Chirality from Heterotic Orbifolds

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Based on:

- M. Fischer, S. Ramos-Sánchez and P. V.: 13xx.xxxx
- S. Groot Nibbelink and P. V.: 1212.4033, accepted by JHEP
- M. Fischer, M. Ratz, J. Torrado and P. V.: 1209.3906, JHEP 1301 (2013) 084

Summary

- Classification of all toroidal orbifold geometries with heterotic $\mathcal{N} = 1$

  $\Rightarrow$ 469 orbifolds

- (50 orbifold geometries with $\mathcal{N} = 2$)
- Computation of Hodge numbers based on SUSY in $d \geq 4$
- 38 orbifold geometries with $h^{1,1} = h^{2,1}$

  $\Rightarrow$ always non–chiral using standard heterotic CFT

- Magnetized orbifolds to create chirality in blow–up

Classification of Toroidal Orbifolds

Space group S

- def. S: discrete subgroup of the group of motions in $\mathbb{R}^6$ with 6 linearly independent translations
- space group elements:

$$g = (\vartheta, \lambda) \text{ for } g \in S$$

acts on $x \in \mathbb{R}^6$ as

$$x \;\mapsto\; g\,x = \vartheta x + \lambda$$

- define orbifold

$$O = \mathbb{R}^6/S \text{ where } x \;\sim\; \vartheta x + \lambda \text{ for all } g \in S$$

define point group: $\vartheta \in P$

Classification of Toroidal Orbifolds

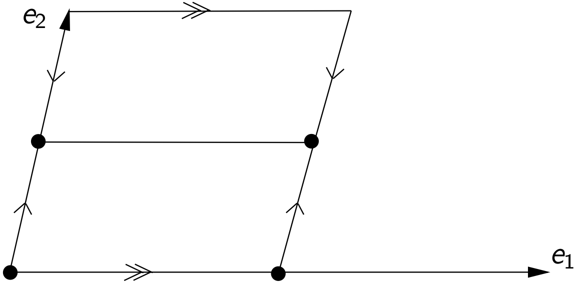
Examples in 2D

- space group generated by

  $(\mathbb{1}, e_1)$, $(\mathbb{1}, e_2)$ and $(\vartheta, 0)$

  where $\vartheta\, e_i = -e_i$

- orbifold: **pillow**



Classification of Toroidal Orbifolds

Examples in 2D

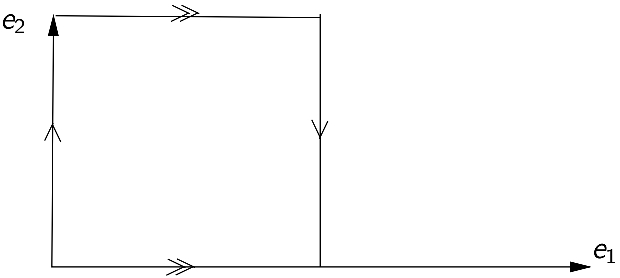
- space group generated by

  $(\mathbb{1}, e_1)$, $(\mathbb{1}, e_2)$ and $(\vartheta, \frac{1}{2}e_1)$

  where $\vartheta\, e_1 = e_1$ and $\vartheta\, e_2 = -e_2$

  roto-translation

- orbifold: **Klein bottle**



Classification of Toroidal Orbifolds

Classification

- CARAT: all space groups in up to 6 dim.
- get $\mathbb{Q}$ classes (point groups) $\Rightarrow$ 7103 point groups in 6D
- check $\mathcal{N} \geq 1$ using GAP
- create $\mathbb{Z}$-classes (lattices)
- create affine classes (inequivalent space groups)

Classification of Toroidal Orbifolds

Summary

- space group S
- lattice Λ
- point group P
- orbifolding group G (includes roto-translations)
- equivalences and SUSY
- results of classification:

60 inequivalent point groups with $\mathcal{N} \geq 1$
186 inequivalent lattices
520 inequivalent toroidal orbifolds with $\mathcal{N} \geq 1$
↳ 162 with Abelian point group
  ↳ 138 with $\mathcal{N} = 1$
↳ 358 with non-Abelian point group
  ↳ 331 with $\mathcal{N} = 1$

Classification of Toroidal Orbifolds

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- results of classification:

22 Abelian (known)
like $\mathbb{Z}_3$, $\mathbb{Z}_4$, $\mathbb{Z}_6 - I$, ...
38 non-Abelian (mostly unknown)
like S_3, D_4, A_4, ...

