Review and recent results in double field theory

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Based on work with:

- C. Hull, B. Zwiebach [2010]
- S. Ki Kwak, B. Zwiebach [2010-2011]
- D. Lüst, B. Zwiebach, [2012-present]

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What is Double Field Theory?

Reformulation (Extension?) of spacetime action for massless string fields:

$$S_{\mathsf{NS}} = \int d^D x \sqrt{-g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} + \frac{1}{4}\alpha' R^{ijkl} R_{ijkl} + \cdots \right]$$

generalized metric and doubled coordinates $X^M = (\tilde{x}_i, x^i)$,

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{pmatrix} \in O(D,D)$$

DFT Action (dilaton density $e^{-2d} = e^{-2\phi}\sqrt{-g}$):

$$S_{\mathsf{DFT}} \; = \int d^{2D} X \, e^{-2d} \, \mathcal{R}(\mathcal{H}, d) \quad \xrightarrow{\tilde{\partial}^i = 0} \quad S_{\mathsf{NS}}\big|_{\alpha' = 0}$$

generalized curvature scalar

$$\mathcal{R} \;\equiv\; 4\, \mathcal{H}^{MN} \partial_M \partial_N d - \partial_M \partial_N \mathcal{H}^{MN} - 4\, \mathcal{H}^{MN} \partial_M d\, \partial_N d + 4 \partial_M \mathcal{H}^{MN}\, \partial_N d$$
$$+ \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL}\, \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL}\, \partial_K \mathcal{H}_{NL}$$

Plan of the talk:

Part I

- Gauge transformations and generalized Lie derivatives
- Generalized coordinate transformations and generalized manifolds
- The role of $O(D, D)$

Part II

- Supersymmetric and Heterotic extensions
- Type II Strings

Part III

- Beyond the strong constraint?
- Massive type IIA
- Generalized Scherk-Schwarz compactification/gauged supergravity
- Conclusions and Outlook

Part I: Gauge transformations and generalized Lie derivatives

Recall g.c.t. from GR:

$$g'_{ij}(x') \;=\; \frac{\partial x^k}{\partial x'^i}\,\frac{\partial x^l}{\partial x'^j}\,g_{kl}(x)$$

Infinitesimally, $x'^i = x^i - \xi^i(x)$, governed by Lie derivatives

$$\delta_\xi g_{ij} = \mathcal{L}_\xi g_{ij} \equiv \xi^k \partial_k g_{ij} + \partial_i \xi^k g_{kj} + \partial_j \xi^k g_{ik}$$

In DFT gauge invariance governed by generalized Lie derivatives

$$\widehat{\mathcal{L}}_\xi \mathcal{H}_{MN} = \xi^P \partial_P \mathcal{H}_{MN} + \left(\partial_M \xi^P - \partial^P \xi_M\right) \mathcal{H}_{PN} + \left(\partial_N \xi^P - \partial^P \xi_N\right) \mathcal{H}_{MP}$$

$$\widehat{\mathcal{L}}_\xi\big(e^{-2d}\big) = \partial_M\big(\xi^M e^{-2d}\big)$$

Invariance and closure, $[\widehat{\mathcal{L}}_{\xi_1}, \widehat{\mathcal{L}}_{\xi_2}] = \widehat{\mathcal{L}}_{[\xi_1,\xi_2]_C}$, modulo strong constraint

$$\eta^{MN}\partial_M\partial_N = 2\tilde{\partial}^i\partial_i = 0 \qquad\qquad \eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Generalized coordinate transformations ?

Generalized g.c.t. that reproduce this infinitesimally:

$$S'(X') = S(X) \qquad\qquad A'_M(X') = \mathcal{F}_M{}^N A_N(X)$$

and analogously on higher tensors, where [O.H., Zwiebach, 1207.4198]

$$\mathcal{F}_M{}^N \equiv \frac{1}{2}\Big(\frac{\partial X^P}{\partial X'^M}\frac{\partial X'_P}{\partial X_N} + \frac{\partial X'_M}{\partial X_P}\frac{\partial X^N}{\partial X'^P}\Big)$$

Setting $X'^M = X^M - \xi^M(X)$ we get $\delta_\xi = \widehat{\mathcal{L}}_\xi$.

