Inflation in the Wigner landscape

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Arxiv:1303.3224

DESY

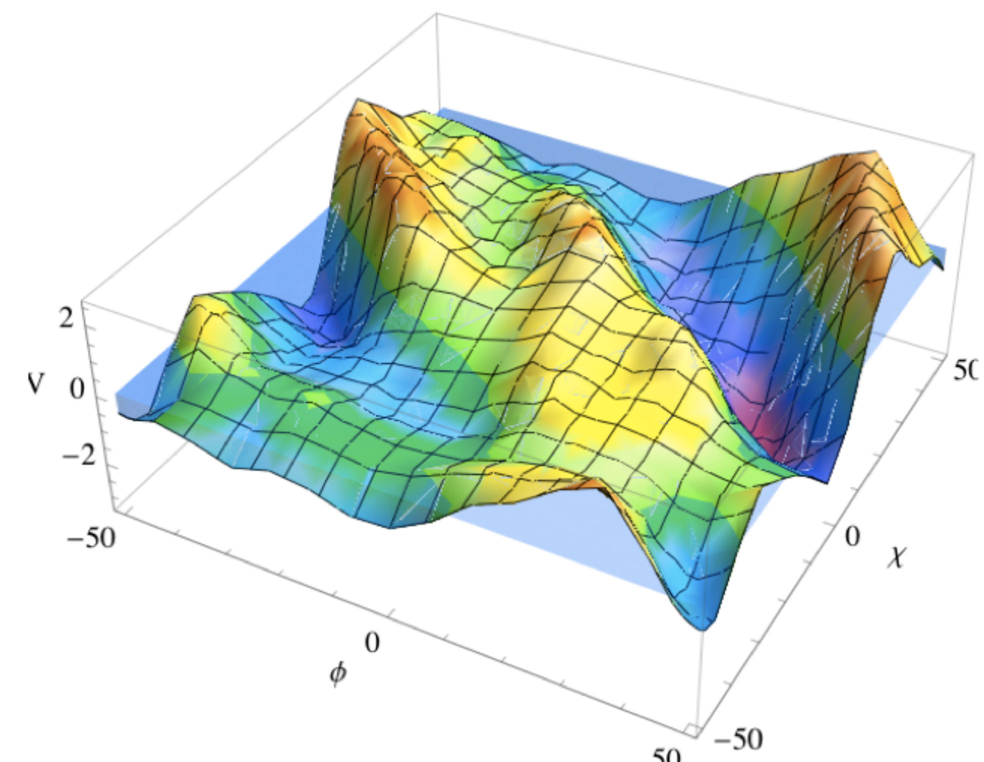
Outline:

- The string landscape
- Random Matrices in physics
- Random supergravity
- Critical points
- Inflation near critical points

The string landscape

Many choices:

- Compactification manifold
- Fluxes
- Brane configuration
- Non-perturbative effects
- Gauge groups
- ...



from [1101.1619]

Looking for vacua<=>scanning over vast parameter space

The string landscape

?Is there an efficient way to do this?

?What general knowledge can be extracted?

?Statistical description of inflation?

Look for generic features using Random Matrix Theory

Random Matrix Theory in physics:

- Nuclear energy levels
- Condensed matter systems

Random SUGRA

F-term potential:

$$V = e^K \left(F_A \bar{F}^A - 3|W|^2 \right)$$

Critical point condition:

$$\partial_A V|_{cp} = 0$$

Hessian matrix:

$$\mathcal{H} = \underbrace{\mathcal{H}_{SUSY} + \mathcal{H}_{K^{(3)}}}_{Wishart+Wishart} + \underbrace{\mathcal{H}_{pure} + \mathcal{H}_{K^{(4)}}}_{Wigner} + \mathcal{H}_{shift}$$

[Ashok&Douglas 2003;Denef&Douglas 2004;Conlon&Quevedo 2004;
Marsh et al.2011/12;Martinez-Pedrera et al 2012]

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Random SUGRA

[1112.3034]

$$\mathcal{H}_{\rm susy} = \begin{pmatrix} Z_a{}^{\bar c}\, \bar Z_{\bar b \bar c} & 0 \\ 0 & \bar Z_{\bar a}{}^{c}\, Z_{bc} \end{pmatrix},$$

$$\mathcal{H}_{\rm pure} = \begin{pmatrix} 0 & U_{ab1}\bar F^{1} - Z_{ab}\overline{W} \\ \overline{U}_{\bar a \bar b \bar 1} F^{\bar 1} - \bar Z_{\bar a \bar b} W & 0 \end{pmatrix},$$

