Heterotic Calabi-Yau Flux Compactifications

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based on work togehter with Andre Lukas and Eirik Svanes

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Overview

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Calabi-Yau domain walls

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Motivation: Why flux on Calabi-Yau?

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- phenomenologically very attractive, recently several hundred standard models (spectrum and more) discovered, see [Anderson, Gray, Lukas, Palti; 1106.4804]
- powerful tools of algebraic geometry available

Models attractive, but: moduli stabilisation is a problem

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Why flux?

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These approaches have been tried in the past:

- geometric flux [Gurrieri, Lukas, Micu 0408121], but: $\mathcal{M}_6$ no longer Calabi-Yau)
- recent progress on this in standard CY compactifications without flux [Anderson, Gray, Lukas, Ovrut]
- ▶ this talk: Calabi-Yau *and* flux

Calabi-Yau compactifications and flux

Make a compactification ansatz:

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then requiring unbroken supersymmetry gives [Wit, Smit, Dass, Nucl.Phys B283 1987]

$$H = 0 \quad \Leftrightarrow \quad \mathcal{M}_6 \text{ is Calabi-Yau}$$

This means either we give up Calabi-Yau or we have no flux.

? Why is there such a tension between Calabi-Yau and flux?

simple argument to illuminate this [Gauntlett, Martelli, Waldram 0302158]: look at dilaton equation of motion

$$\nabla^2 \mathrm{e}^{-2\phi} = \mathrm{e}^{-2\phi} * (H \wedge *H)$$

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integrating gives

$$\underbrace{-\int_{X_6} \mathrm{d}_6 \left(\mathrm{e}^{4A} *_6 \mathrm{d}_6 \mathrm{e}^{-2\phi} \right)}_{=0} = \int_{X_6} \mathrm{e}^{4A} \mathrm{e}^{-2\phi} (H \wedge *_6 H) = \| \mathrm{e}^{2A} \mathrm{e}^{-\phi} H \|^2$$

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$$\Rightarrow H = 0 \qquad \forall \mathcal{M}_6 \text{ compact}$$

i.e. we did not use Calabi-Yau, nor supersymmetry! (of course, α' corrections avoid this argument, however: again no CY! [Strominger])

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Now the dilaton equation of motion gives after integrating

$$-\partial_y^2 \mathrm{e}^{-2\phi} - \partial_y \mathrm{e}^{-2\phi} \frac{\partial_y V}{V} = \frac{1}{\mathrm{V}} \|\mathrm{e}^{-\phi} H\|^2$$

$\rightarrow$ giving ϕ an appropriate y dependence allows for non-zero flux!

Calabi-Yau domain walls

Supersymmetry conditions

We have motivated the existence of Calabi-Yau domain walls with flux. Let us make this more explicit:

$$\left(\nabla_M + \frac{1}{8}\mathcal{H}_M\right)\epsilon = 0 \qquad \left(\nabla\!\!\!/\phi + \frac{1}{12}\mathcal{H}\right)\epsilon = 0$$

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using $6d$ spinors we define the forms on $\mathcal{M}_6$:

$$J := \eta_-^\dagger \gamma_{uv}\eta_- \, e^{uv} \qquad \Omega := \eta_+^\dagger \gamma_{uvw}\eta_- \, e^{uvw}$$

with the domain wall ansatz $\mathcal{M}_{10} = \mathcal{M}_3 \times \mathbb{R} \ltimes \mathcal{M}_6$ this leads to

$$\mathrm{d}\Omega_- = 2\mathrm{d}\phi \wedge \Omega_- \qquad J \wedge \mathrm{d}J = J \wedge J \wedge \mathrm{d}\phi$$

$$J \wedge H = *\mathrm{d}\phi \qquad \mathrm{d}J = 2\phi'\Omega_- - \Omega'_- - 2\mathrm{d}\phi \wedge J + *H$$

$$\mathrm{d}\Omega_+ = J \wedge J' - \phi' J \wedge J + 2\mathrm{d}\phi \wedge \Omega_+ \qquad \Omega_- \wedge H = 2\phi' * 1$$

