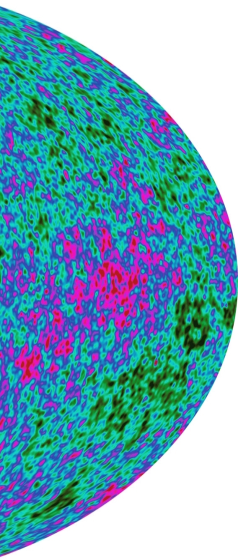

Holography and the very early universe

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Introduction

The aim of this work is to obtain a holographic description of the very early universe, the period usually associated with inflation.

- This holographic description includes:
- Conventional inflation.
- New models for the very early universe that have a weakly coupled holographic dual QFT. Such universe would be non-geometric at early times.

Introduction

The observables we will discuss and compute are the standard cosmological observables that are currently being measured :

- Power spectra
- Non-gaussianities
- As we will see the holographic viewpoint leads to new and *falsifiable* models for the early universe and to a considerable new insight about conventional inflation.

References

The talk is based on work with Paul McFadden

- Holography for Cosmology, arXiv:0907.5542
- The Holographic Universe, arXiv:1007.2007
- Observational signatures of holographic models of inflation, arXiv:1010.0244
- Holographic Non-Gaussianity, arXiv:1011.0452
- Cosmological 3-point correlators from holography, arXiv:1104.3894
- R. Easther, R. Flauger, P. McFadden, KS, Constraining holographic inflation with WMAP, arXiv:1104.2040.
- A. Bzowski, P. McFadden, KS, Holographic predictions for cosmological 3-point functions, arXiv:1112.1967.
- A. Bzowski, P. McFadden, KS, Holography for inflation using conformal perturbation theory, arXiv:1211.????

Outline

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Thm 1: Background solutions [KS, Townsend (2006)]

The underlying framework for this work is gravity coupled to a scalar field Φ with a potential $V(\Phi)$.

- There is 1-1 correspondence, the **Domain-wall/Cosmology correspondence**, between

FRW solutions of the theory with potential $V(\Phi)$	$\leftrightarrow$	Domain-wall solutions of the theory with potential $-V(\Phi)$.

- This correspondence can be understood as analytic continuation.
- An example of this correspondence is the analytic continuation from de Sitter to Anti de Sitter. This theorem shows that this relation is not accidental.

Inflation/holographic RG correspondence

- A special case of the correspondence is that between inflationary backgrounds and holographic RG flow spacetimes.
- Inflationary spacetimes can either
  - approach de Sitter spacetime at late times,

$$ds^2 \to ds^2 = -dt^2 + e^{2t} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

  - approach power-law scaling solutions at late times ,

$$ds^2 \to ds^2 = -dt^2 + t^{2n} dx^i dx^i, \quad (n > 1) \qquad \text{as} \quad t \to \infty$$

- These backgrounds are in 1-1 correspondence with holographic RG flows, either asymptotically AdS or asymptotically power-law.

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Thm 2: Fluctuations [McFadden, KS (2009-2012)]

- Not only the background solutions are in correspondence but also arbitrary fluctuations around them map to each other.
- The fluctuations describe a scalar mode ζ and a transverse traceless mode, which we will describe using a helicity basis, $\gamma^{(\pm)}$.
- We explicit checked to second order in perturbation theory that the fluctuations map to each other provided

$$\kappa \to -i\kappa, \qquad q \to -iq$$

where κ^2 is Newton's constant and q is the magnitude of the momentum of the fluctuation.

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Thm 3: The quantum story [McFadden, KS ('09-'12)]

- We are interested in computing cosmological observables, like the power spectra and non-Gausianities.
- These can be obtained from the late-time behavior of in-in correlators.
- The power-spectra are obtained from 2-point functions, $\langle\zeta\zeta\rangle, \langle\gamma^{s_1}\gamma^{s_2}\rangle$.
- Non-Gaussianities are obtained from 3-point functions, $\langle\zeta\zeta\zeta\rangle, \langle\zeta\zeta\gamma^{s}\rangle, \langle\zeta\gamma^{s_1}\gamma^{s_2}\rangle, \langle\gamma^{s_1}\gamma^{s_2}\gamma^{s_3}\rangle$.
- We computed these correlators for general inflationary spacetimes.