- $\eta'_{MN} = \eta_{MN} \;\Rightarrow\; \mathcal{F} \in O(D,D)$, but <u>not</u> gauged $O(D,D)$!
- $x^{i\prime} = x^{i\prime}(x)$, $\tilde{x}'_i = \tilde{x}_i$ leads to usual g.c.t.,

  $\tilde{x}'_i = \tilde{x}_i - \tilde{\xi}_i(x)$, $x^{i\prime} = x^i$ leads to $b_{ij} \to b_{ij} + \partial_i\tilde{\xi}_j - \partial_j\tilde{\xi}_i$
- composition according to BCH of C-bracket

Generalized manifold

Generalized coordinate transformations $\Rightarrow$ generalized manifold
('patched together' by generalized coordinate transformations)

- similar to idea of T-fold (patching by $O(d,d)$)
  [Hellerman, McGreevy, Williams, hep-th/0208174; Hull, hep-th/0406102]

- however, different in general:

$$A'(X') = \mathcal{F}A(X)\ , \quad \text{with} \quad X'^M \neq \mathcal{F}^M{}_N X^N \quad \text{in general}$$

- in presence of abelian isometries (torus)
  $\Rightarrow$ fields independent of coordinates x^i, $i = 1, \ldots, d < D$
  $\Rightarrow$ full $O(d,d)$ particular generalized g.c.t.:
  on top of geometric subgroup $GL(d) \ltimes \mathbb{R}^{\frac{1}{2}d(d-1)}$ also 'hidden' part

$$x'^i = x^i + \frac{1}{2} f^{ij} \tilde{x}_j \quad \rightarrow \quad h = \begin{pmatrix} 1 & 0 \\ f & 0 \end{pmatrix} \in O(d,d)$$

[differs from 'naive' ansatz by $\frac{1}{2}$, but $A'(X') = A'(X) = hA(X)$]

What's the role of $O(D, D)$?

usual expectation:

– T-duality group $O(d, d)$ only present for torus

– $2D$-dimensional manifold with metric $\eta_{MN} = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}$ very special

$\Rightarrow$ preferred coordinates

Thus: DFT tied to torus-like backgrounds? No, because

– doubled 'manifold' generalized $\rightarrow \eta_{MN}$ constant in all local charts

– usual g.c.t. of D-dim. submanifold is subgroup of generalized g.c.t.

$\Rightarrow$ no constraints on 'physical' D-dimensional space

Non-trivial examples beyond T-folds? Not yet clear [in progress with Lüst, Zwiebach]

Analogy with general relativity

$$S = \int dx \sqrt{-g} R \; = \; \int dx \sqrt{-g} \Big(-\frac{1}{4} g^{ik} g^{jl} g^{pq} \partial_p g_{ij} \partial_q g_{kl} \; + \; \text{more terms} \Big)$$

manifestly $GL(D, \mathbb{R})$ invariant, but proper symmetry diffeomorphism group;
$GL(d)$ reappears, e.g., as rigid symmetry on T^d; so does $O(d, d)$ in DFT

Part II: Supersymmetric and Heterotic Extensions

(Generalized) vielbein formalism required [Siegel (1993), O.H. & Ki Kwak (2010)]

$$\mathcal{H}^{MN} = \hat{\eta}^{AB} E_A{}^M E_B{}^N \,, \qquad \hat{\eta}_{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & \eta_{\bar{a}\bar{b}} \end{pmatrix}$$

local $SO(1,9)_L \times SO(1,9)_R$ Lorentz symmetry

Gauge fixing to diagonal subgroup

$$E_A{}^M = \begin{pmatrix} E_{ai} & E_a{}^i \\ E_{\bar{a}i} & E_{\bar{a}}{}^i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e_{ia} + b_{ij} e_a{}^j & e_a{}^i \\ -e_{i\bar{a}} + b_{ij} e_{\bar{a}}{}^j & e_{\bar{a}}{}^i \end{pmatrix}$$

<u>Fermions:</u> singlets under $O(10,10)$ and $\widehat{\mathcal{L}}_\xi$

[Coimbra, Strickland-Constable, Waldram, 1112.3989; O.H., S. Ki Kwak, 1111.7293]

Ψ_a : vector of $SO(1,9)_L$, spinor of $SO(1,9)_R$

ρ : spinor of $SO(1,9)_R$,

ϵ : spinor of $SO(1,9)_R$

$\mathcal{N} = 1$ supersymmetric Lagrangian

$$\mathcal{L} = e^{-2d}\Big(\mathcal{R}(E,d) - \bar{\Psi}^a\gamma^{\bar{b}}\nabla_{\bar{b}}\Psi_a + \bar{\rho}\gamma^{\bar{a}}\nabla_{\bar{a}}\rho + 2\bar{\Psi}^a\nabla_a\rho\Big)$$

