Blackfolds as fluids and materials

Workshop on Geometry and Physics, LMU, Munich, November 21, 2012 Niels Obers, NBI

1210.5197 (with J. Armas)

1209.2127 (PRL) (with J. Armas, J. Gath)

1110.4835 (JHEP) (with J. Armas, J. Camps, T. Harmark)

+ earlier work:

0912.2352 (JHEP), 0910.1601 (JHEP), 0902.0427 (PRL) + 1106.4428 (JHEP) (with R. Emparan, T. Harmark, V. Niarchos)

1012.5081 (PRD) (with J. Armas)

0708.2181 (JHEP) (with R. Emparan, T. Harmark, V. Niarchos, M.J. Rodriguez)

Motivations for Blackfold approach

- New insights into GR/geometry
- find BHs in higher dimensions and discover their properties
- effective theory that integrates out gravitational degrees of freedom
- AdS/CFT (fluid/gravity) inspired new way to look at gravity
- find universal features of black branes in long wave length regime described by "every day" physics
- reduce complicated gravitational physics to simple response coefficients

Reviews:

Harmark,Niarchos,NO Emparan,Reall/Emparan NO/Emparan,Harmark,Niarchos,NO Harmark,NO(to appear)

Intro +overview

- blackfold (BF) method developed in the last five years
 - effective theory of black brane dynamics

 dynamics of BHs in higher dimensions (especially D >5) too complex to be described by conventional methods

 BF is an effective description for black objects in terms of a fluid living on a dynamical worldvolume validity: horizon exhibits two (or more) widely separated length scales (not possible in 4D pure Einstein gravity)

but: can infer properties that go beyond by extrapolation e.g. instabilities, horizon topology changing mergers, hints for new (extremal) exact solutions, checks on numerics

- reveals that black branes have both "fluid" and "solid" properties
 for multi-charged branes of Sugra/ST: anisotropic fluids
- when applied to D-branes in ST: description of thermal D-branes that goes
 beyond the DBI description _____ new method for thermal probe branes

Plan

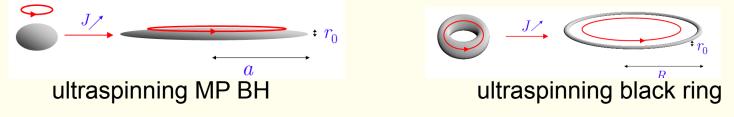
- Introduction : Separation of scales in higher-dimensional black holes
- Effective worldvolume theory: Blackfold (BF) approach
- Fluid and Solid properties of black branes
 - Intrinsic perturbations: viscosities
 - Extrinsic perturbations: relativistic Young-modulus, piezo electric moduli
- Discussion and outlook

In this talk: restrict mostly to asymptotically flat BH solutions of pure gravity and supergravity (including those relevant to ST) but:

- interesting parallels with BHs in KK spaces
- techniques are easily applied to AdS/dS space

Hierarchy of scales

- reason for new dynamics in higher-dimensional black holes:
 - possibility of different length scales along the horizon



- \longrightarrow natural framework:
 - organize black holes according to scale hierachy



Kerr behavior



mergers, connections
 -numerics
 e.g. Dias, Figueras, Monteiro, Santos, Emparan
 -conifold-type transitions
 Kol/Emparan,Haddad



develop new tool:

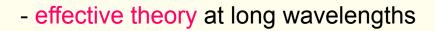
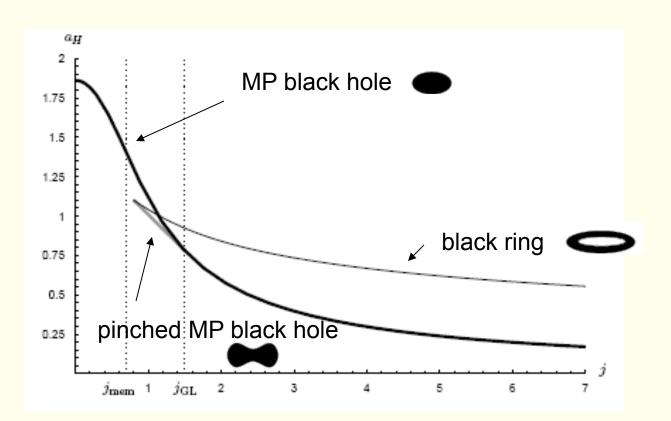