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e.g. many non-factorizable ones like Lie lattice $F_4 \times SU(3)$ or E_6, but many unknown more, i.e. not all lattices are Lie lattices

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Classification of Toroidal Orbifolds

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- results of classification:

including roto-translations
only classified for $\mathbb{Z}_2 \times \mathbb{Z}_2$ before

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non-local GUT breaking:
21 space groups from $\mathbb{Z}_2 \times \mathbb{Z}_2$
6 space groups from $\mathbb{Z}_2 \times \mathbb{Z}_4$
4 space groups from $\mathbb{Z}_3 \times \mathbb{Z}_3$

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Hodge numbers $(h^{1,1}, h^{2,1})$

- Standard embedding $\Rightarrow$ # **27** and $\overline{\mathbf{27}}$ gives Hodge numbers
- Many more heterotic orbifold compactifications due to gauge embedding
  (shift, Wilson lines, discrete torsion $\Leftrightarrow$ "flux background")
- For Abelian orbifolds: use "orbifolder"

  H.P. Nilles, S. Ramos-Sánchez, P. V. and A. Wingerter 2011
- For non–Abelian orbifolds: use SUSY in $d \geq 4$

Hodge numbers $(h^{1,1}, h^{2,1})$ from SUSY in $d \geq 4$

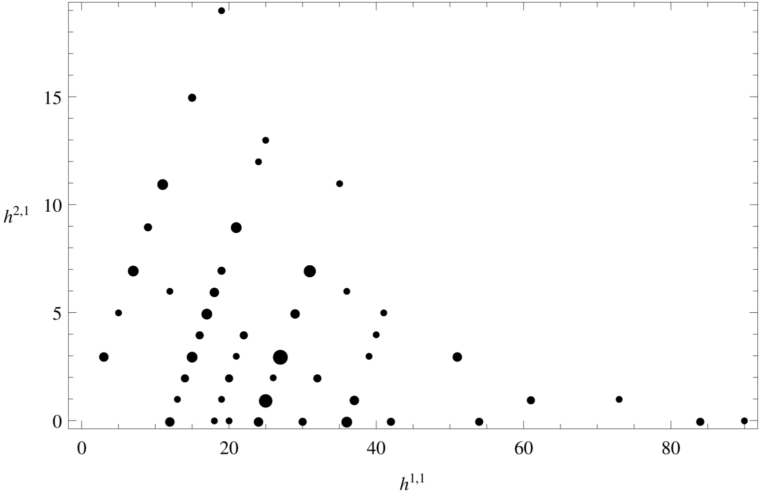
- $(h^{1,1}, h^{2,1})$ from untwisted and twisted sectors
- Untwisted sector: count invariant moduli
- Twisted sectors: fixed points and fixed tori:

$$\begin{aligned} \mathcal{N} = 2 \text{ in 4D} &\Rightarrow (1,1) \\ \mathcal{N} = 1 \text{ in 4D} &\Rightarrow (1,0) \end{aligned}$$

- Fixed point: 4D $\mathcal{N} = 1 \Rightarrow (1,0)$
- Fixed torus: 6D $\mathcal{N} = 1$ equal to 4D $\mathcal{N} = 2 \Rightarrow (1,1)$
  But, further orbifold action:

$$\begin{aligned} \text{fixed torus is orbifolded} &\Rightarrow \mathcal{N} = 1 \text{ in 4D, hence } (1,0) \\ \text{fixed torus is not orbifolded} &\Rightarrow \mathcal{N} = 2 \text{ in 4D, hence } (1,1) \end{aligned}$$

$h^{2,1}$ vs. $h^{1,1}$ for Abelian orbifold geometries



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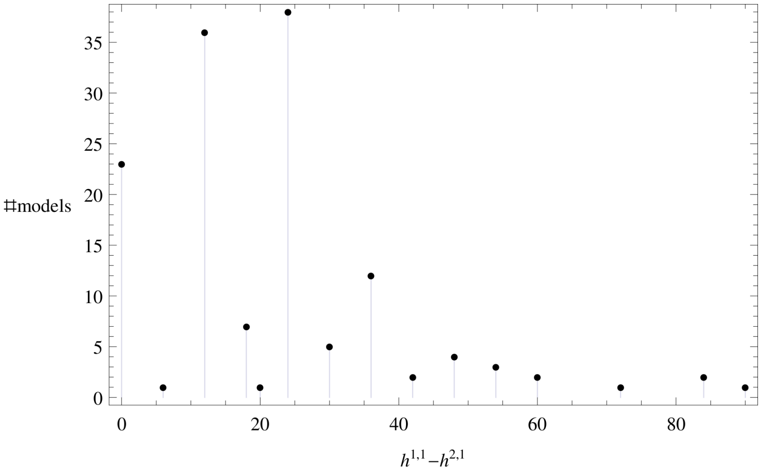
Number of generations for Abelian orbifold geometries

