Explicit complex structure moduli stabilization in IIB flux compactifications

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Outline:

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3. Scanning all vacua with paramotopy
4. Kähler uplifted de Sitter vacua
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1. Introduction: De Sitter vacua in type IIB

Motivation: Construct explicit de Sitter vacua in type IIB/F-theory

Cosmology:

- Acceleration of the universe on large scales is observed.
- Simplest explanation: dS space with small cosmological constant.

The setup we use:

- Non-perturbative effects $W = A_i\, e^{-a_i\, T_i}$
  [Kachru,Kallosh,Linde,Trivedi '03]
- Leading α'-corr. to the Kähler pot. $K = -2\ln(\hat{\mathcal{V}}(T_i) + \alpha'^3 \hat{\xi}(\tau))$
  [Becker,Becker,Haack,Louis '02]
- Quantized RR and NS-NS fluxes $\int F_3, H_3 \in \mathbb{Z}$
  $\Rightarrow$ **SUSY vacua for the U_a and τ** [Giddings,Kachru,Polchinski '01]

Moduli stabilization in the large volume limit

$\mathcal{N} = 1$, 4D effective potential:

$$V = e^K \left(K^{\alpha\bar{\beta}} D_\alpha W \overline{D_\beta W} - 3|W|^2 \right)$$

For $\hat{\xi} \ll \hat{\mathcal{V}}$ this is to 0-th order the positive semi-definite potential

[Balasubramanian,Berglund,Conlon,Quevedo'05],[Westphal,MR'11]

$$V_{flux} = e^K \left(K^{\tau\bar{\tau}} \left| D_\tau W_0 \right|^2 + K^{a\bar{b}} D_a W_0 \overline{D_b W_0} \right) + \mathcal{O}\left(\frac{\hat{\xi}}{\hat{\mathcal{V}}}\right)$$

due to no-scale structure [Cremmer,Ferrara,Kounnas,Nanopoulos'83]

- **Every SUSY extremum for the τ, U_a is a minimum for $\hat{\mathcal{V}} \gg \hat{\xi}$!**
- **Separation of scales $\Rightarrow$ Stabilize U_a first!**

2. The complex structure moduli of $\mathbb{CP}_{11169}[18]$

$$\mathbb{CP}_{11169}: \ (x_1, x_2, x_3, x_4, x_5) \sim (\lambda x_1, \lambda x_2, \lambda x_3, \lambda^6 x_4, \lambda^9 x_5)$$

e.g. $\quad x_1^{18} + x_2^{18} + x_3^{18} + x_4^3 + x_5^2 = 0 \qquad (h^{1,1} = 2 \text{ and } h^{2,1} = 272)$

Complex structure moduli stabilization of $\mathbb{CP}_{11169}[18]$ (I)

Need to find $f = \int F_3 \in \mathbb{Z}$ and $h = \int H_3 \in \mathbb{Z}$ with

$$D_a W = 0\,, a = 1, .., h^{2,1} = 272$$ [Giddings,Kachru,Polchinski'01]

Effective theory completely determined by $\mathcal{N} = 2$ prepotential:

$$\mathcal{G}(U_1, \ldots, U_{h^{2,1}}) = \sum_{i+j\leq 3} c_{ij} U_a^i U_b^j + \xi + \mathcal{G}_{\text{instanton}}(e^{-2\pi U_1}, \ldots, e^{-2\pi U_{h^{2,1}}})$$

- $W_0 = \int (F_3 - \tau H_3) \wedge \Omega(U_a) = (f - \tau h) \cdot (\mathcal{G}_{U_a}, U_a)$ [Gukov,Vafa,Witten'00]
- $K_{cs} = -\ln \left[i(\bar{U}_a \mathcal{G}_{U_a} - U_a \bar{\mathcal{G}}_{U_a}) \right]$
- **Large complex structure limit:** $\left| \dfrac{\mathcal{G}_{\text{instanton}}}{\mathcal{G}_{\text{cubic}}} \right| \leq \epsilon_{LCS}$