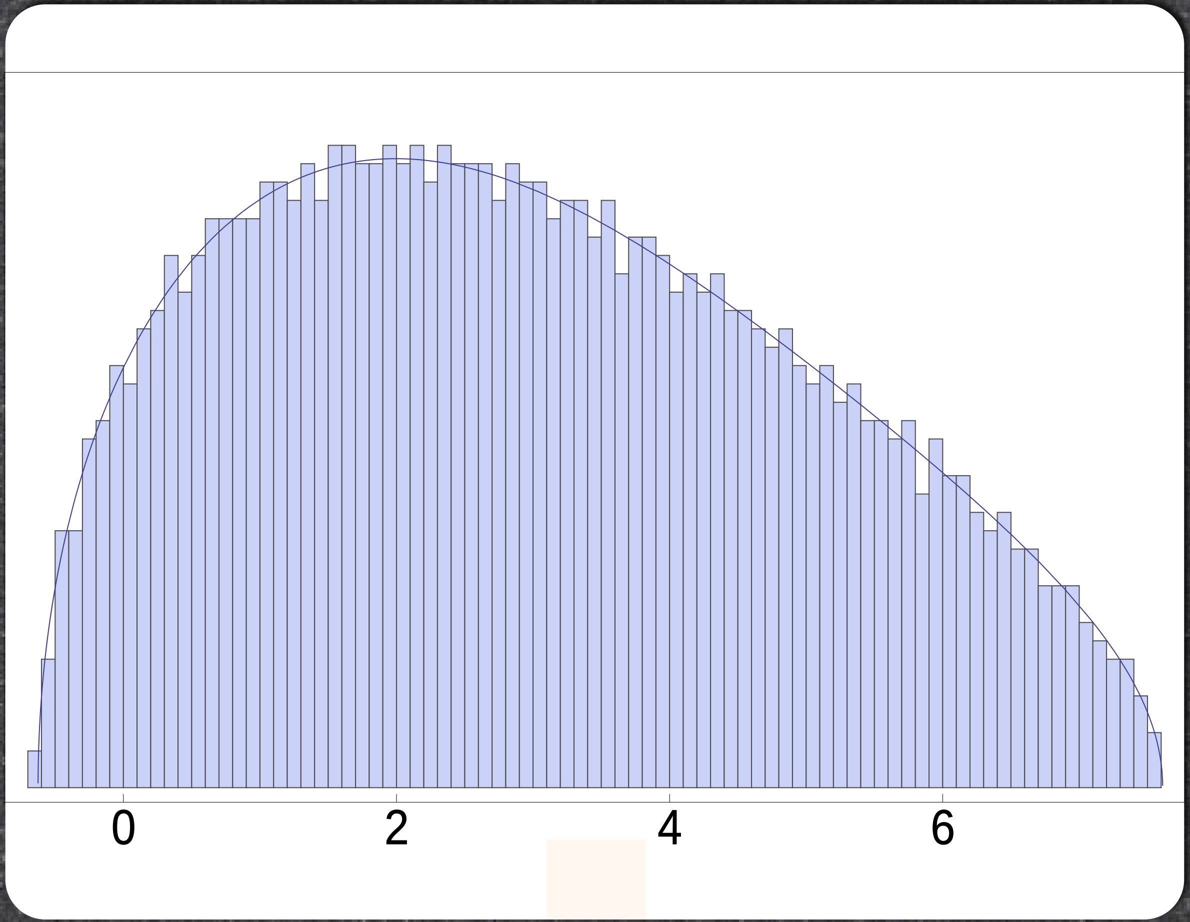
$$\mathcal{H}_{K^{(4)}} = F^2 \begin{pmatrix} -K_{a\bar b 1 \bar 1} & 0 \\ 0 & -K_{b \bar a 1 \bar 1} \end{pmatrix},$$

$$\mathcal{H}_{K^{(3)}} = F^2 \begin{pmatrix} K_{a1}{}^{e}\, K_{\bar b \bar 1 e} & 0 \\ 0 & K_{\bar a \bar 1}{}^{\bar e}\, K_{b 1 \bar e} \end{pmatrix},$$

$$\mathcal{H}_{\rm shift} = \mathbb{1}\left(F^2 - 2|W|^2\right) - F^2 \delta_a{}^{1} \delta_{\bar b}{}^{\bar 1} - F^2 \delta_{\bar a}{}^{\bar 1} \delta_b{}^{1}\,.$$

