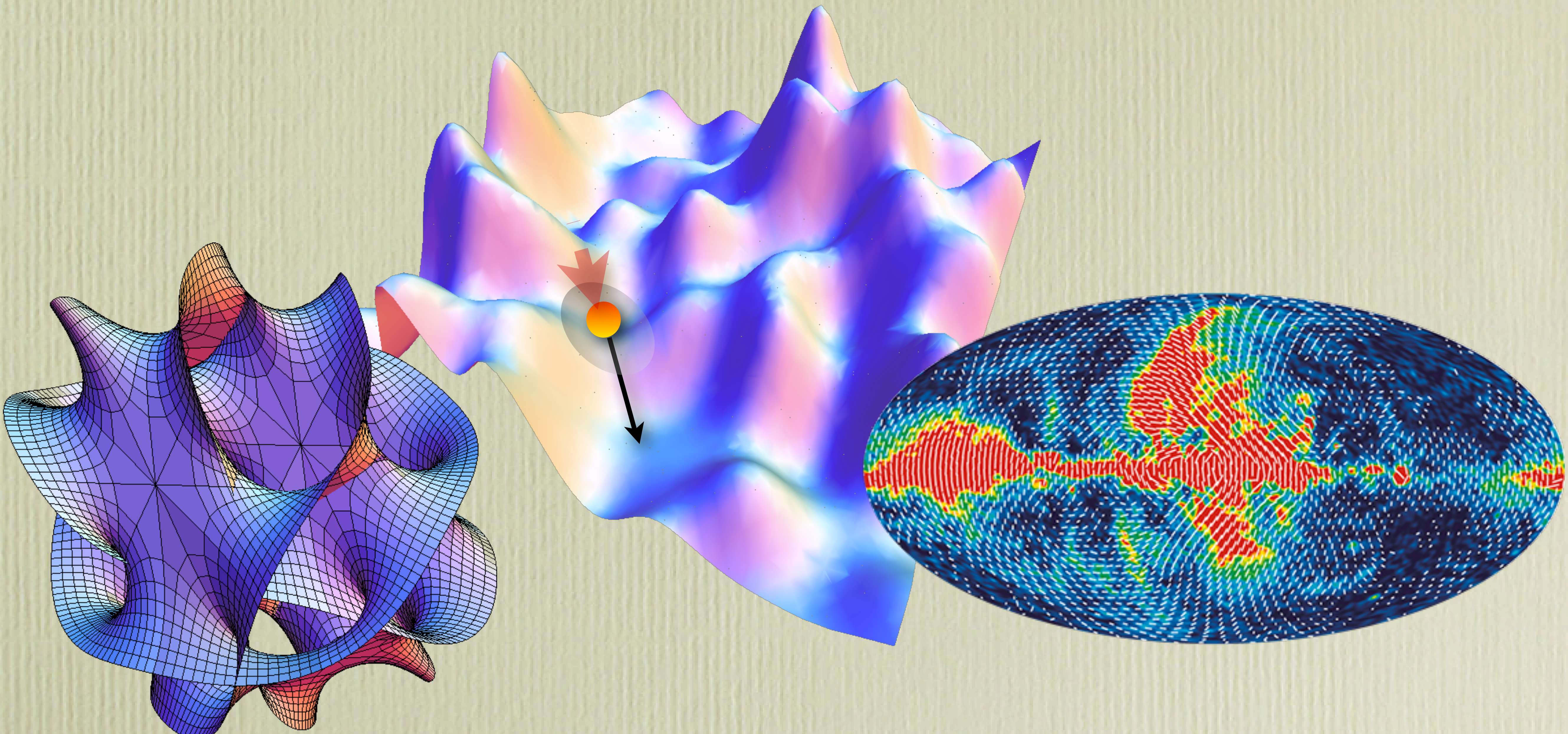
Inflationary cosmology after PLANCK



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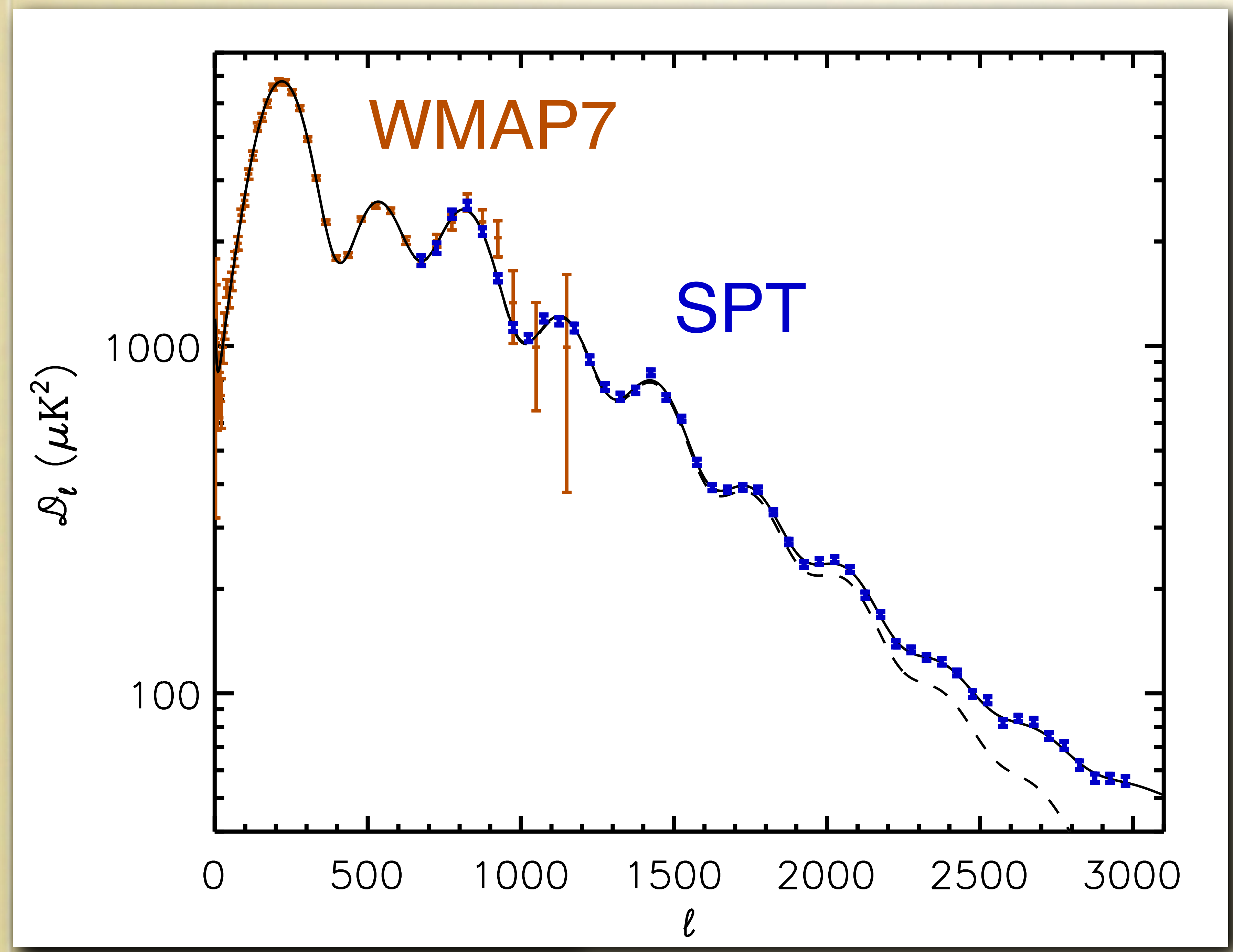
yesterday < 12 pm CET: $\mathrm{SPT} + \mathrm{WMAP\ 7yr} + \mathrm{BAO} + H_0$