Complex structure moduli stabilization of $\mathbb{CP}_{11169}[18]$ (II)

- Discrete symmetry $\mathbb{Z}_6 \times \mathbb{Z}_{18}$: U_a , $a = 1, .., h^{2,1}_{\text{inv.}} = 2$ and $\tilde{U}_a$, $a = 3, .., 272$ [Greene,Plesser'89], [Candelas,Font,Katz,Morrison'94]
- Switch on flux only on $h^{2,1}_{\text{inv.}} = 2$ [Giryavets,Kachru,Tripathy,Trivedi '03]
- Symmetry $\Rightarrow D_{\tilde{U}_a} W_0 = 0$ at $\tilde{U}_a = 0$ for $a = 3, \ldots, 272$
- **$\Rightarrow$ Only need to solve $\mathbf{D_\phi W|_{\tilde{U}_a=0} = 0}$ for $\boldsymbol{\phi = \tau, U_1, U_2}$**
- $\Rightarrow V \sim |D_a W|^2$ ensures stable minimum for all 272 U_a in the large volume limit!

3. Scanning all vacua with paramotopy

Paramotopy

see e.g. [Li '03], [Sommese,Wampler '05]

Have to solve polynomial system $P(x) = (p_1(x), \ldots, p_m(x))^T = 0$ with $x = (x_1, \ldots, x_m)^T$

- Maximal number of isolated solutions in $\mathbb{C}^m$: $\prod_{i=1}^{m} d_i$, with d_i the degree of the ith polynomial. (*Classical Bézout bound*)

- Construct homotopy $H(x,t) = \gamma(1-t)Q(x) + t\, P(x)$, with e.g. $Q(x) = \left(x_1^{d_1} - 1, \ldots, x_m^{d_m} - 1\right)$ $\Rightarrow$ **Easy to solve.**

- Follow paths $H(x,t) = 0$ for $0 \leq t \leq 1$ for all solutions to $Q(x) = 0$

$\Rightarrow$ **Will find all solutions to P(x) = 0**

The scan

Use to solve $D_\phi W(f) = 0$ for all fluxes f with $L \sim f^2 \leq L_{max}$

[Martinez-Pedrera,Mehta,Westphal,MR '12]

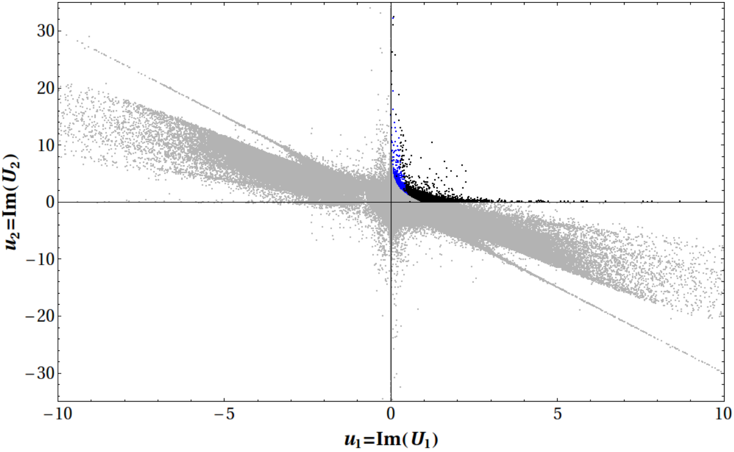
- The 10D IIB action is $SL(2,\mathbb{Z})$ invariant: $\tau \to \frac{a\tau + b}{c\tau + d}$,

  $G_3 = F_3 - \tau H_3 \to \frac{G_3}{c\tau + d}$ with $a, b, c, d \in \mathbb{Z}$, $ad - bc = 1$

- Make sure to only consider physically inequivalent configurations!

- Our scan: $\sim$ 50.000 parameter points ($L \leq 35$).