Random SUGRA

Typical spectrum:



- Typical spectra contain (many) tachyons
- Local minima and inflationary c.p. are highly atypical
- Large fluctuations of extreme eigenvalues:

$$P_{min} \sim e^{-cN_f^p + \mathcal{O}(N)}$$

Random SUGRA

- Looking for rare events >>> computationally expensive
- For vacua analysis [1112.3034] & [1207.2763]

Is there a cheaper/faster way?

- Study the Wigner ensemble since:
  - we know joint pdf
  - we can use known analytical results

$$H_{SUGRA} \sim \text{Wigner}$$

The Wigner Ensemble

Ensemble of matrices:

$$M = A + A^{\dagger}$$

Joint prob. distrib.function

$$dP(\lambda_1, ..., \lambda_{N_f}) = exp\left(-\frac{1}{\sigma^2}\sum_{i=1}^{N_f}\lambda_i^2\right)\prod_{i<j}(\lambda_i - \lambda_j)^2$$

Minima:

$$P(min) = \int_0^{\infty}\prod_i \mathrm{d}\lambda_i dP(\lambda_1, \ldots, \lambda_{N_f})$$

Dyson: 1D gas of charged particles

The Wigner Ensemble

Semi circle law:

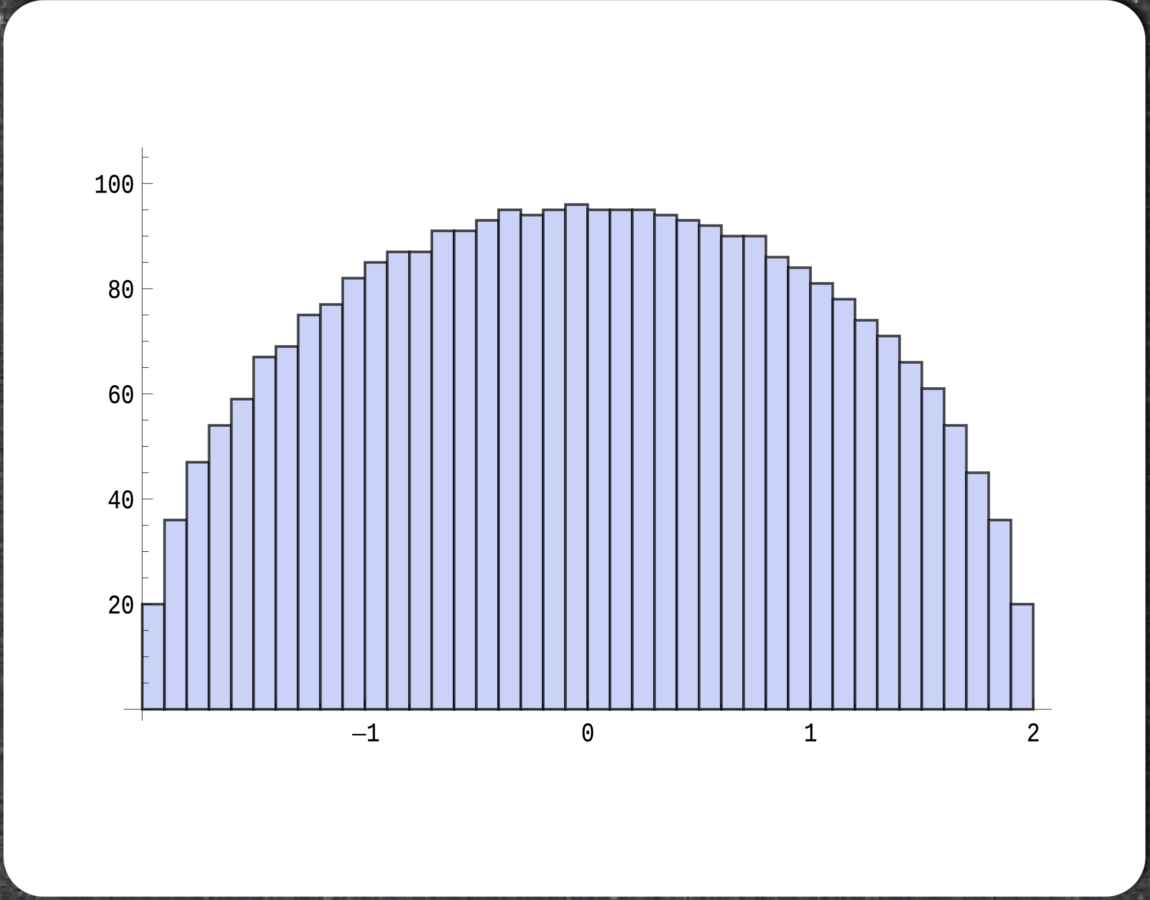
$$\rho(\lambda) = \frac{1}{2\pi N_f \sigma^2}\sqrt{4N_f\sigma^2 - \lambda^2}$$