Thm 3: The quantum story: the QFT side

- By Theorem 1, corresponding to any of the inflationary spacetimes, there is a corresponding holographic RG flow spacetime.
- By standard gauge/gravity duality, these spacetimes are dual to a QFT.
- We used standard gauge/gravity duality to compute the 2-point and 3-point function of the stress energy tensor, $\langle T_{ij}T_{mn}\rangle, \langle T_{ij}T_{mn}T_{pq}\rangle$.

Thm 3: Holographic formulae for cosmology

- By comparing the cosmological results to the QFT results one finds that the former can be expressed in terms of the latter provided

$$\kappa \to -i\kappa, \qquad q \to \bar{q} = -iq$$

- The cosmological 2-point functions are given by

$$\langle \zeta(q)\zeta(-q)\rangle = \frac{-1}{8\mathrm{Im}[B(\bar{q})]}, \qquad \langle \hat{\gamma}^{(s)}(q)\hat{\gamma}^{(s')}(-q)\rangle = \frac{-\delta^{ss'}}{\mathrm{Im}[A(\bar{q})]},$$

where $\langle T_{ij}(\bar{q})T_{kl}(-\bar{q})\rangle = A(\bar{q})\Pi_{ijkl} + B(\bar{q})\pi_{ij}\pi_{kl}$.

Holographic formulae: 3-point functions

- $\langle\zeta(q_1)\zeta(q_2)\zeta(q_3)\rangle$

$$= -\frac{1}{256}\Big(\prod_i \mathrm{Im}[B(\bar{q}_i)]\Big)^{-1} \times \mathrm{Im}\Big[\langle T(\bar{q}_1)T(\bar{q}_2)T(\bar{q}_3)\rangle + (\text{semi}-\text{local terms})\Big],$$

- $\langle\zeta(q_1)\zeta(q_2)\hat{\gamma}^{(s_3)}(q_3)\rangle$

$$= -\frac{1}{32}\Big(\mathrm{Im}[B(\bar{q}_1)]\mathrm{Im}[B(\bar{q}_2)]\mathrm{Im}[A(\bar{q}_3)]\Big)^{-1}$$

$$\times\, \mathrm{Im}\Big[\langle T(\bar{q}_1)T(\bar{q}_2)T^{(s_3)}(\bar{q}_3)\rangle + (\text{semi}-\text{local terms})\Big],$$

[McFadden, KS (2010), (2011)]

Holographic formulae: 3-point functions

- $\langle \zeta(q_1)\hat{\gamma}^{(s_2)}(q_2)\hat{\gamma}^{(s_3)}(q_3)\rangle$

$$= -\frac{1}{4}\Big(\mathrm{Im}[B(\bar{q}_1)]\mathrm{Im}[A(\bar{q}_2)]\mathrm{Im}[A(\bar{q}_3)]\Big)^{-1}$$

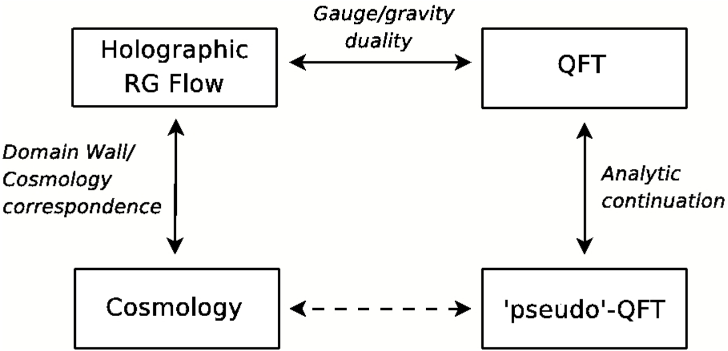
$$\times\,\mathrm{Im}\Big[\langle T(\bar{q}_1)T^{(s_2)}(\bar{q}_2)T^{(s_3)}(\bar{q}_3)\rangle + (\text{semi}-\text{local terms})\Big],$$

- $\langle \hat{\gamma}^{(s_1)}(q_1)\hat{\gamma}^{(s_2)}(q_2)\hat{\gamma}^{(s_3)}(q_3)\rangle$

$$= -\Big(\prod_i \mathrm{Im}[A(\bar{q}_i)]\Big)^{-1}\times\,\mathrm{Im}\Big[2\langle T^{(s_1)}(\bar{q}_1)T^{(s_2)}(\bar{q}_2)T^{(s_3)}(\bar{q}_3)\rangle + (\text{semi}-\text{local terms})\Big]$$

[McFadden, KS (2011)]

Summary


Gauge/gravity duality
Holographic RG Flow
QFT
Domain Wall/ Cosmology correspondence
Analytic continuation
Cosmology
'pseudo'-QFT

Outline

Conjecture [McFadden, KS ('09-'12)]

- Theorem 3 was derived under the assumption that gravity is weakly coupled. In this case the dual QFT is strongly coupled.