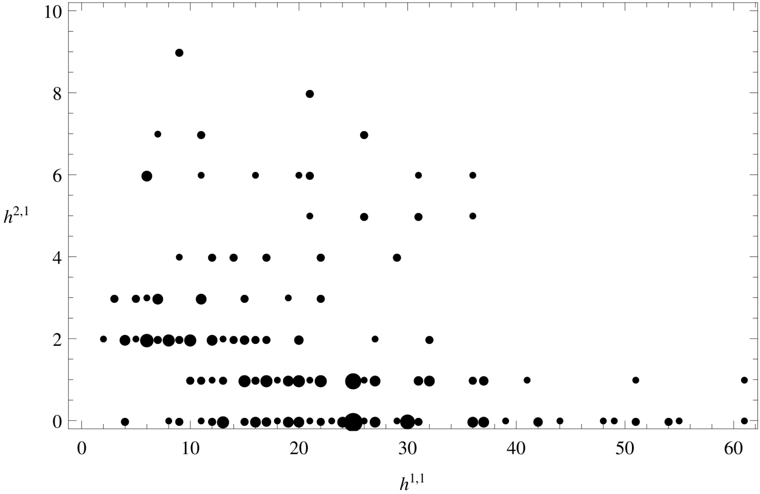


M. Fischer, M. Ratz, J. Torrado and P. V. 2012

Number of generations for Abelian orbifold geometries

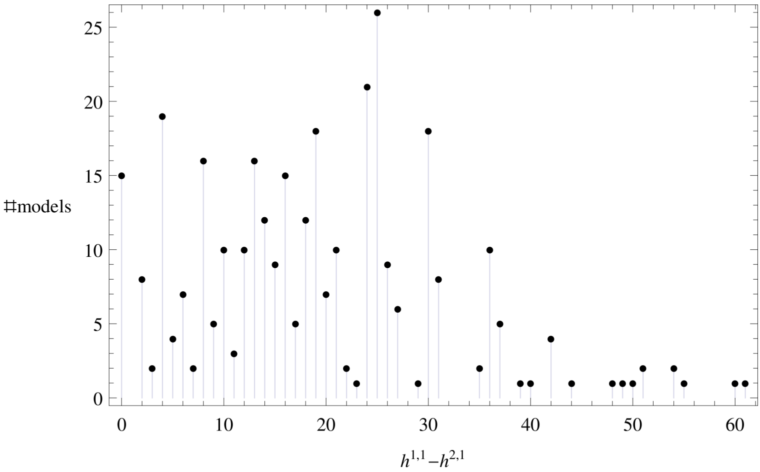
- $h^{1,1} - h^{2,1}$ always divisible by six
- Only exception: $(h^{1,1}, h^{2,1}) = (20, 0)$
- No geometry with three generations
  $\Rightarrow$ discrete Wilson lines needed for three generations

$h^{2,1}$ vs. $h^{1,1}$ for non–Abelian orbifold geometries



(preliminary) M. Fischer, S. Ramos-Sánchez and P. V. 2013

Number of generations for non–Abelian orbifold geometries



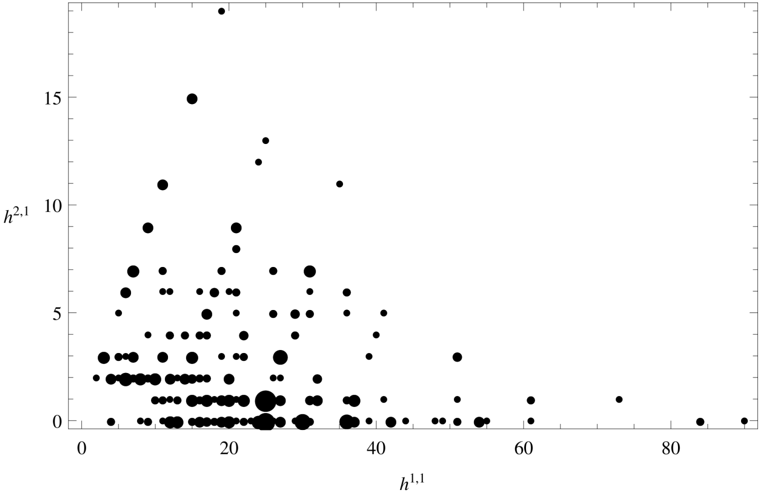
(preliminary) M. Fischer, S. Ramos-Sánchez and P. V. 2013

