mathbf{1} \div \mathbf{5}$ of effective potential graphs above the instability threshold $M_{\mathrm{H}}^{\mathrm{inst}} = 134.27$ GeV up to $M_{\mathrm{H}} = 184.3$ GeV: occurrence of a metastable vacuum followed for high M_{H} by the formation of a negative slope branch. Local peaks of $\widehat{V}$ situated at $t = 34 \div 35$ grow with M_{H} for $M_{\mathrm{H}} \lesssim 160$ GeV and start decreasing for larger M_{H}.

Probability peak – maximum of Einstein frame potential in $\varGamma_-(\varphi) = 24\pi^2 \frac{M_P^4}{\hat{V}(\varphi)}$

RG

$$\varphi_0 \frac{d\varGamma_-}{d\varphi_0} = \frac{d\varGamma_-}{dt_0} \simeq -\frac{6\xi^2}{\lambda}\left(\boldsymbol{A_I} + \frac{64\pi^2 M_P^2}{\xi Z^2 \varphi_0^2}\right) = 0$$

$$\varphi_0^2 = -\frac{64\pi^2 M_P^2}{\xi \boldsymbol{A_I} Z^2}\bigg|_{t=t_0} > 0$$

Quantum width of the peak:

$$\frac{\Delta\varphi^2}{\varphi_0^2} = \left(\frac{d^2\varGamma_-}{dt^2}\right)^{-1} = -\frac{\lambda}{12\xi^2}\frac{1}{\boldsymbol{A_I}}\bigg|_{t=t_0} \sim 10^{-10}$$

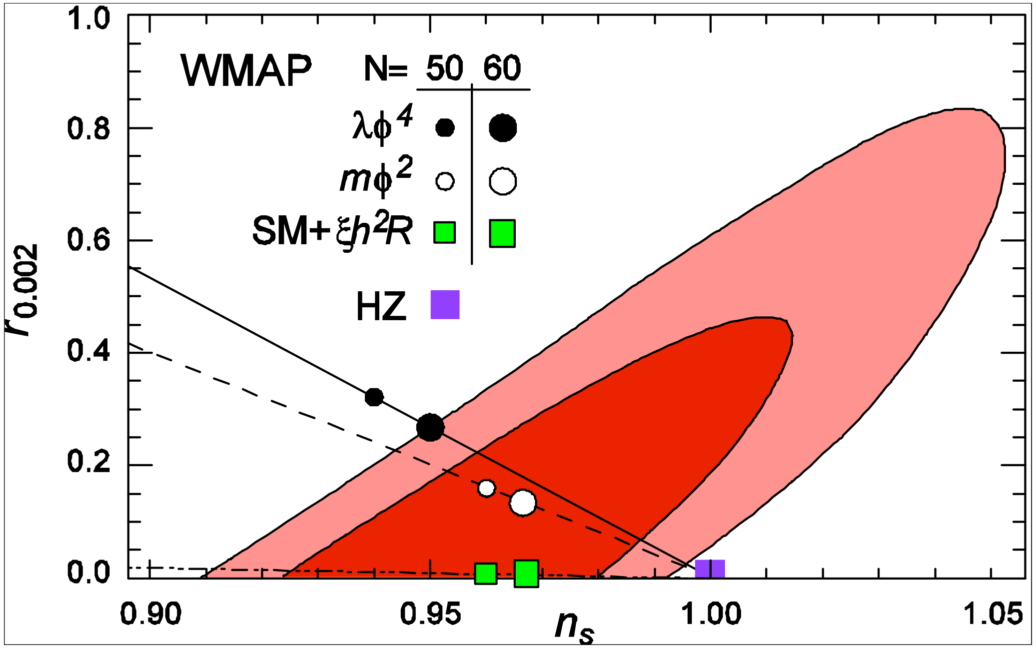
Conclusions

A complete cosmological scenario is obtained:

i) formation of initial conditions for the inflationary background (a sharp probability peak in the inflaton field distribution) and
ii) the ongoing generation of the WMAP compatible CMB perturbations on this background. in the Higgs mass range

$$135.6 \text{ GeV} \lesssim M_H \lesssim 184.5 \text{ GeV}$$

Effect of heavy SM sector and RG running --- small negative anomalous scaling: analogue of asymptotic freedom $A_I < 0$



Problems and prospects

Comparison with: F. L. Bezrukov & M. Shaposhnikov, Phys. Lett. B 659 (2008) 703;
F. L. Bezrukov, A. Magnon & M. Shaposhnikov, arXiv: 0812.4950;
F. Bezrukov & M. Shaposhnikov, arXiv: 0904.1537 [hep-ph]
A. De Simone, M. P. Hertzberg & F. Wilczek, arXiv: 0812.4946 [hep-ph]

Gauge, parametrization (Cartesian vs radial configuration space coordinates) and frame (Jordan vs Einstein) dependence of results

Rigorous definition of quantum CMB parameters as gauge-invariant physical observables