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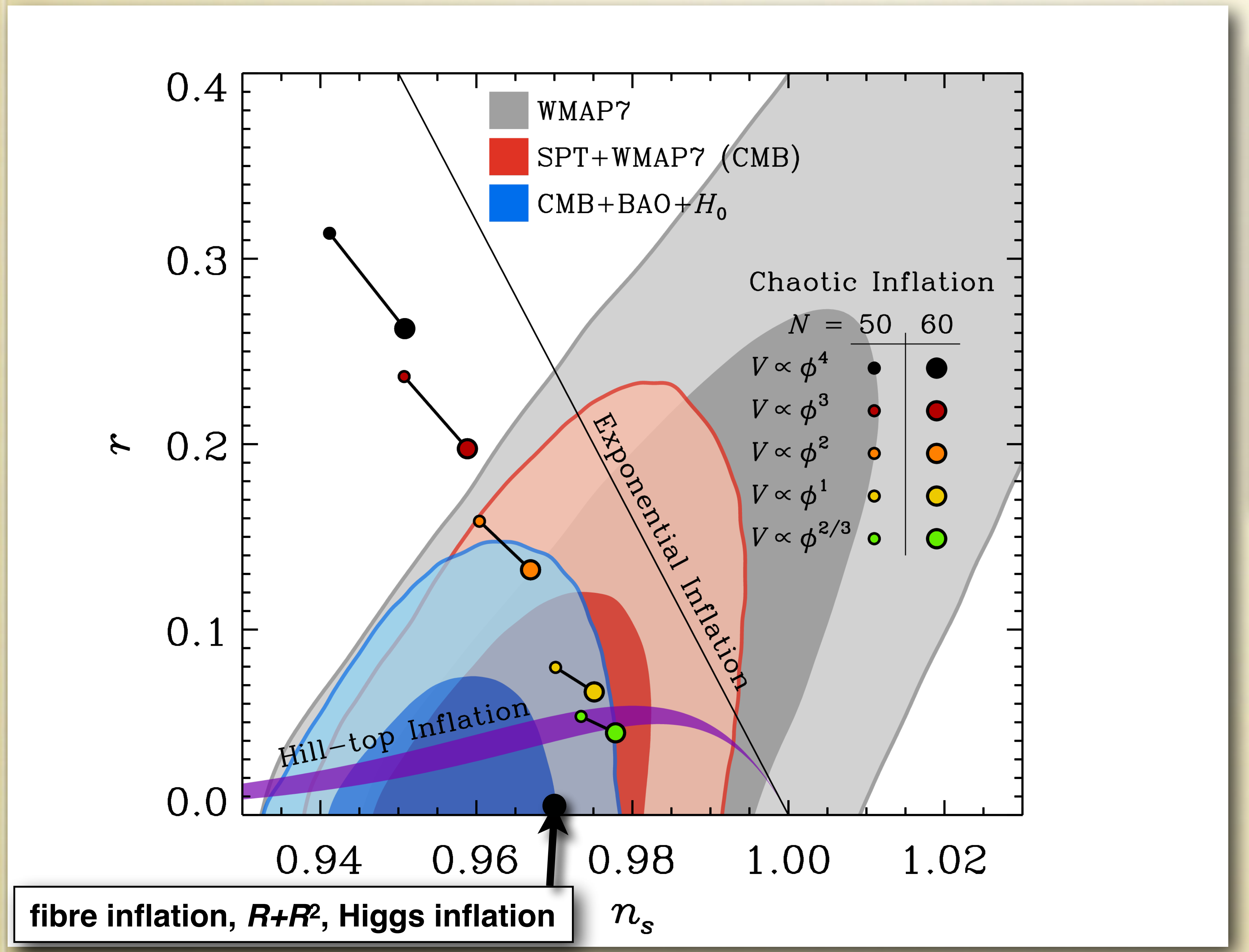


$$n_s = 0.9538 \pm 0.0081 \ (68\%)$$

$$r < 0.11 \ (95\%)$$

$$\Omega_k = -0.0059 \pm 0.004$$

$$-10 < f_{NL}^{local} < 74$$

$$-214 < f_{NL}^{equil} < 266$$

Inflation ...

- inflation: period quasi-exponential expansion of the very early universe

  (solves horizon, flatness problems of hot big bang ...)

  e.g. 3-curvature: $\rho_k \sim \frac{1}{a^2} \sim a^2 \rho_{rad.}$, $\Omega_k \equiv \frac{\rho_k}{\rho_{crit.}} \lesssim 0.01$ today

  $\Rightarrow \quad \rho_k \sim e^{-60} \rho_{rad.}$ at GUT epoch

- driven by the vacuum energy of a slowly rolling light scalar field:

  e.o.m.: $\ddot{\phi} + 3H\dot{\phi} + V' = 0$

Inflation ...

- slow-roll inflation:

scale factor grows exponentially : $a \sim e^{Ht}$ if : $\ddot{\phi} \ll \dot{\phi}$

$$\Rightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \quad , \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \simeq \frac{V''}{V} \ll 1$$

with the Hubble parameter $H^2 = \frac{\dot{a}^2}{a^2} \simeq const. \sim V$

Inflation ...

- inflation generates metric perturbations: scalar (us) & tensor

$$\mathcal{P}_R \sim \frac{H^2}{\epsilon} \sim \left(\frac{\delta\rho}{\rho}\right)^2 \quad \text{and} \quad \mathcal{P}_T \sim H^2 \sim V$$

$$\sim k^{n_S - 1}$$

window to GUT scale &
direct measurement of inflation scale

but caveat: inflaton w/ pseudo-scalar couplings to light vector fields can source additional B-modes

[Barnaby, Namba & Peloso '11; Senatore, Silverstein & Zaldarriaga '11]
[Barnaby, Moxon, Namba, Peloso, Shiu & Zhou '12]

Inflation ...

- inflation generates metric perturbations: scalar (us) & tensor

$$\mathcal{P}_R \sim \frac{H^2}{\epsilon} \sim \left(\frac{\delta\rho}{\rho}\right)^2 \quad \text{and} \quad \mathcal{P}_T \sim H^2 \sim V$$

$$\sim k^{n_S - 1}$$

- observables: $\mathcal{P}_R|_{k=aH}$

$$n_S \equiv \left.\frac{d\ln\mathcal{P}_R}{d\ln k}\right|_{k=aH} = 1 - 6\epsilon + 2\eta|_{N_e=50\ldots60}$$

$$r \equiv \left.\frac{\mathcal{P}_T}{\mathcal{P}_R}\right|_{k=aH} = 16\epsilon|_{N_e=50\ldots60}$$

Inflation ...