$$\Omega_+ \wedge H = 0 \; ,$$

The Killing spinor equations then reduce for a Calabi-Yau (i.e. $\mathrm{d}J = 0$, $\mathrm{d}\Omega = 0$) to the following flow equations

$$\begin{aligned} \Omega'_+ &= 2\phi'\Omega_+ - H \\ J \wedge J' &= \phi' J \wedge J \\ \Omega_- \wedge H &= 2\phi' \; *1 \, , \end{aligned}$$

together with the constraint

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$\longrightarrow$ existence of solutions guaranteed by [Hitchin math.DG 0107101]

Expand forms into a basis of harmonic three- and two-forms, $\{\alpha_A, \beta^B\}$ and ω_i

$$\Omega = Z^A\Big[\alpha_A - \mathcal{G}_{AB}(Z^A)\beta^B\Big]$$
$$J = v^i\omega_i$$
$$H = \mu^A\alpha_A + \epsilon_B\beta^B$$

Here Z^A are the $h^{2,1}+1$ projective complex moduli and $\mathcal{G}$ their prepotential, v^i the $h^{1,1}$ Kähler moduli and μ^A, ϵ_A y-independent flux parameters.

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The flow equations for the forms can be easily integrated if we define $X^A := \mathrm{e}^{-2\phi}Z^A$ and a new coordinate z via $\mathrm{d}y/\mathrm{d}z = \mathrm{e}^{2\phi}$:

$$\text{Re } X^A = -\mu^A z - \gamma^A$$
$$\text{Re } \mathcal{G}_B(X^A) = \epsilon_B z + \eta_B$$
$$v^i = e^{\phi} v_0^i \ ,$$

with integration constants $\{\gamma^A, \eta_B\}$, such that $\gamma^A \epsilon_A + \eta_B \mu^B = 0$.

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Constructing explicit solutions

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- for any given Calabi-Yau such a solution can be constructed
- it allows for any harmonic flux to be present (no α' corrections needed)

Low energy phenomenology

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$\mathcal{N} = 1$ SUGRA, moduli fields (S, T^i, X^A) [Gurrieri, Lukas, Micu 0408121]

$$K = -\ln \mathrm{i}(\bar{S} - S) - \ln 8V - \ln \mathrm{i}(X^A \bar{\mathcal{G}}_A - \bar{X}^A \mathcal{G}_A)$$

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new: superpotential

$$\boxed{W = \epsilon_A X^A + \mu^A \mathcal{G}_A}$$

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We also know something about the asymptotics:

weak coupling limit as $y \rightarrow \infty$

X^A approach constant value as $y \rightarrow \infty$

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Therfore, want to take the viewpoint:

$$\begin{array}{ccc} \text{CY max. sym.} & \longrightarrow & \text{CY domain wall} \\ \downarrow & & \downarrow \\ \text{some nice model} & \longrightarrow & \text{some nice model with W} \end{array}$$

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However, why shouldn't the domain wall itself be lifted in this process, leading to a vacuum with a maximally symmetic spacetime in $4d$?
In fact, for half-flat domain walls it was shown that the domain wall can indeed be lifted to a maximally symmetric and stable vacuum (moduli at consistent values) [M.K., Lukas, Matti, Svanes 1210.5933]

Conclusions and Outlook

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- this results in a model which has a domain wall, so it seems no realistic phenomenology possible

$\rightarrow$ however, we believe that past work has shown that there is justified reason to believe that such a model can be lifted to a maximally symmetric vacuum

Outlook: End of a paradigm?

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What if we take a different point of view:

$\boxed{!?}$ start with some ansatz that allows for enough freedom to stabilise all moduli but the dilaton (= switch on a superpotential)

$\rightarrow$ lift to non-perturbative stable vacuum, with a maximally symmetric spacetime

After all, it is the final lifted vacuum which we want to look like our universe.

Outlook

Still many things left to do:

- study lifting and moduli stabilisation for an explicit model
- try supersymmetric cosmic string, black hole, ...

Thank you!

Thank you very much!