Large fluctiuations:

$$P(\forall \lambda > \xi) = e^{-2\Phi(\xi) N_f^2}$$

Dean&Majumdar [condmat/0609651].

Typical spectrum:



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Inflation & Wigner Landscape

At a critical point $\epsilon = 0$

To get inflationary c.p. look for

$$\eta_{sr} = \frac{m^2}{H^2} \ll 1$$

Typically: $V_F \sim m_{3/2}^2 M_P^2$ and $m^2 \sim m_{3/2}^2$

$$\eta \sim \mathcal{O}(1)$$

Inflation is rare

Looking for $m \ll m_{3/2}$ when $V_F \sim m_{3/2}^2 M_P^2$

$\Leftrightarrow$ $m \ll 1$ when $V_F \sim M_P^4$

Vacua and Inflation

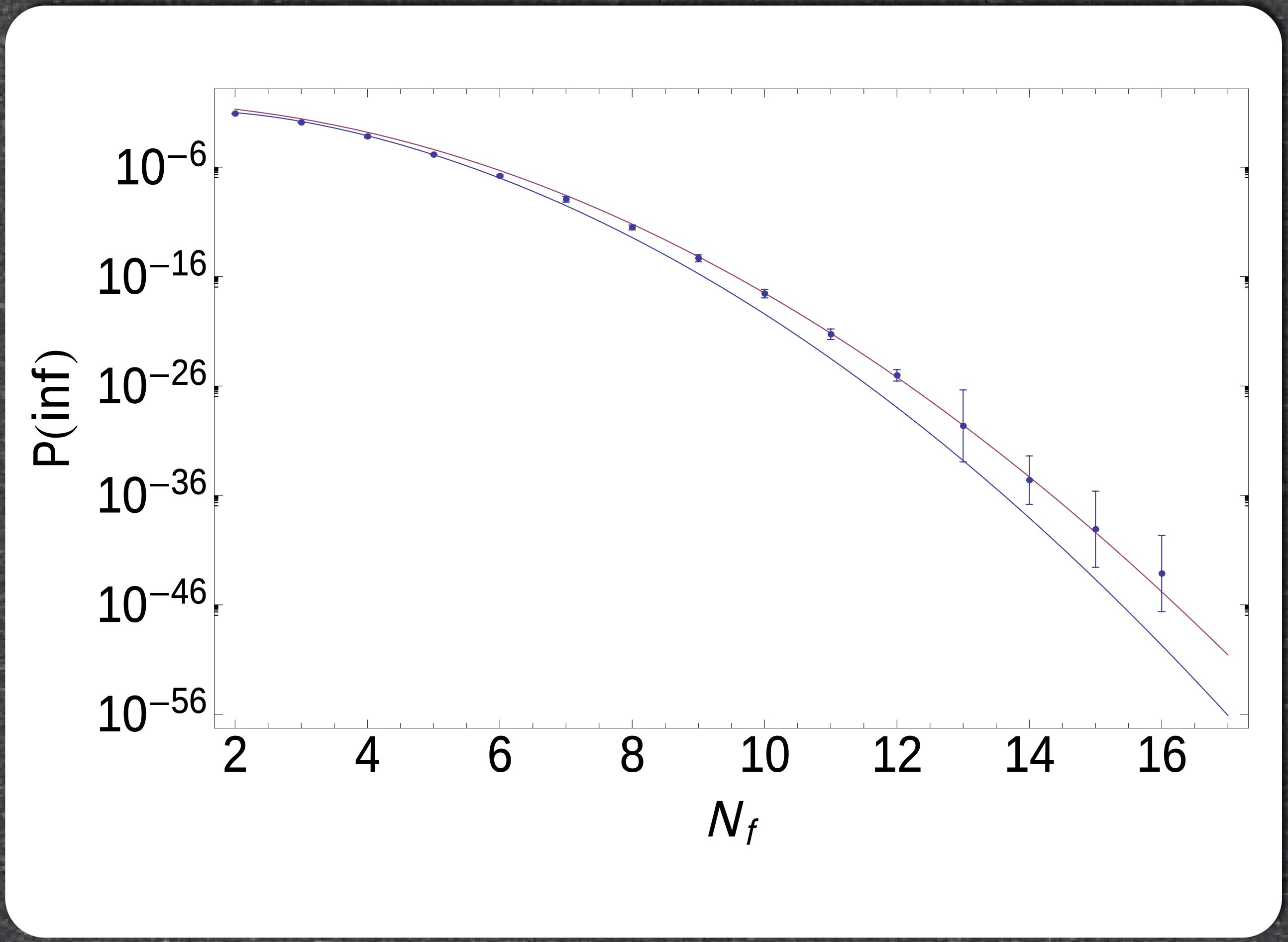
Inflationary c.p.: $\forall \lambda \geq -\eta \sim -\mathcal{O}(0.1)$