- description exact in ε , η: Mukhanov-Sasaki equation for variable $v = zR$, $z = a\dot{\phi}/H$; can choose gauge where

curvature perturbation

tensor perturbations

$$\delta\phi = 0 \quad , \quad g_{ij} = a^2[(1 - 2R)\delta_{ij} + h_{ij}] \quad , \quad \partial_i h_{ij} = h^i_i = 0$$

$$\text{quadratic action for } R \text{ from } S_{EH} \quad \Rightarrow \quad \boxed{v''_k + \left(k^2 - \frac{z''}{z}\right) v_k = 0}$$

- higher-order interactions of *R* - 3-point function:

$$\langle R_{\vec{k}_1} R_{\vec{k}_2} R_{\vec{k}_3} \rangle \sim \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\, B_R(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

- for purely Gaussian fluctuations 3-point function vanishes - “non-Gaussianity” has triangular shape in *k*-space

Inflation ...

- magnitude of non-Gaussianity:

$$f_{NL} \equiv \frac{5}{18} \frac{B_R(k,k,k)}{\mathcal{P}_R(k)^2}$$

- Maldacena's result for single-field slow-roll:

$$f_{NL} = \mathcal{O}(\epsilon, \eta)$$

intuitively sensible:
ε, η describe deviation from free-field action, which has purely Gaussian wave functions

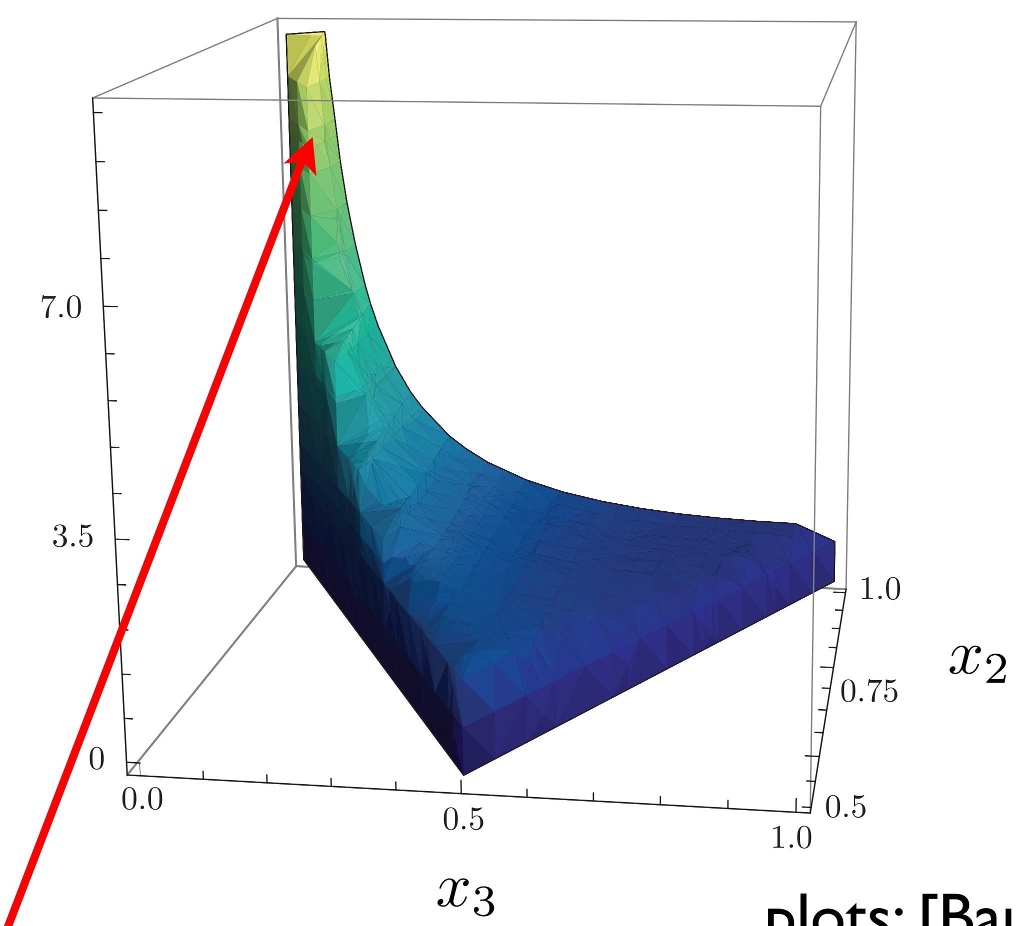
very recently:

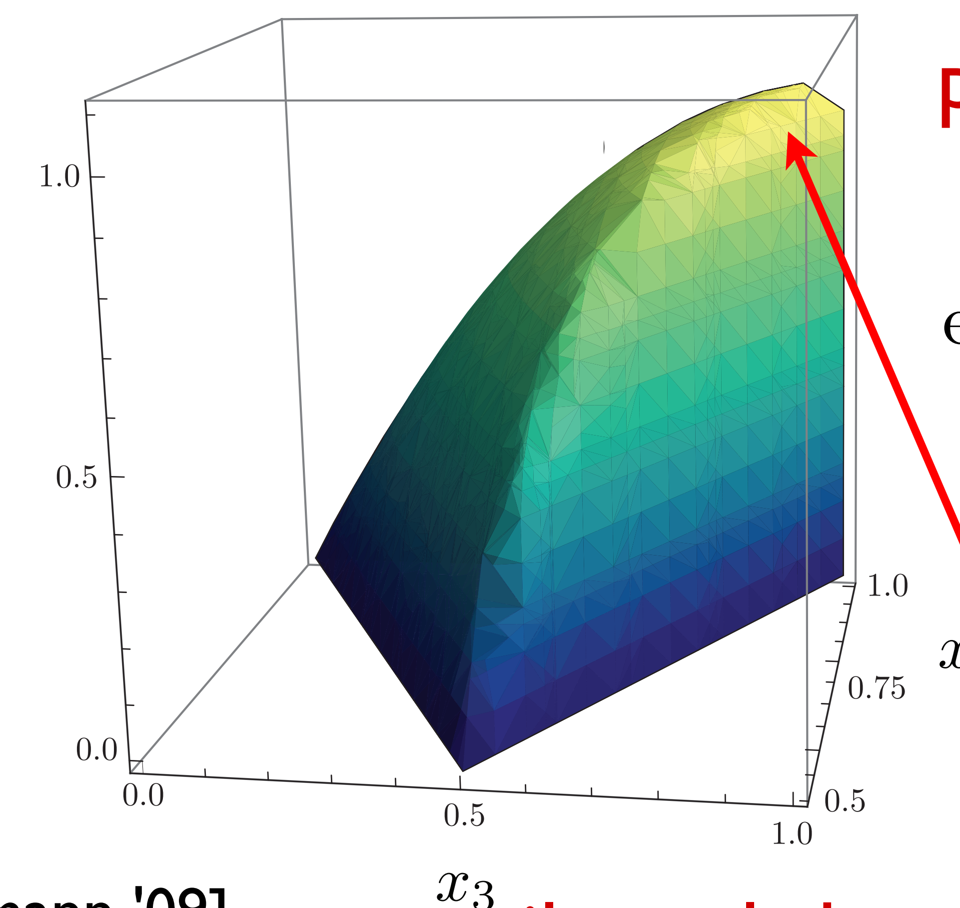
- if R non-constant outside horizon -- large <u>local</u> f_{NL} for ~ 10 out of 60 e-folds [Chen, Firouzjahi, Namjoo & Sasaki '13]

Inflation ...