$\mathcal{N} = 1$ supersymmetry transformations

$$E_{\bar{b}}{}^M\delta_\epsilon E_{aM} = \frac{1}{2}\bar{\epsilon}\gamma_{\bar{b}}\,\Psi_a \qquad \delta_\epsilon d = -\frac{1}{4}\bar{\epsilon}\rho \qquad \delta_\epsilon\Psi_a = \nabla_a\epsilon \qquad \delta_\epsilon\rho = \gamma^{\bar{a}}\nabla_{\bar{a}}\epsilon$$

Proof of supersymmetric invariance: variation of bosonic term

$$e^{2d}\,\delta_\epsilon\mathcal{L}_{\mathsf{B}} \;=\; \frac{1}{2}\bar{\epsilon}\rho\mathcal{R} + \bar{\epsilon}\gamma^{\bar{b}}\Psi^a\mathcal{R}_{a\bar{b}}$$

variation of fermionic terms

$$\begin{aligned}
e^{2d}\,\delta_\epsilon\mathcal{L}_{\mathsf{F}} &= -2\bar{\Psi}^a\gamma^{\bar{b}}\nabla_{\bar{b}}\nabla_a\epsilon + 2\bar{\rho}\gamma^{\bar{a}}\nabla_{\bar{a}}\big(\gamma^{\bar{b}}\nabla_{\bar{b}}\epsilon\big) + 2\nabla^a\bar{\epsilon}\,\nabla_a\rho + 2\bar{\Psi}^a\nabla_a\big(\gamma^{\bar{b}}\nabla_{\bar{b}}\epsilon\big)\\
&= -2\bar{\Psi}^a\Big[\gamma^{\bar{b}}\nabla_{\bar{b}},\nabla_a\Big]\epsilon + 2\bar{\rho}\,\Big(\gamma^{\bar{a}}\nabla_{\bar{a}}\gamma^{\bar{b}}\nabla_{\bar{b}} - \nabla^a\nabla_a\Big)\,\epsilon\\
&= \bar{\Psi}^a\gamma^{\bar{b}}\mathcal{R}_{a\bar{b}}\epsilon - \frac{1}{2}\bar{\rho}\mathcal{R}\epsilon = -\frac{1}{2}\bar{\epsilon}\rho\mathcal{R} - \bar{\epsilon}\gamma^{\bar{b}}\Psi^a\mathcal{R}_{a\bar{b}}
\end{aligned}$$

Thus: $\quad \delta_\epsilon\big(S_{\mathsf{B}} + S_{\mathsf{F}}\big) = 0$

Add abelian vector multiplets: $SO(1,9+n)\times SO(1,9) \subset O(10+n,10)$

($n=16$: heterotic string truncated to Cartan of $E_8\times E_8$ or $SO(32)$)

Frame field: $A=(a\,,\ \bar{a}) = (\underline{a}\,,\ \underline{\alpha}\,,\ \bar{a})\ ,\ \underline{a}=0,\ldots,9\ ,\ \underline{\alpha}=1,\ldots,n$

$$E_A{}^M = \frac{1}{\sqrt{2}}\begin{pmatrix} e_{i\underline{a}} - e_{\underline{a}}{}^k c_{ki} & -e_{\underline{a}}{}^k A_k{}^\beta & e_{\underline{a}}{}^i \\ \sqrt{2}A_{i\underline{\alpha}} & \sqrt{2}\delta_{\underline{\alpha}}{}^\beta & 0 \\ -e_{i\bar{a}} - e_{\bar{a}}{}^k c_{ki} & -e_{\bar{a}}{}^k A_k{}^\beta & e_{\bar{a}}{}^i \end{pmatrix}$$

where $c_{ij} = b_{ij} + \frac{1}{2}A_i{}^\alpha A_{j\alpha}$

Additional gauginos χ_α encoded in

$$\Psi_a \ = \ \big(\Psi_{\underline{a}}\,,\ \Psi_{\underline{\alpha}}\big) \ \equiv \ \big(e_{\underline{a}}{}^i\Psi_i\,,\ \tfrac{1}{\sqrt{2}}\chi_{\underline{\alpha}}\big)$$

Formally same Lagrangian and supersymmetry variations as above!