Conjecture: The holographic formulae hold also when the dual QFT is weakly coupled.

- In these models the very early universe is non-geometric. Spacetime emerges only at late times. Late time here means the beginning of hot big bang cosmology.

Outline

New holographic models

To specify the model we need to specify the dual QFT. The two classes of asymptotic behaviors correspond to two classes of dual QFT's.

- asymptotically de Sitter $\rightarrow$ QFT is deformation of a CFT
- asymptotically power-law $\rightarrow$ QFT is super-renormalizable

We will first summarize the phenomenology of the second case.

Dual QFT

- A class of models exhibiting is given by the following super-renormalizable theory:

$$S = \frac{1}{g_{YM}^2} \int d^3x \mathrm{tr} \Big[\frac{1}{2} F_{ij}^I F^{Iij} + \frac{1}{2} (D\phi^J)^2 + \frac{1}{2} (D\chi^K)^2 + \bar{\psi}^L \not{D} \psi^L + \lambda_{M_1 M_2 M_3 M_4} \Phi^{M_1} \Phi^{M_2} \Phi^{M_3} \Phi^{M_4} + \mu_{M L_1 L_2}^{\alpha\beta} \Phi^M \psi_\alpha^{L_1} \psi_\beta^{L_2} \Big].$$

  $\Phi^M = \{\phi^I, \chi^K\}$, χ^K: conformal scalars, ϕ^I: minimally coupled scalars, ψ^L: fermions

- To extract predictions we need to compute n-point functions of the stress energy tensor analytically continue the result and insert them in the holographic formulae.

Phenomenology

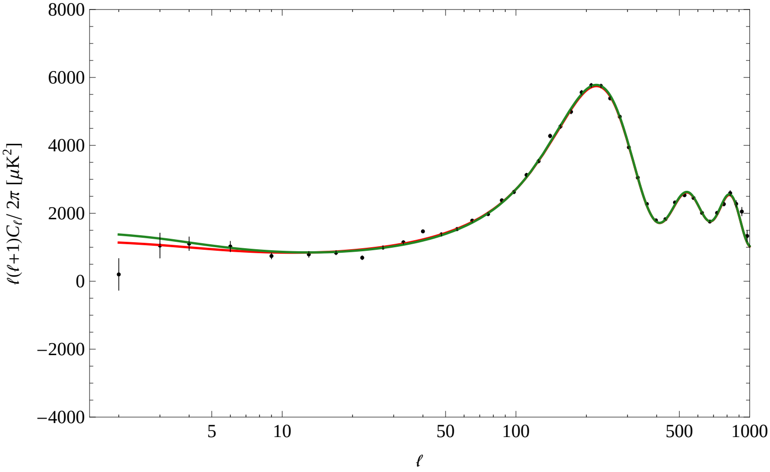
We worked out all cosmological observables for this class of theories.

- Prediction are different from those of conventional inflationary models, yet they are compatible with current data.
- The scalar power spectrum is given by

$$\Delta^2_{\mathcal{R}}(q) = \Delta^2_{\mathcal{R}} \frac{1}{1 + (gq_*/q)\ln|q/gq_*|},$$

  where q^* is a reference scale.
- The smallness of the amplitude $\Delta^2_{\mathcal{R}}$ is due to the fact that we are considering a large N theory.
- The small deviation from scale invariance is due to the fact that g, the coupling constant of the dual QFT, is very small!
- Non-gaussianities also exhibit interesting universal structure.

Angular power spectrum: ΛCDM vs holographic model



Red: ΛCDM, Green: holographic model

Confronting with data

- The scalar power spectrum is significantly different than that of conventional slow-roll models so given the success of ΛCDM one may wonder whether these holographic models are compatible with current data.
- We undertook a dedicated data analysis [Easther, Flauger, McFadden, KS (2011) (related work appeared in [Dias (2011)]) to custom-fit this model to WMAP and other astrophysical data.
- This model is compatible with WMAP and is competitive to ΛCDM model: a model selection analysis using Bayesian evidence shows that current data does not favor one or the other model.
- Results from the Planck satellite should be able to rule in or out this class of models!