- different shapes B_R - peak at different triangle configurations (k_1,k_2,k_3)

- can plot B_R as a function of: $x_2 = \dfrac{k_2}{k_1} \leq 1 \quad , \quad x_3 = \dfrac{k_3}{k_1} \leq 1$





plots: [Baumann '09]

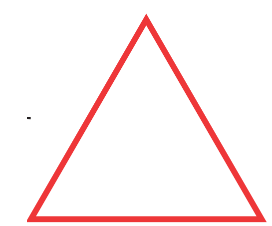
peaks at:

squeezed

local shape:
multi-field inflation, e.g. curvaton

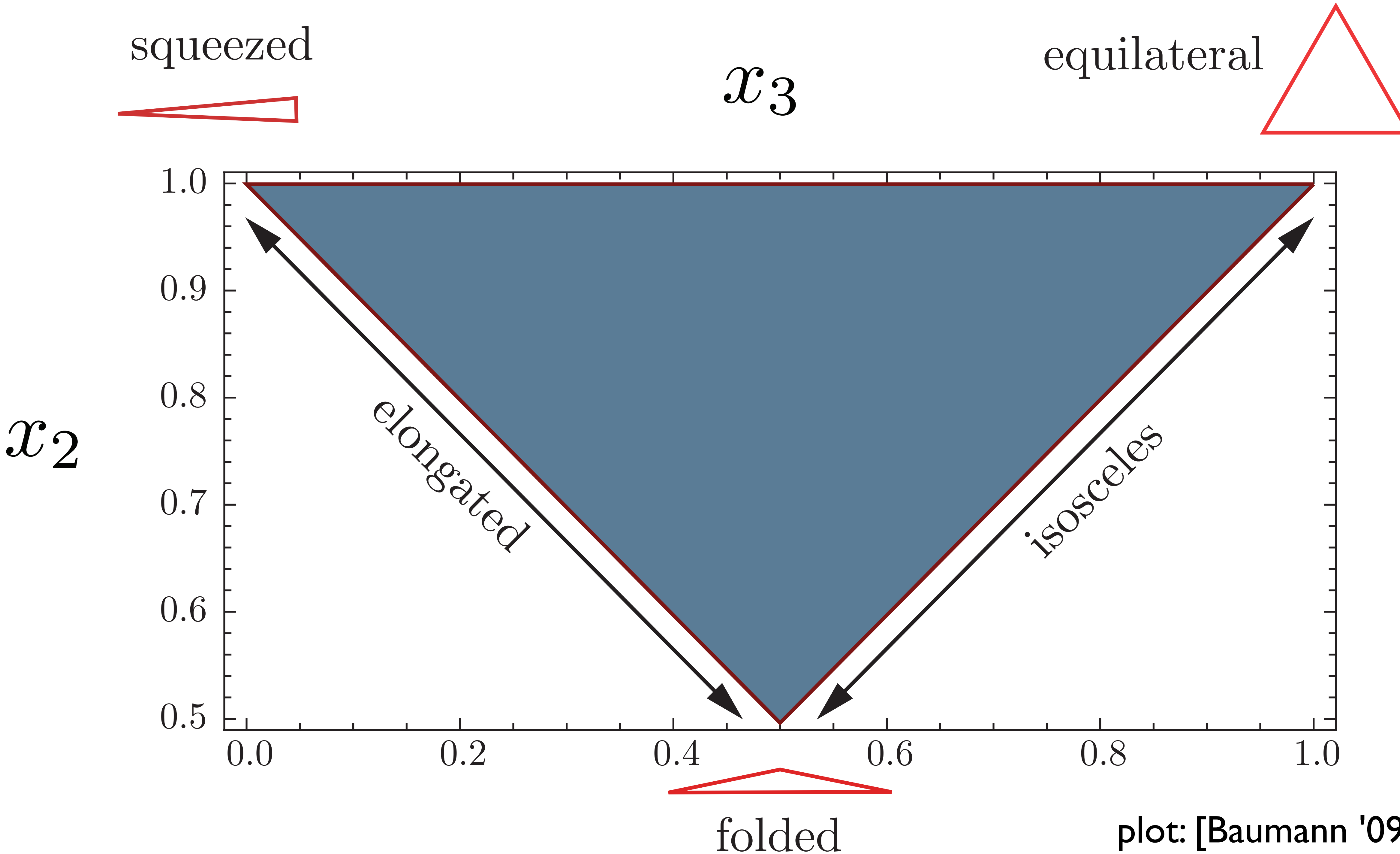
peaks at:

equilateral



equilateral shape:
single-field <u>*non*</u>-slow roll, e.g. DBI-inflation

Inflation ...



plot: [Baumann '09]

Inflation ... field range vs tensor mode power

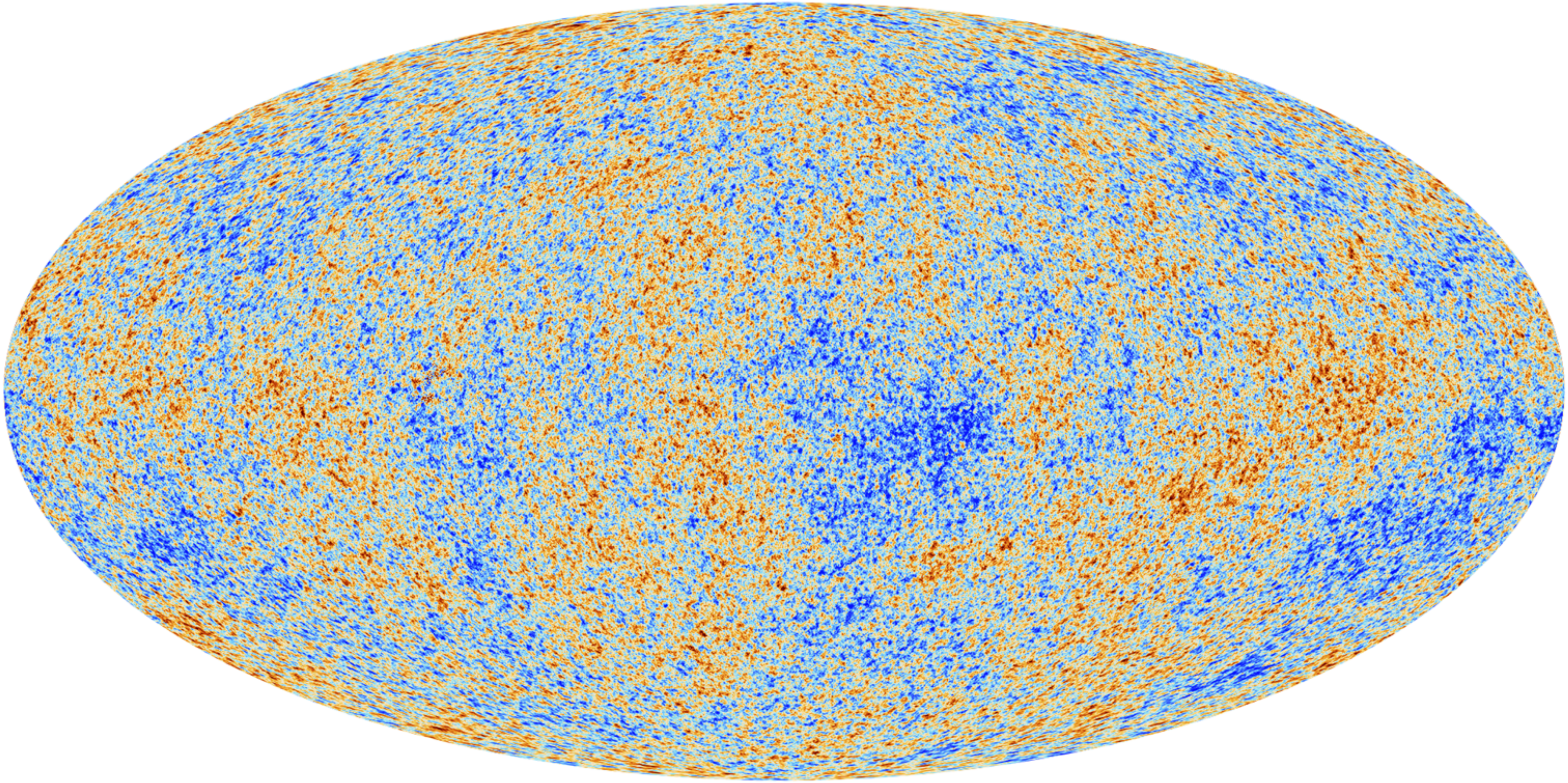
- if field excursion sub-Planckian, no measurable gravity waves: [Lyth '97]