$\rightarrow$ reduces to standard action and SUSY rules setting $\tilde{\partial}^i = 0$

Comparison: standard $\mathcal{N} = 1$ supergravity action

$$
\begin{aligned}
S \;=\; \int d^{10}x\, e\, e^{-2\phi}\Big[\Big(& R + 4\partial^i\phi\,\partial_i\phi - \frac{1}{12}\widehat{H}^{ijk}\widehat{H}_{ijk} - \frac{1}{4}F_{ij}F^{ij}\Big) \\
& - \bar\psi_i\gamma^{ijk}D_j\psi_k - 2\bar\lambda\gamma^i D_i\lambda - \frac{1}{2}\bar\chi^\alpha \not{D}\chi_\alpha \\
& + 2\bar\psi^i(\partial_i\phi)\gamma^j\psi_j - \bar\psi_i(\not{\partial}\phi)\gamma^i\lambda - \frac{1}{4}\bar\chi_\alpha\gamma^i\gamma^{jk}F_{jk}{}^\alpha\big(\psi_i + \frac{1}{6}\gamma_i\lambda\big) \\
& + \frac{1}{24}\widehat{H}_{ijk}\Big(\bar\psi_m\gamma^{mijkn}\psi_n + 6\bar\psi^i\gamma^j\psi^k - 2\bar\psi_m\gamma^{ijk}\gamma^m\lambda + \frac{1}{2}\bar\chi^\alpha\gamma^{ijk}\chi_\alpha\Big) \\
& + \text{quartic fermions}\Big]
\end{aligned}
$$

where

$$
\widehat{H}_{ijk} \;=\; 3\left(\partial_{[i}b_{jk]} - A_{[i}{}^\alpha\partial_j A_{k]\alpha}\right)
$$

Comparison: standard $\mathcal{N} = 1$ supersymmetry rules

$$
\begin{aligned}
\delta_\epsilon e_i{}^a &= \frac{1}{2}\bar\epsilon\,\gamma^a\psi_i - \frac{1}{4}\bar\epsilon\,\lambda\, e_i{}^a\,, \\
\delta_\epsilon\phi &= -\,\bar\epsilon\lambda \quad , \quad \delta_\epsilon A_i{}^\alpha \;=\; \frac{1}{2}\bar\epsilon\,\gamma_i\chi^\alpha \quad , \\
\delta_\epsilon\chi^\alpha &= -\,\frac{1}{4}\gamma^{ij}F_{ij}{}^\alpha\epsilon \\
\delta_\epsilon\psi_i &= D_i\epsilon - \frac{1}{8}\gamma_i(\partial\!\!\!/\phi)\epsilon + \frac{1}{96}(\gamma_i{}^{klm} - 9\delta_i{}^k\gamma^{lm})\widehat{H}_{klm}\epsilon\,, \\
\delta_\epsilon\lambda &= -\,\frac{1}{4}(\partial\!\!\!/\phi)\epsilon + \frac{1}{48}\gamma^{ijk}\widehat{H}_{ijk}\epsilon\,, \\
\delta_\epsilon b_{ij} &= \frac{1}{2}(\bar\epsilon\,\gamma_i\psi_j - \bar\epsilon\,\gamma_j\psi_i) - \frac{1}{2}\bar\epsilon\,\gamma_{ij}\lambda + \frac{1}{2}\bar\epsilon\gamma_{[i}\chi^\alpha A_{j]\alpha}\,.
\end{aligned}
$$

Type II Double Field Theory

NS-NS: dilaton d, lift of $\mathcal{H} \in O(10,10)$ to $\mathbb{S} \in Spin(10,10)$

RR: Majorana-Weyl spinor χ of $O(10,10)$

Action:

$$S = \int dx d\tilde{x} \left(e^{-2d} \mathcal{R} + \frac{1}{4} (\partial\!\!\!/ \chi)^\dagger \, \mathbb{S} \, \partial\!\!\!/ \chi \right)$$

Dirac operator in terms of raising and lowering operators ψ_i, ψ^i of $O(10,10)$

$$\partial\!\!\!/ \equiv \psi^i \partial_i + \psi_i \tilde{\partial}^i \quad \Rightarrow \quad \partial\!\!\!/^2 = \frac{1}{2} \eta^{MN} \partial_M \partial_N = 0$$

(Self-)duality constraint (C: charge conjugation matrix)

$$\partial\!\!\!/ \chi = -\mathcal{K} \, \partial\!\!\!/ \chi \qquad \mathcal{K} \equiv C^{-1} \mathbb{S}$$

Reduces to democratic type IIA (or IIB) supergravity for $\tilde{\partial}^i = 0$, where conventional RR p-forms $C^{(p)}$ encoded as

$$\chi = \sum_p \frac{1}{p!} C_{i_1 \dots i_p} \, \psi^{i_1} \dots \psi^{i_p} |0\rangle$$

Curiously: encodes also exotic type IIA* and IIB* [Hull, hep-th/9806146]

Part III: Can the strong constraint be relaxed?