Minima rarer than inflationary c.p.

The steeper c.p. are the most abundant

$$\frac{P(inf)}{P(min)} = (e^{2\Delta c N_f^2} - 1)e^{2\widetilde{\Delta} c N_f^2} \sim e^{2\eta\Phi'(0)N_f^2} + \mathcal{O}(\eta^2).$$

Inflation in the Wigner Landscape



Exact result:

$$P(inf) = e^{-2\Phi(-\eta)N_f^2} - e^{-2\Phi(\eta)N_f^2}.$$

Best fit:

$$P(inf) \sim e^{-(0.402\pm0.02)N_f^2}.$$

Inflation is exponentially rare

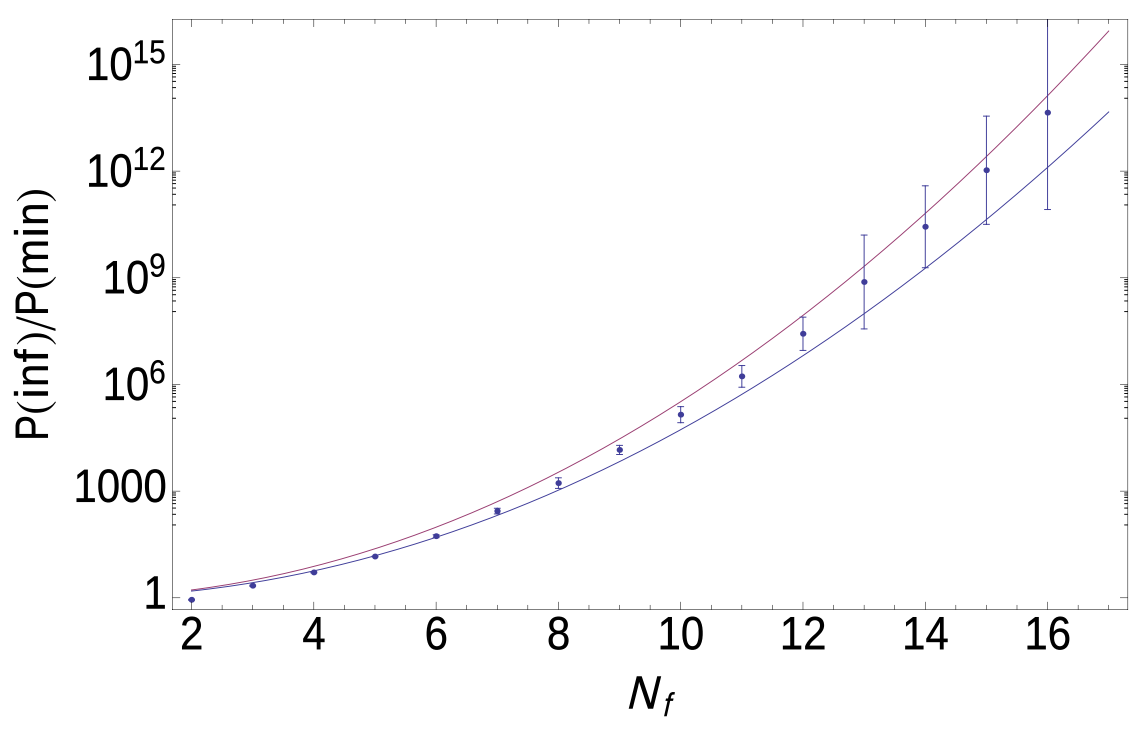
but...