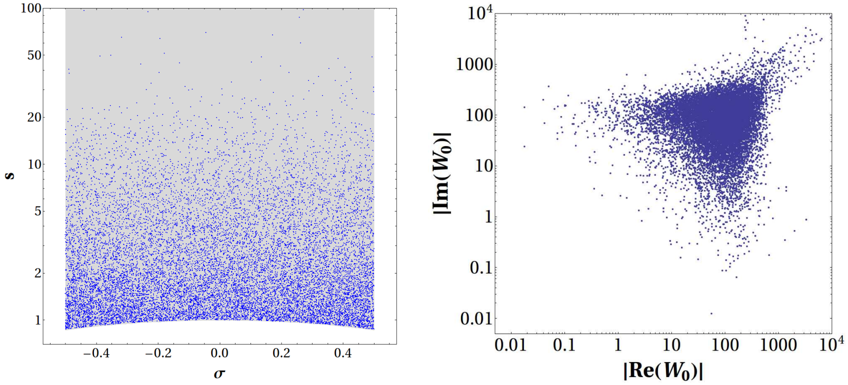
Scan results: The limit of large complex structure



$$\left|\frac{\mathcal{G}_{\text{instanton}}}{\mathcal{G}_{\text{cubic}}}\right| \leq \epsilon_{LCS} = \begin{cases} 10^{-1} & \text{(blue)} \Rightarrow 25.000 \text{ of } 500.000 \\ 10^{-2} & \text{(black)} \Rightarrow 15.000 \text{ of } \quad '' \end{cases}$$

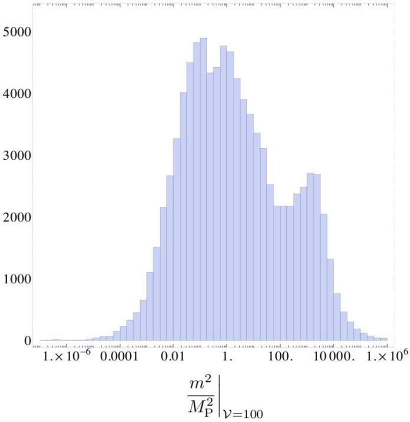
Scan results: $\tau = \sigma + i\,s$ and W_0

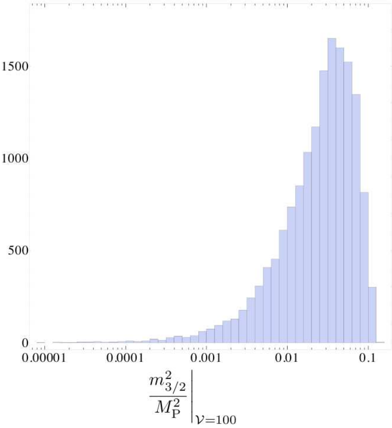
- $SL(2,\mathbb{Z})$ fundamental domain: $-\frac{1}{2} \leq \mathrm{Re}(\tau) \leq \frac{1}{2}$ and $|\tau| > 1$
- Transformations: $\tau \to \tau + b\,,\ G_3 \to G_3$ and $\tau \to -1/\tau\,,\ G_3 \to G_3/\tau$



Scan results: Mass scales

Masses $\left.\frac{M^2}{M_{\mathrm{P}}^2}\right|_{\hat{\mathcal{V}}=100}\left(\frac{100}{\hat{\mathcal{V}}}\right)^2$ for 6 real moduli fields and gravitino:

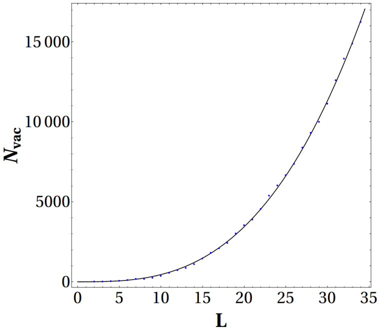




Scan results: Number of vacua N_{vac}

- $N_{vac}^{\text{stat}} = \frac{(2\pi L)^3}{3!} \int \det(-\mathcal{R} - \mathbf{1} \cdot \omega)$
  $\simeq 0.03\, L^3$
  [Ashok,Douglas'04],[Denef,Douglas,Florea'04]