Absolutely, if we take the full closed string theory into account

$\Rightarrow$ closed SFT on torus is truly doubled, subject only to (weak) level-matching constraint $\tilde{\partial}^i \partial_i = 0$ $(p \cdot w = 0)$ [Kugo & Zwiebach, hep-th/9201040]

$\Rightarrow$ in full string theory doubled coordinates undoubtedly physical

More interesting question:

Can the strong constraint be relaxed on massless string fields only?

Subtle: consistent gauge variation δ_ξ requires $\tilde{\partial}^i \partial_i (\delta_\xi \Phi) = 0$, so

$$\delta_\xi \Phi = \xi \cdot \Phi \quad \longrightarrow \quad \delta_\xi \Phi = \Big[\xi \cdot \Phi \Big]$$

where $[\ \ ]$ projects out Fourier modes with $p \cdot w \neq 0$ $\Rightarrow$ non-locality

gauge algebra and invariance involved [partial results with Hull and Zwiebach]

However: more mild relaxations in massive and gauged deformations

Massive Type IIA: Romans theory

Massive type IIA obtained for

$$C^{(1)}(x,\tilde{x}) \;=\; C_i(x)dx^i + m\tilde{x}_1 dx^1$$

Ansatz consistent because gauge transformations can be re-written

$$\delta_\xi \chi \;=\; \not{\xi}\,\not{\partial}\chi$$

so that linear $\tilde{x}$ dependence drops out.
General field strengths

$$F = \not{\partial}\chi = (\psi^i\partial_i + \psi_i\tilde{\partial}^i)\chi = F_{m=0} + \psi_i\tilde{\partial}^i(m\tilde{x}_1)\psi^1|0\rangle$$

lead to non-trivial 0-form field strength

$$F^{(0)} \;=\; m$$

$\Rightarrow$ '(-1)-form' $\equiv$ 1-form depending on $\tilde{x}$ [Lavrinenko, Lu, Pope, Stelle (1999)]

$\Rightarrow$ Type II DFT reduces to (democratc formulation of) massive Type IIA

Generalized Scherk-Schwarz compactification

Scherk-Schwarz Reduction of DFT in generalized metric form.

[Aldazabal, Baron, Marques & Nunez; Geissbuhler (2011)]

$$\mathcal{H}_{MN}(x,\mathbb{Y}) \;=\; U^{A}{}_{M}(\mathbb{Y})\mathcal{H}_{AB}(x)U^{B}{}_{N}(\mathbb{Y}) \;, \qquad U \in O(D,D)$$

Flux components in lower-dimensional (4D) theory directly given by

$$F_{ABC} \;=\; 3\eta_{D[A}(U^{-1})^{M}{}_{B}(U^{-1})^{N}{}_{C]}\partial_{M}U^{D}{}_{N}$$

[see also: Andriot, O.H., Larfors, Lüst, Patalong & Blumenhagen, Deser, Plauschinn, Rennecke]

yields gauged supergravities with 'non-geometric fluxes'

however, not all gaugings obtained because of strong constraint

$\Rightarrow$ relaxation of strong constraint? [Grana & Marques (2012)]

Intriguing first steps, but complete picture still elusive

Summary & Outlook

Most conservatively:

- Strong constraint solved by

$$\partial_M = \begin{cases} \partial_i & \text{if } M = i \\ 0 & \text{else} \end{cases} \quad .$$

  but technically, ∂_i, g, b and ϕ never used!

- (very economic!) *reformulation* of low-energy action for string theory
  $\Rightarrow$ geometry can be thought of as 'generalized geometry' [Hitchin, Gualtieri]
  (to the extent it had been developed)

Concrete reasons for more:

- Full closed string field theory *is* a truly doubled field theory

- mild relaxations of strong constraint possible
  $\rightarrow$ massive IIA & gauged supergravity

Parallel developments:

- U-duality invariant versions of (truncated) M-theory
  [Hillmann, Berman & Perry, Coimbra et. al.]
  so far not valid for full 11-dimensional supergravity
- Generalized Scherk-Schwarz reductions in M-theory versions
  [Berman, Musaev & Thompson]

Main open problem:

- Encoding higher-derivative α' corrections [Riem^2]
- almost certainly requires extension of framework [O.H. & Zwiebach (2011)]
  $\rightarrow \alpha'$ corrections of T-duality rules and/or gauge transformations?
- indeed, gauge structure in CSFT α'–corrected