Outline

Holographic slow-roll inflation [BMS, to appear]

By Theorem 3 we know that all standard slow-roll results should also be derivable from a strongly coupled QFT.

- What are the properties of the dual QFT?
- To what extend the slow-roll cosmological observables are fixed by the underlying (broken) conformal invariance?
- What is the phenomenology of corresponding weakly coupled models?

A holographic model for slow-roll inflation

- We define the model by giving the fake superpotential/Hubble function [Townsend (1984)][Townsend, KS (1999)] ... [Bond, Salopek (1990)],

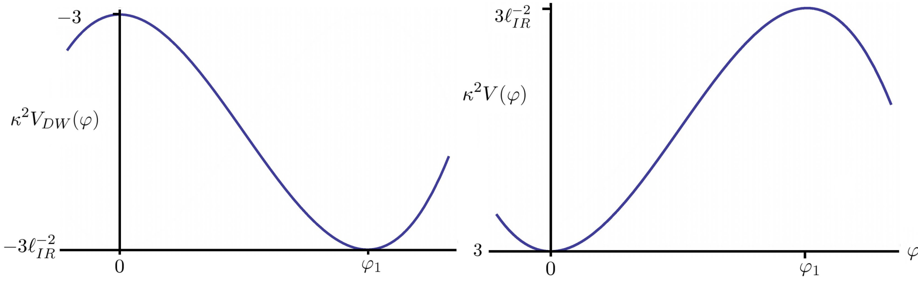
$$W(\Phi) = -2 - \frac{1}{2}\lambda\Phi^2 - \frac{1}{3}c\Phi^3,$$

  where we take $\lambda \ll 1$ and c to be of order 1.
- The background equations can be integrated exactly,

$$\phi(t) = \frac{3\lambda/c}{1+\exp(\lambda t)},$$

$$a(t) = \Big(1+\exp(\lambda t)\Big)^{-\lambda^2/3c^2} \exp\left[t(1+\frac{\lambda^3}{3c^2}) + \frac{\lambda^2\exp(\lambda t)}{3c^2(1+\exp(\lambda t))^2}\right],$$

Holographic RG vs Cosmology



Holographic interpretation

- On the domain-wall side, the solution has the interpretation as a deformation of the UV CFT by a relevant operator of dimension $\Delta_{UV} = 3 - \lambda$,

$$L = L^{UV}_{CFT} + \frac{2\lambda}{c} O_{\Delta_{UV}}$$

- This flows in the IR to a new CFT and in the vicinity of the IR fixed point the deforming operator has dimension $\Delta_{IR} = 3 + \lambda + O(\lambda^4)$.

$$L = L^{IR}_{CFT} - \frac{2\lambda}{c} O_{\Delta_{IR}}$$

- Since $\lambda \ll 1$ one can analyze the theory using conformal perturbation theory. This can be done either around the UV or the IR fixed point.

Cosmology

- On the cosmology side this describes a "Hilltop" inflationary model.
- One can compute the slow-roll parameters at horizon exit,

$$\epsilon_* = \frac{2\lambda^4}{c^2}\frac{q^{2\lambda}}{(1+q^\lambda)^4} + O(\lambda^7), \quad \eta_* = -\lambda + \frac{2\lambda}{1+q^\lambda} + O(\lambda^4),$$

- Cosmological observables can now be computed by applying standard formulas. For example, [Steward, Lyth (1993)]

$$\Delta_S^2 = \frac{q^3}{2\pi^2}\langle\langle\zeta(q)\zeta(-q)\rangle\rangle = \frac{H_*^2}{8\pi^2\epsilon_*}\left(1 + 2b\eta_* + O(\lambda^2)\right)$$

Holography for slow-roll inflation

- The holographic formulas express the cosmological observables in terms of correlation functions of the dual QFT.
- Using conformal perturbation theory we can express the correlation functions of the dual QFT in terms of CFT correlation functions.
- These CFT correlation functions are uniquely fixed by conformal invariance up to a few constants.
- If we fix these constant to be those computed by AdS/CFT at the fixed point, then we recover exactly the slow-roll results both for the power spectra and the non-gaussianities!
- This includes both scalar and tensor modes as well as all non-gaussianities.