- $N_{vac} \simeq (0.50 \pm 0.04)\, L^{2.94 \pm 0.03}$



Scan results: Tuning of the cosmological constant

- Untuned cosmological constant (cc): $\Lambda \sim \frac{m_{3/2}^2}{\hat{\mathcal{V}}}$
- Spacing in cc: $\frac{\Delta\Lambda}{\Lambda} \sim 2\,\frac{\Delta m_{3/2}}{m_{3/2}} \sim \frac{C}{L^{a\,(h^{2,1}_{\rm eff}+1)}}$
- Fit, **Extrapolate** $\Rightarrow \frac{\Delta\Lambda}{\Lambda} \simeq (6.0 \pm 0.3)\, L^{-(0.95\pm0.005)\,(h^{2,1}_{\rm eff}+1)}$

$h^{2,1}_{\rm eff}$	L	$\Delta\Lambda/\Lambda$
2	34	$7 \cdot 10^{-3} \pm 5 \cdot 10^{-4}$
2	500	$5 \cdot 10^{-5} \pm 4 \cdot 10^{-6}$
40	34	$3 \cdot 10^{-58} \pm 2 \cdot 10^{-58}$
40	500	$10^{-102} \pm 10^{-102}$

4. Kähler uplifted de Sitter vacua

Uplifting to de Sitter

$W_0 \ll 1$, α'-correction negligible [KKLT '03]

$W_0 \neq 0$, α'-correction [Balasubramanian,Berglund '05]

$\hat{\mathcal{V}} >>> \hat{\xi}$, W_0 arbitrary → LVS

$\hat{\mathcal{V}} >> \hat{\xi}$, $W_0 \sim \mathcal{O}(1-100)$ → Kähler uplifting

KKLT

- $\bar{D}3$ branes
- F-terms from matter fields [Lebedev,Nilles,Ratz'06]
- F-terms from metastable vacua in gauge theories [Intriligator,Seiberg,Shih'07]

LVS [Balasubramanian, Berglund,Conlon,Quevedo '05]

- $\bar{D}3$ branes
- D-terms [Burgess,Kallosh,Quevedo'03, Haack,Krefl,Lüst,Van Proyen,Zagermann'06]
- [Cicoli,Krippendorf,Mayrhofer,Quevedo,Valandro'12] F-terms from dilaton dep. non-pert. effects [Cicoli,Maharana,Quevedo,Burgess'12]

Kähler uplifting [Westphal '06]

- F-terms from Kähler moduli + α'-correction sufficient for dS

Kähler moduli: Constraints on a consistent global model

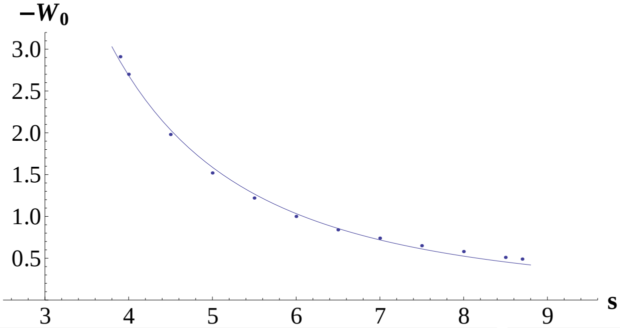
- Contribution of gaugino condensation to the superpotential, $A \neq 0$:
  - Rigid divisor? [Witten'96]
  - Can it be 'rigidified' by gauge flux $\mathcal{F}$? [Martucci'06, Bianchi,Collinucci,Martucci'11]
- Swiss-cheese?
- $N_1 \gg 1$ enforces factorization of D7 brane equation in coordinates $u_i \neq u_1$ [Cicoli,Mayrhofer,Valandro'11]?
- Flux: Freed-Witten anomalies?[Minasian,Moore'96, Freed,Witten'97]
- Chiral matter at brane intersections that might destroy $A \neq 0$ [Blumenhagen,Moster,Plauschinn'08]?
- Stabilization inside the Kähler cone?
- D3 tadpole: $Q^{D7-\text{stacks}} + Q^{O7} = Q^{\mathcal{F}} + Q^{RR,NS-NS} + Q^{D3-\text{branes}}$?