F.G.Pedro, 23 March 2014, Munich

Inflation vs. Vacua

Inflation is more abundant than minima!

$$\frac{P(inf)}{P(min)} \sim e^{0.1 N_f^2}$$



If we are sitting at a minimum there are many inflationary inflection points around us

F.G.Pedro, 23 March 2014, Munich

Comparison with full Hessian

Minima:

$$P(min) \sim e^{0.29N_f^{1.5}} \sim e^{0.06N_f^2}$$

(Full SUGRA)

>>

$$P(min) \sim e^{0.55N_f^2}$$

(Wigner)

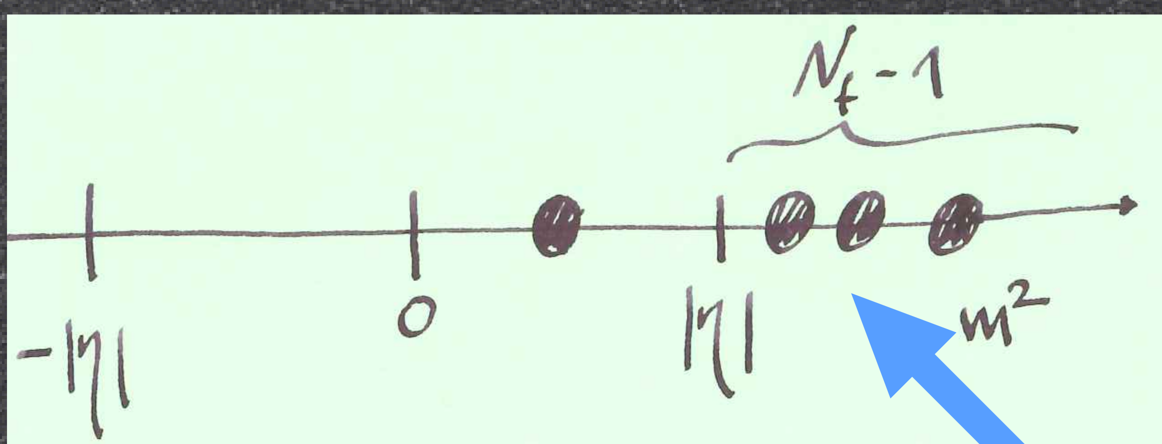
Difference due to shape of spectrum

Qualitatively similar, c.p. in the full case are much more likely

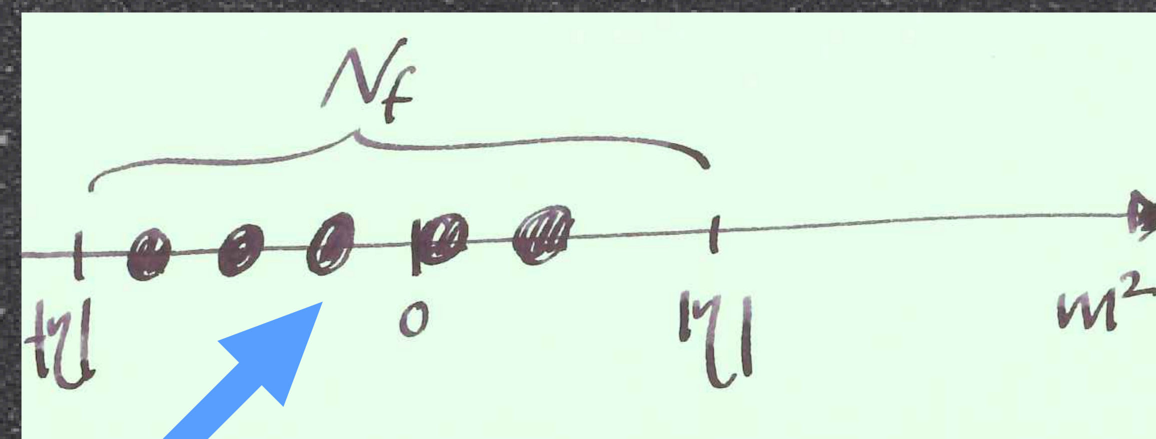
Dynamics of inflation

For a given Nf, how many fields are dynamical?