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_R} = 16\epsilon \leq 0.003 \left(\frac{50}{N_e}\right)^2 \left(\frac{\Delta\phi}{M_\mathrm{P}}\right)^2$$

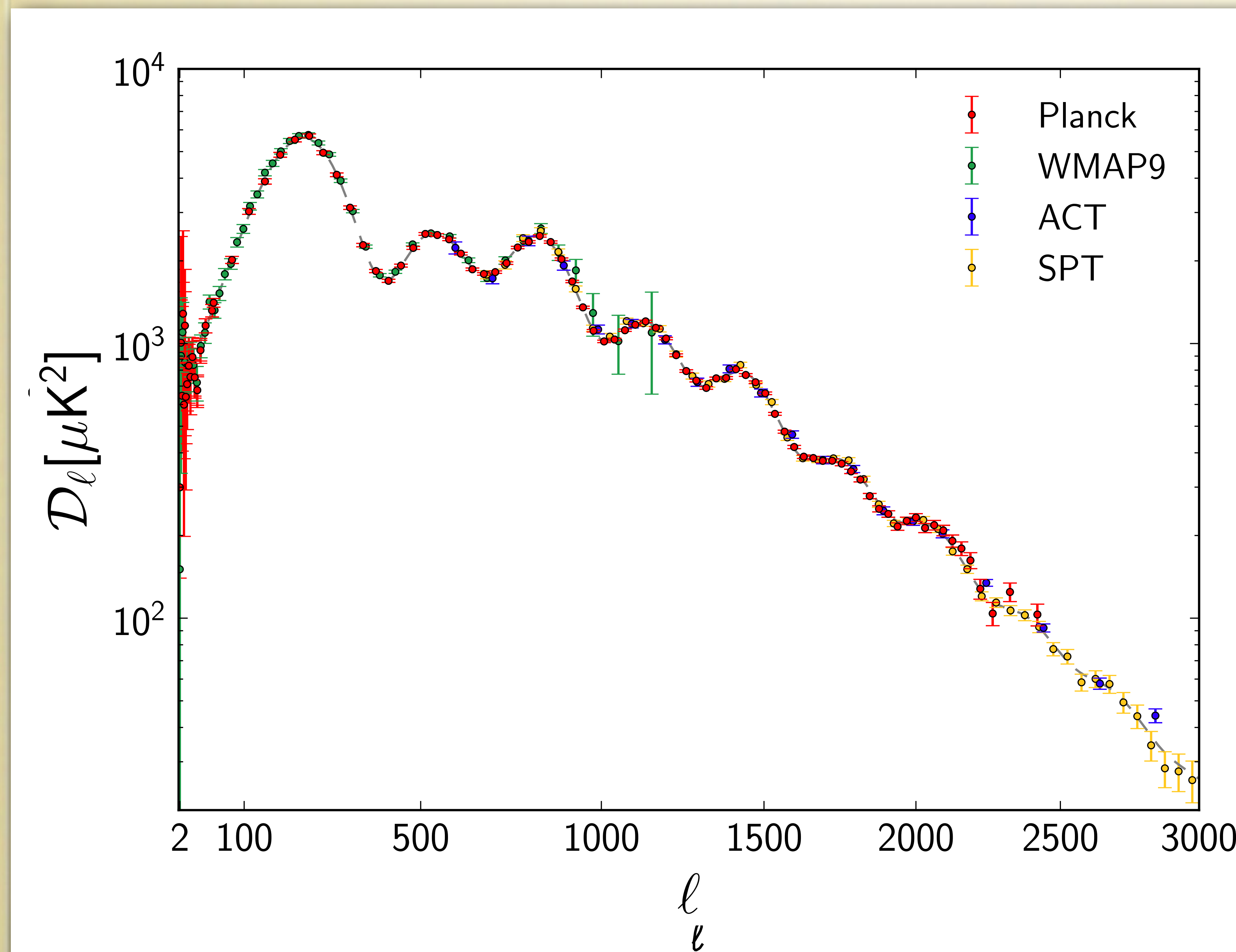
there are exceptions to this general rule:

- if ε evolves non-monotonically, *r* can be enhanced for $O(M_P)$ field range - this is non-generic & needs tuning [Ben-Dayan & Brustein '09] [Hotchkiss, Mazumdar & Nadathur '11]

- if inflaton does not provide graceful exit: hybrid inflation - then ε may decrease from initially larger values



yesterday ≈ 12 pm CET: PLANCK + WMAP + highL + BAO



$$n_s = 0.9608 \pm 0.0054 \; (68\%)$$

$$r < 0.11 \; (95\%)$$

$$\Omega_k = -0.0004 \pm 0.00036 \; (68\%)$$

$$f_{NL}^{local} = 2.7 \pm 5.8 \; (68\%)$$

$$f_{NL}^{equil} = -42 \pm 75 \; (68\%)$$

$$f_{NL}^{orth} = -25 \pm 39 \; (68\%)$$

$$N_{eff} = 3.32^{+0.54}_{-0.52} \; (95\%)$$

$$\sum m_\nu < 0.28\,\text{eV} \; (95\%)$$

15.5 months of temperature data

no B-mode/E-mode polarization yet!

full release of polarization and all 30 months of temperature data in 2014

the upshot:

inflation is *fully consistent* with *single-field slow-roll* !!

in particular, the constraint on local NG implies: effects of multiple fields can only be of %-level in f_{NL} compared to their natural size - inflation is effectively single-field

rules also out a part of ekpyrotic alternatives to inflation:

ekpyrotic/cyclic models: for 'ekpyrotic conversion'
predict: $\left|f_{NL}^{local}\right| = 5/12 \cdot c_1^2 > 10$
because $10 < c_1 < 20$ to match power spectrum
$\Rightarrow$ ruled **out** by Planck f_{NL}^{local} bound!!

alternative:
kinetic conversion
serverely constrained

single field models ...