Kähler uplifted de Sitter vacua (I)

Global model for Kähler moduli stabilization in a de Sitter vacuum on $\mathbb{CP}_{11169}[18]$ via non-perturbative effects: [Louis,Valandro,Westphal,MR '12]

$$V = e^K \left(K^{T_i \bar{T}_j} \left[W_{T_i} \overline{W_{T_j}} + W_{T_i} \cdot \overline{W K_{T_j}} \right] + 3\hat{\xi} \frac{\hat{\xi}^2 + 7\hat{\xi}\hat{\mathcal{V}} + \hat{\mathcal{V}}^2}{(\hat{\mathcal{V}} - \hat{\xi})(\hat{\xi} + 2\hat{\mathcal{V}})^2} |W|^2 \right)$$

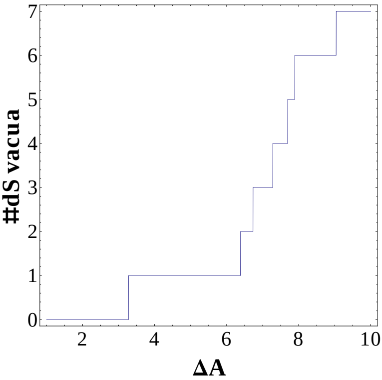
Numerically: Stable de Sitter solutions for $A_1, A_2 \sim 1$

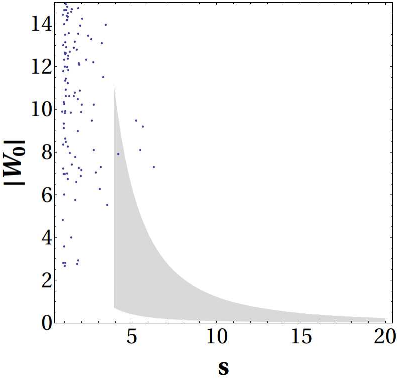


Kähler uplifted de Sitter vacua (II)

Parametrize missing knowledge of A_1, A_2 by ΔA with scaling relations:

$$W_0 \to W_0 \cdot \Delta A\,,\;\; A_1 \to A_1 \cdot \Delta A\,,\;\; A_2 \to A_2 \cdot \Delta A \;\;\Rightarrow V \to V \cdot \Delta A^2$$





(Uplifting can be applied in the shaded region for $\Delta A = 4$)

5. Conclusions

Conclusions

- *All* solutions to polynomial equations can be found using Paramotopy (highly parallelizable! $\Rightarrow$ 3000 days on 3000 cores $\Rightarrow$ 1 day!)
- $\Rightarrow$*All* flux vacua of reduced moduli space have been constructed for given D3-tadpole ($L = 35$) in the large complex structure limit
- $g_s \lesssim 1$ and $W_0 \sim \mathcal{O}(10^1 - 10^3)$ are preferred in our solutions
- N_{vac} and $\Delta\Lambda/\Lambda \sim 10^{-100}$ for $h^{2,1}_{\text{eff}} = 40$ and $L = 500$ consistent with semi-analytical predictions [Ashok,Douglas'04],[Denef,Douglas,Florea'04]
- Consistent global model of Kähler uplifting on $\mathbb{CP}_{11169}[18]$ $\Rightarrow\sim 10^{-4}$ flux vacua can be uplifted