Single field:



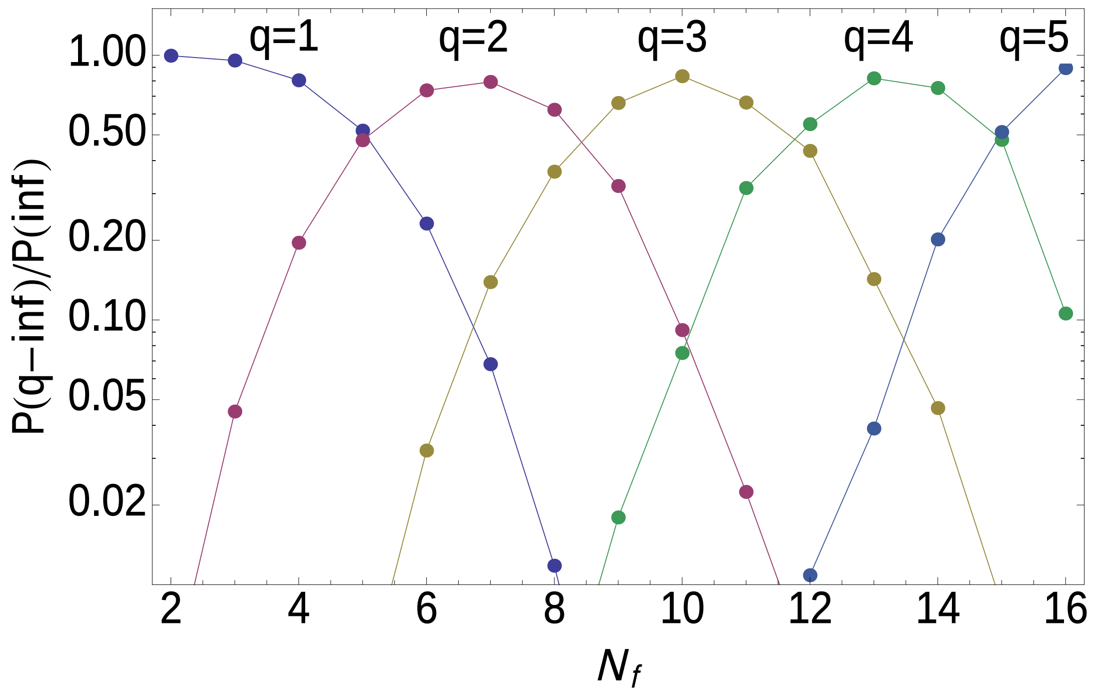
N_f field:



Strong repulsion

Most probable configuration between extremes

Dynamics of inflation



Inflation and Strings

[Cicoli&Quevedo:1108.2659]
[Burgess&McAllister:1108.2660]

Large field: $\Delta\phi > M_P$ e.g. axion monodromy

Small field: $\Delta\phi < M_P$ e.g. Kahler moduli IIB

Tensor-to-scalar ratio: $r = 16\epsilon$

Lyth: $$\frac{\Delta\phi}{M_P} \sim \mathcal{O}(1)\sqrt{\frac{r}{0.01}}$$

Large field: $r > 0.01$

Small field: $r < 0.01$

Large vs Small field

Assume:

- IIB flux compactifications.
- Large field = Axion monodromy

Drake Eqs.:

[Westphal 1206.4034]

$$N_{small} \sim N_{CY} \times N_{c.p.} \times \beta_{min} \times \beta_{flat-saddle}$$

$$N_{large} \sim N_{CY} \times N_{c.p.} \times \beta_{min} \times \beta_{axion-monod}$$

$$\beta_{flat-saddle} \gg 1$$

$$\beta_{flat-saddle} \gg \beta_{axion-monod}$$

Small field dominates:

$$\frac{P_{\Delta\phi_{60} > M_P}}{P_{\Delta\phi_{60} < M_P}} = \frac{N_{large}}{N_{small}} \sim \frac{\beta_{axion-monod}}{\beta_{flat-saddle}} < 1$$

Summary:

- Exponentially more inflationary c.p. than local minima,
- Multiple dynamical fields,
- Small field dominates over large field,
- No tensor modes at current level,

What next?

- Almost critical points may play an important role
- Generalization to full random SUGRA
- How to connect critical points?
- What observational signals to expect?