Scalar Non-gaussianities

- The scalar non-gaussianity for slow-roll models has been worked by [Maldacena (2003)]. For the model at hand and to leading order λ the answer is

$$\langle\langle \zeta(q_1)\zeta(q_2)\zeta(q_3)\rangle\rangle = \frac{H_*^4 \eta_*}{16\epsilon_*^2}\left(\frac{1}{q_1^3 q_2^3} + \frac{1}{q_2^3 q_3^3} + \frac{1}{q_1^3 q_3^3}\right)$$

- In this limit the non-Gaussianity is purely of a local type with $f_{NL} = 5\eta^*/6$.
- We would like to reproduce this expression holographically.

Sketch of holographic computation

- The holographic formula relates $\langle\langle \zeta(q_1)\zeta(q_2)\zeta(q_3)\rangle\rangle$ with $\langle\langle T(q_1)T(q_2)T(q_3)\rangle\rangle$ and $\langle\langle T(q)T(-q)\rangle\rangle$ where T is the trace of the stress energy tensor.
- Ward identities of the 3d theory relate $\langle\langle T(q)T(-q)\rangle\rangle$ to $\langle\langle O(q))(-q)\rangle\rangle$ and $\langle\langle T(q_1)T(q_2)T(q_3)\rangle\rangle$ to $\langle\langle O(q_1)O(q_2)O(q_3)\rangle\rangle$.
- We need to compute these correlators in the theory specified by the action

  $$S = S_{CFT} + \lambda \int d^3 x O$$

  where the operator O has dimension $(3-\lambda)$.
- Since $\lambda << 1$ we can use conformal perturbation theory.

Conformal perturbation theory

- Let's discuss first the 2-point function

$$\langle O(x_1)O(x_2)\rangle = \langle O(x_1)O(x_2)e^{-\lambda\int O}\rangle_{CFT}$$
$$= \langle O(x_1)O(x_2)\rangle_{CFT} - \lambda\int d^3x\langle O(x_1)O(x_2)O(x)\rangle_{CFT} + \cdots$$

- Naively: only the terms displayed are universal and all higher order terms are negligible as $\lambda \to 0$.
- This turn out to be incorrect: *all higher order terms contribute and their leading order contribution as $\lambda \to 0$ is universal*.

Conformal perturbation theory

- One can show that

$$I_n = \int d^3z_1 \dots d^3z_n \langle O(x_1)O(x_2)O(z_1)\dots(z_n)\rangle_{CFT}.$$

in the limit $\lambda \to 0$ equals to

$$I_n \sim \frac{1}{\lambda^n}|x_{12}|^{(n+2)\lambda-6}$$

- This behavior is a manifestation of a (new?) conformal anomaly of correlators of dimension 3.

Resummed correlators

One can resum these corrections to obtain:

- 2-point function

$$\langle O(x_1)O(x_2)\rangle = c_2|x_{12}|^{2\lambda-6}\left[1+b|x_{12}|^{\lambda}\right]^{-4}+\dots$$

  where c_2 is the normalization of the conformal 2-point function and b depends on the coefficient of the deformation.

- 3-point function

$$\langle O(x_1)O(x_2)O(x_3)\rangle = c_3\prod_{i<j}|x_{ij}|^{-(3-\lambda)}\left[1+b|x_{ij}|^{\lambda}\right]^{-2}+\dots$$

  where c_3 is the constant characterizing the conformal 3-point function.

Holographic non-gaussianity

- Insert these expressions in the holographic formulas.
- Use for the constants c_2, c_3, b the values at the fixed point obtained via AdS/CFT.
- Slow-roll scalar non-gaussianity.

- These results are universal and hold also beyond the regime of validity of gravity: *the CFT in conformal perturbation theory can have couplings of any strength*.
- The only freedom left is a few constants like c_2, c_3 etc.

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Conclusions

- Inflation is holographic: standard observables such as power spectra and non-Gaussianities can be expressed in terms of (analytic continuation of) correlation functions of a dual QFT.
- The QFT dual to slow-roll inflation is a deformation of a CFT.
- Slow-roll results are essentially fixed by conformal invariance.
- There are new holographic models based on perturbative QFT that describe a universe that started in a non-geometric strongly coupled phase.
- A class of such models based on a super-renormalizable QFT was custom-fit to data and shown to provide a competitive model to ΛCDM. Data from the Planck satellite should permit a definitive test of this holographic scenario.

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