- monomial large-field, (n = 2/3 , 1 , 2 , 3 , 4):

$$V(\phi) = \lambda M_{\rm pl}^4 \left(\frac{\phi}{M_{\rm pl}}\right)^n$$

$$n_s = 1 - \frac{n+2}{2N_e} \quad , \quad r = \frac{4n}{N_e}$$

$$\Delta\phi(N_e) = \sqrt{2nN_e}\, M_{\rm P}$$

- natural (axion) inflation:

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right]$$

$$f \gtrsim 1.5\, M_{\rm P} \quad : \quad \text{large-field } (m^2\phi^2) : \quad n_s = 1 - \frac{2}{N_e} \quad , \quad r = \frac{8}{N_e}$$

$$f \lesssim 1.5\, M_{\rm P} \quad : \quad \text{small-field} : \quad n_s \approx 1 - \frac{M_{\rm P}^2}{f^2} \quad , \quad r \to 0$$

single field models ...

- hill-top small-field:

$$V(\phi) \approx \Lambda^4 \left(1 - \frac{\phi^p}{\mu^p} + ...\right)$$

$$p = 2 \;:\; \text{large-field, fits Planck for } \mu \gtrsim 9\, M_{\mathrm{P}}$$

$$p \geq 3 \;:\; \text{small-field}: \;\; n_s = 1 - \frac{2}{N_e}\frac{p-1}{p-2} \;,\;\; r \to 0 \;,\;\; \text{fits Planck for } p \geq 4$$

- D-term hybrid inflation - disfavored if $U(\phi)$ is curving upward, like $m^2\phi^2$:

$$V(\phi,\chi) = \Lambda^4 \left(1 - \frac{\chi^2}{\mu^2}\right)^2 + U(\phi) + \frac{g^2}{2}\phi^2\chi^2$$

$$U(\phi) = \alpha_{\mathrm{h}}\Lambda^4 \ln\left(\frac{\phi}{\mu}\right)$$

$$n_s = 1 - \frac{1 + 3\alpha_h/2}{N_e}$$

$$r = \frac{8\alpha_h}{N_e}$$

single field models ...

- R+R^2 / Higgs inflation / fibre inflation in LVS string scenarios:

$$S = \int d^4x \sqrt{-g} \frac{M_{\rm pl}^2}{2}\left(R + \frac{R^2}{6M^2}\right) \quad \Rightarrow \quad V(\phi) = \frac{3M^2}{4}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2$$

$$\text{or fibre inflation}: \quad V(\phi) \sim \left(1 - \frac{4}{3} e^{-\sqrt{\frac{1}{3}}\phi}\right)$$

$$n_s = 1 - 8\frac{4N_e + 9}{(4N_e + 3)^2} \quad , \quad r = \frac{192}{(4N_e + 3)^2}$$

shades of difficulty ...

- observable tensors link levels of difficulty:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_R} = 16\epsilon \leq 0.003 \left(\frac{50}{N_e}\right)^2 \left(\frac{\Delta\phi}{M_{\rm P}}\right)^2$$ [Lyth '97]

- $r << O(1/N_e^2)$ models:

$$\Delta\phi \ll \mathcal{O}(M_P) \quad \Rightarrow$$

Small-Field inflation ... needs control of leading dim-6 operators

➡ enumeration & fine-tuning reasonable

- $r = O(1/N_e^2)$ models:

$$\Delta\phi \sim \mathcal{O}(M_P) \quad \Rightarrow$$

needs *severe* fine-tuning of all dim-6 operators, or accidental cancellations

- $r = O(1/N_e)$ models:

$$\Delta\phi \sim \sqrt{N_e} M_P \gg M_P \quad \Rightarrow$$

Large-Field inflation ... needs suppression of all-order corrections

➡ symmetry is essential!

shades of difficulty ...

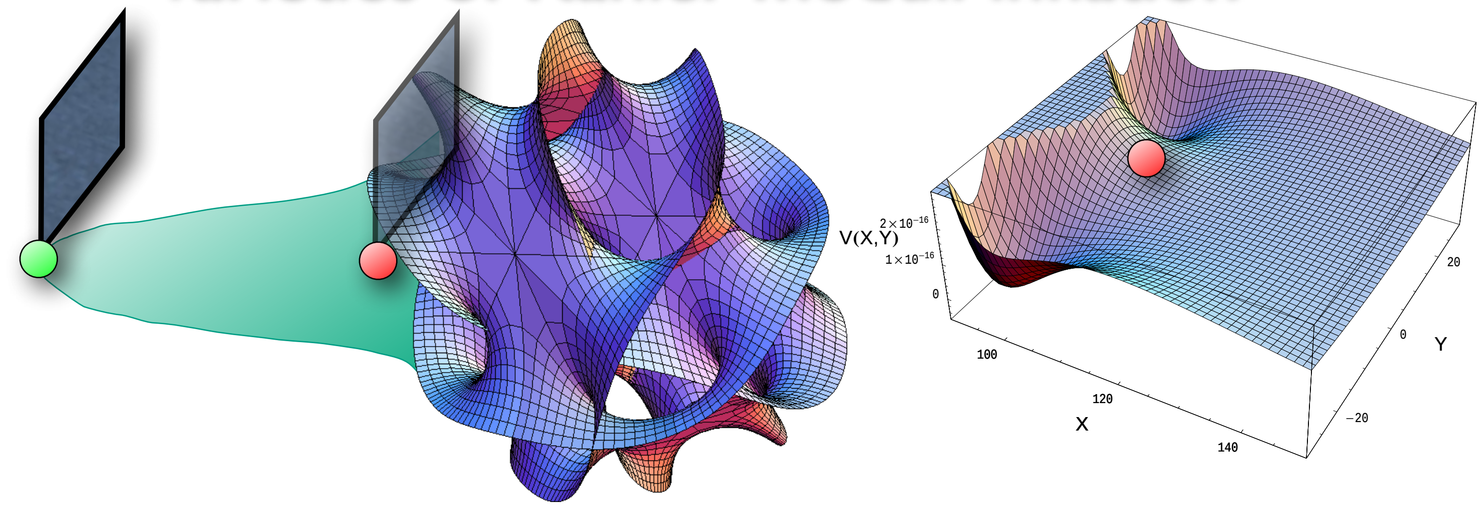
- observable tensors link levels of difficulty:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_R} = 16\epsilon \leq 0.003 \left(\frac{50}{N_e}\right)^2 \left(\frac{\Delta\phi}{M_\mathrm{P}}\right)^2$$ [Lyth '97]

- $r << O(1/N_e^2)$ models:

$$\Delta\phi \ll \mathcal{O}(M_P) \quad \Rightarrow$$

warped D-brane inflation & DBI; varieties of Kahler moduli inflation



- $r = O(1/N_e^2)$ models:

$$\Delta\phi \sim \mathcal{O}(M_P) \quad \Rightarrow$$

fibre inflation in LARGE volume scenarios (LVS)

- $r = O(1/N_e)$ models:

$$\Delta\phi \sim \sqrt{N_e} M_P \gg M_P \quad \Rightarrow$$

shades of difficulty ...