Number of generations for Abelian orbifold geometries

- No pattern for $h^{1,1} - h^{2,1}$
- Two geometries with three generations:

$$
\begin{aligned}
T^6/A_4 & \quad \text{orbifold with} \quad (h^{1,1}, h^{2,1}) = (6,3) \\
T^6/S_4 & \quad \text{orbifold with} \quad (h^{1,1}, h^{2,1}) = (5,2)
\end{aligned}
$$

$h^{2,1}$ vs. $h^{1,1}$ for all orbifold geometries



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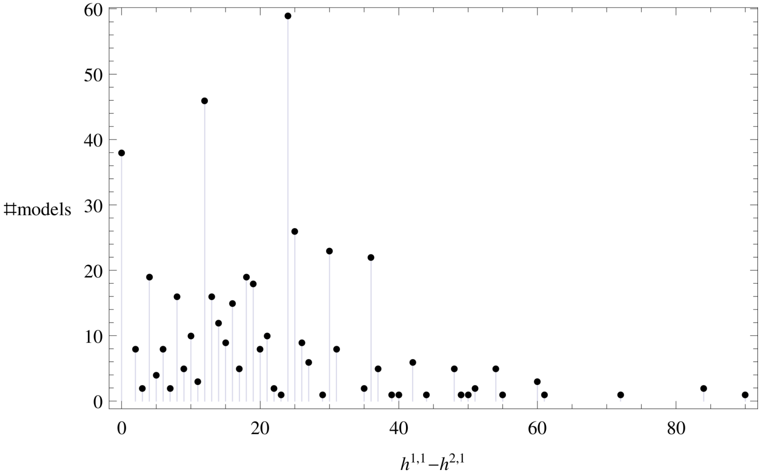

(preliminary) M. Fischer, S. Ramos-Sánchez and P. V. 2013

Number of generations for all orbifold geometries

- 38 orbifold geometries with $h^{1,1} = h^{2,1}$:

23	$T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$	orbifolds
11	T^6/S_3	orbifolds
1	T^6/D_4	orbifold
3	T^6/A_4	orbifolds

- Here: chiral spectrum not possible using standard CFT
- Remark:
  (heterotic) $h^{1,1} = h^{2,1}$ implies (spontaneously broken) $\mathcal{N} = 2$

  A. Kashani-Poor, R. Minasian, H. Triendl arXiv:1301.5031
- Conjecture: $h^{1,1} = h^{2,1}$ and $h^{1,1}_T = h^{2,1}_T = 0$ implies (spontaneously broken) $\mathcal{N} = 4$

Schoen manifold with line bundles as resolved magnetized orbifold

S. Groot Nibbelink and P. V. 2012

- Schoen manifold = resolved $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with roto–translation (DW(0-2)) R. Donagi and K. Wendland 2008
- Heterotic DW(0-2) $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold always non–chiral (SUSY in $d \geq 4$)
- Blow–up thereof also non–chiral

Schoen manifold with line bundles as resolved magnetized orbifold

S. Groot Nibbelink and P. V. 2012

- But: heterotic Schoen manifold with line bundles can be chiral

  using $\mathrm{SU}(5)$ bundles: e.g. R. Donagi, B. Ovrut, T. Pantev and D. Waldram 2000
- Way–out: resolved $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with magnetized tori (flux on i-th torus B_i, $i = 1, 2, 3$)
- Description only known in SUGRA limit
- Conjecture how torus–flux enters CFT modular invariance and string mass conditions:

$$V^2 = \frac{3}{2} \text{ mod } 1 \quad \Rightarrow \quad V^2 = \frac{3}{2} + \frac{1}{4} B_i \cdot B_3 \text{ mod } 1$$

$$\frac{1}{2}(p+V)^2 + \tilde{N} - \frac{3}{4} = 0 \quad \Rightarrow \quad \frac{1}{2}(p+V)^2 + \tilde{N} - \frac{3}{4} - \frac{1}{8} B_i \cdot B_3 = 0$$

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