- observable tensors link levels of difficulty:

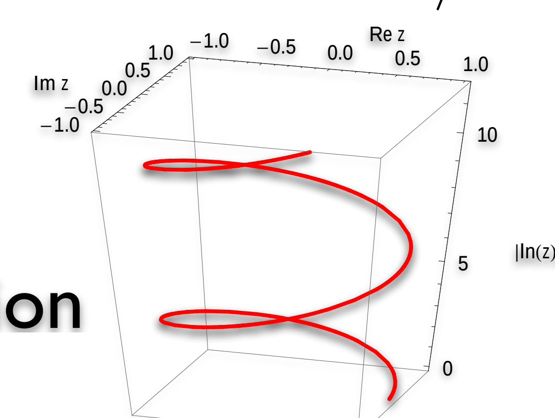
$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_R} = 16\epsilon \leq 0.003 \left(\frac{50}{N_e}\right)^2 \left(\frac{\Delta\phi}{M_{\rm P}}\right)^2$$ [Lyth '97]

observable tensors: $r > 0.01$

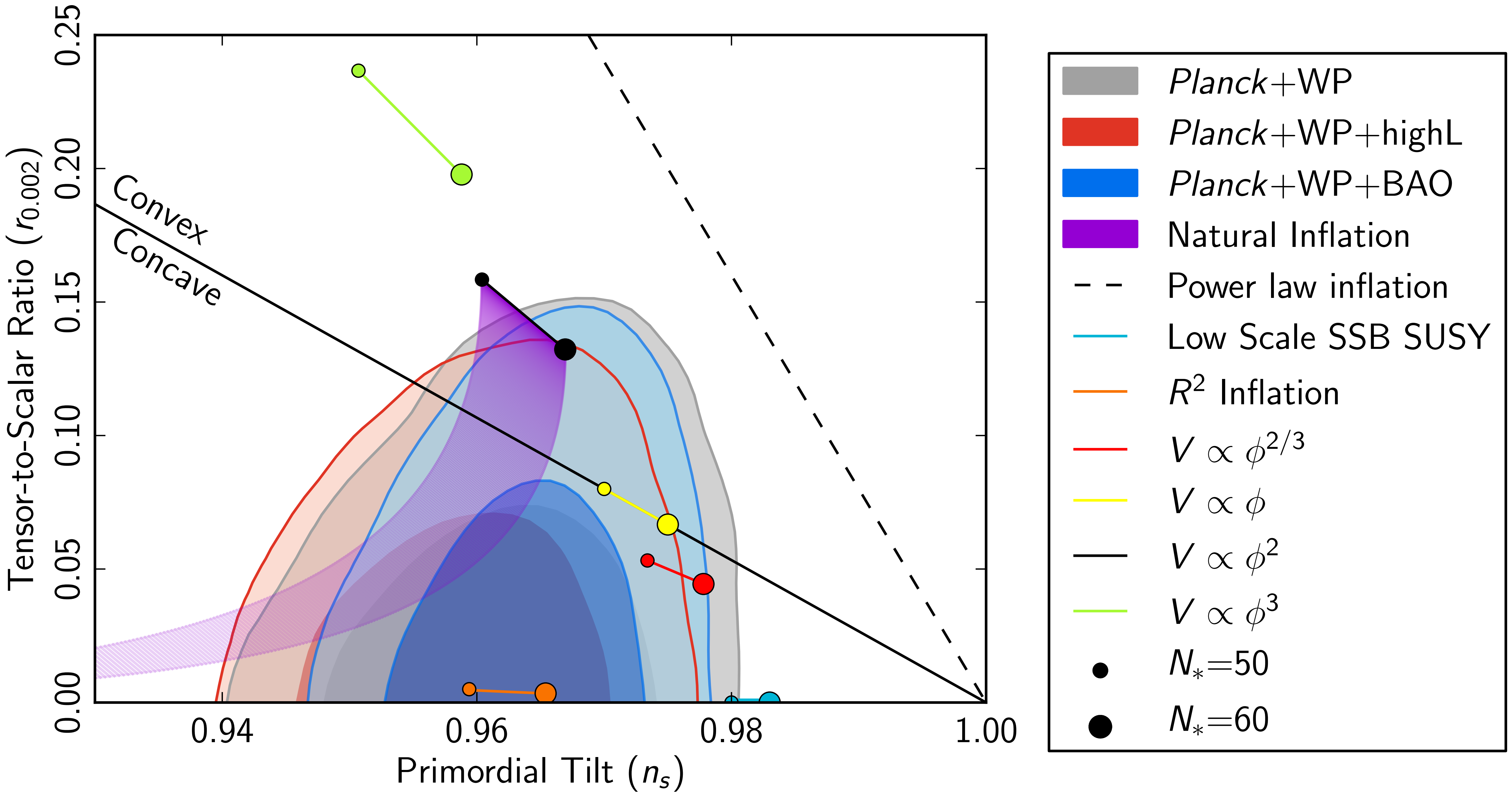
- $r = O(1/N_e)$ models:

$\Delta\phi \sim \sqrt{N_e} M_P \gg M_P \quad \Rightarrow$ axion monodromy inflation

$T_{D5/NS5}$



PLANCK ...



Tensor-to-Scalar Ratio ($r_{0.002}$)
0.00
0.05
0.10
0.15
0.20
0.25
Convex
Concave
0.94
0.96
0.98
1.00
Primordial Tilt (n_s)
Planck+WP
Planck+WP+highL
Planck+WP+BAO
Natural Inflation
Power law inflation
Low Scale SSB SUSY
R^2 Inflation
$V \propto \phi^{2/3}$
$V \propto \phi$
$V \propto \phi^2$
$V \propto \phi^3$
$N_*=50$
$N_*=60$

PLANCK ...

Planck Collaboration: Constraints on inflation

Model	Instantaneous entropy generation		Restrictive entropy generation		Permissive entropy generation	
	$\ln[\mathcal{E}/\mathcal{E}_0]$	$\Delta\chi^2_{\rm eff}$	$\ln[\mathcal{E}/\mathcal{E}_0]$	$\Delta\chi^2_{\rm eff}$	$\ln[\mathcal{E}/\mathcal{E}_0]$	$\Delta\chi^2_{\rm eff}$
$n = 4$	−14.9	25.9	−18.8	27.2	−13.2	17.4
$n = 2$	−4.7	5.4	−7.3	6.3	−6.2	5.0
$n = 1$	−4.1	3.3	−5.4	2.8	−4.9	2.1
$n = 2/3$	−4.7	5.1	−5.2	3.1	−5.2	2.3
Natural	−6.6	5.2	−8.9	5.5	−8.2	5.0
Hilltop	−7.1	6.1	−9.1	7.1	−6.6	2.4
ΛCDM	−4940.7	9808.4	...	...	...	...

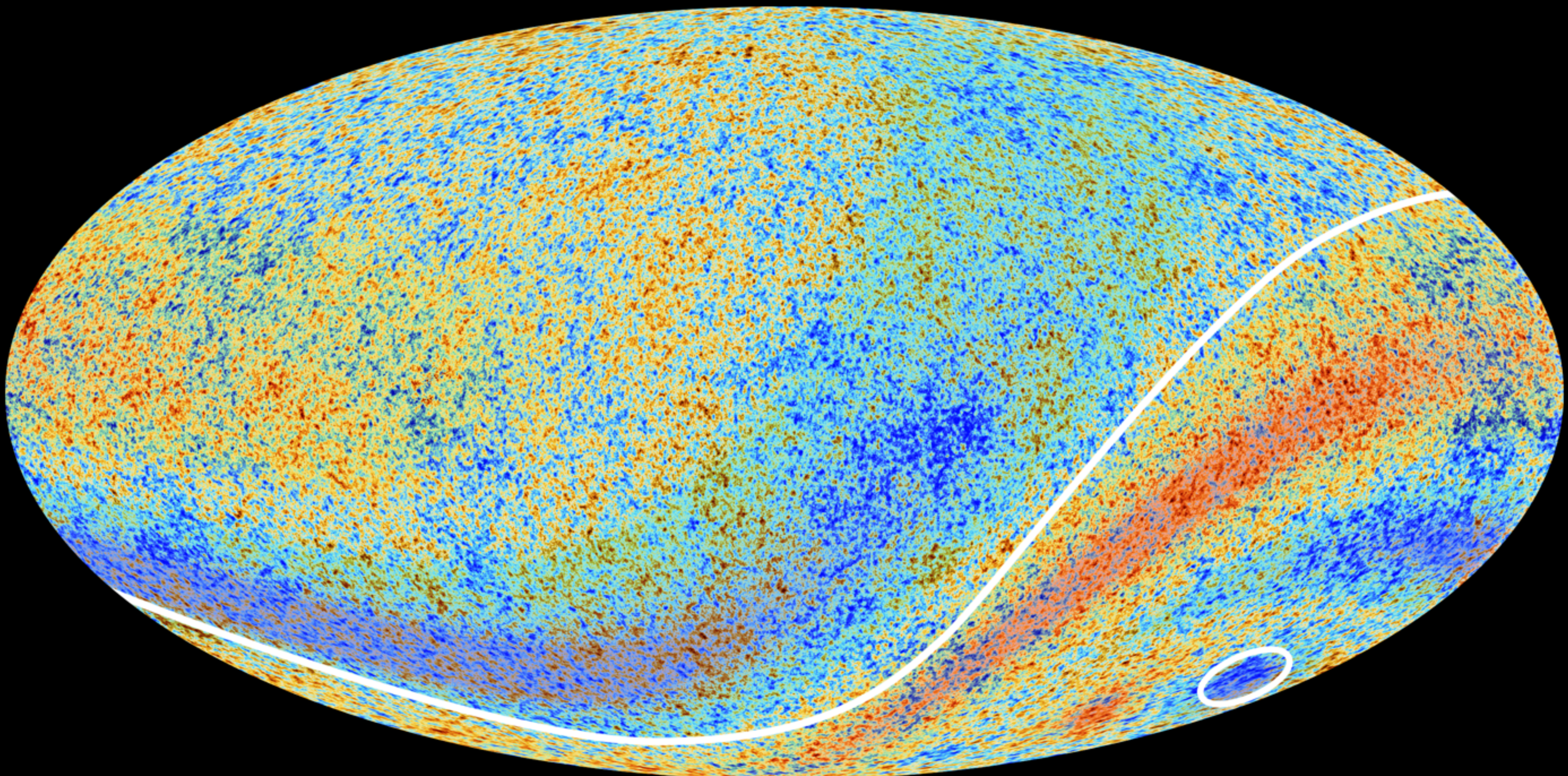
Table 7. Inflationary model comparison results. For each model and set of assumptions concerning entropy generation [(1), (2), (3)], the natural logarithm of the Bayesian evidence ratio as well as $\Delta\chi^2_{\rm eff}$ for the best-fit model in each category are indicated, relative to the ΛCDM concordance model (denoted by subscript "0"); $\ln\mathcal{E}_0$ and $-2\ln\mathcal{L}_0$ for the latter are also given.

PLANCK ... large-scale anomalies !!

- hemispheric asymmetry of mean power and temperature ~ 3 σ
- quadrupole - octopole alignment
- cold spot ~ 3 σ
- fit Planck data from high-precision data at l > 100, then predict from that power at l < 30: *too low* power at low-l, 10% deficit, ~ 2.7 σ

theory task: explain!

PLANCK ... large-scale anomalies !!



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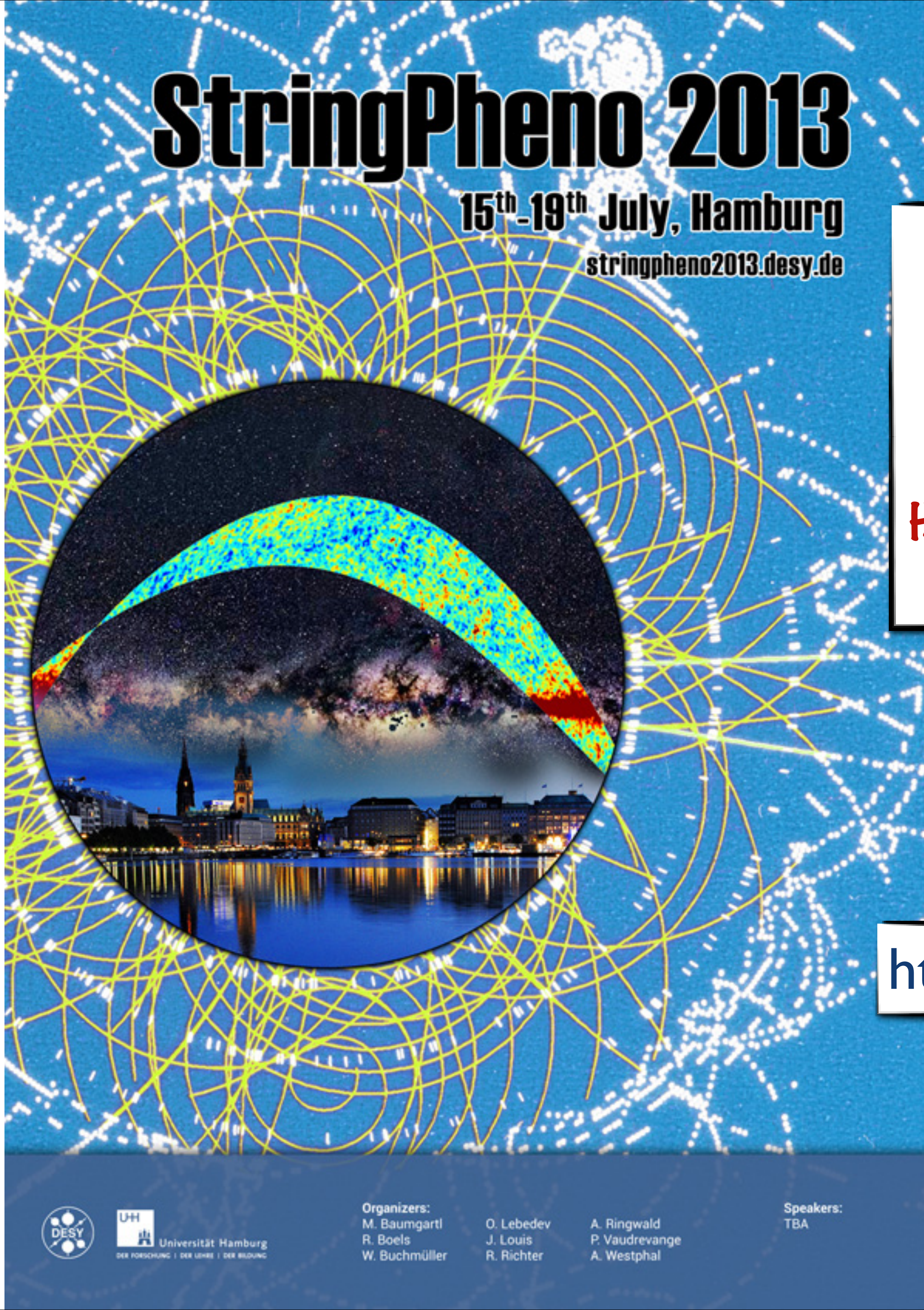

Welcome to Hamburg in July 2013!!

Hope to see you all there and then!

http://stringpheno2013.desy.de/