Illustration of hiarchies of scales

- Kerr regime
- regime of mergers and connections

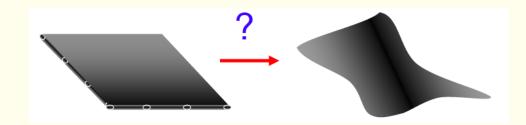
 i) new solution branches (via 0-modes_
 ii) topology changing transition
- BF regime (ultraspinning)



Blackfolds: A new framework

effective fluid living on a dynamical worldvolume:

- based on bending/vibrating of (flat) black branes

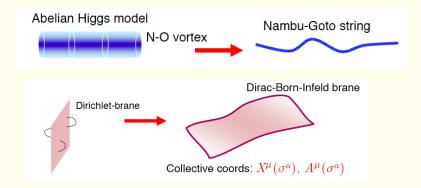


blackfold = black brane
wrapped on a compact
submanifold of spacetime



very much like other solitonic objects:

 Nielsen-Olesen vortices and NG strings



- open strings and DBI action

for blackfolds in addition: worldvolume thermodynamics

Main ingredients

- Ingredients:
- classical brane dynamics (Carter)
- long wavelengths: dynamics of fluid that lives on dynamical worldvolume
- black branes correspond to specific type of fluid

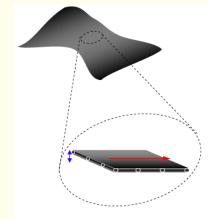
to leading order: perfect fluid

$$T^{ab} = (\varepsilon + P)u^a u^b + P\gamma^{ab}$$



for charged black branes of sugra: novel type of (an)isotropic charged fluids

black hole looks locally like a flat black brane



- identify collective coordinates of the brane
- blackfold equations of motion follow from conservation laws (stress tensor, currents,...)

notation: spacetime worldvolume

$$X^{\mu}, \mu, \nu \dots = 0, \dots, D-1$$

 $\sigma^{a}, a, b \dots = 0, \dots, p$ $n = D - p - 3$

Effective worldvolume theory – leading order

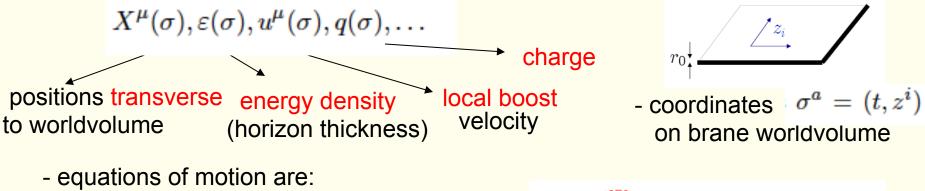
similar to effective theories for other extended objects:

difference: - short-distance d.o.f. = gravitational short-wavelength modes

- extended objects posses black hole horizon

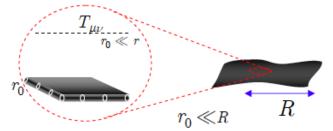
start from flat black brane solution:

- identify symmetries + conserved charges (and 0-modes from SSB): energy, momentum, charges, ...
- construct worldvolume densities as functions of these: $T_{\mu\nu}, J_{\mu}, \dots$ (integrate thickness)
- promote collective coordinates to slowly varying quantities



$$\bar{\nabla}_{\mu}T^{\mu\nu} = 0, \bar{\nabla}_{\mu}J^{\mu} = 0, \dots$$

validity:



Metric and stress tensor (neutral black branes)

metric of neutral (flat) black boosted branes

$$ds^{2} = \left(\eta_{ab} + \frac{r_{0}^{n}}{r^{n}}u_{a}u_{b}\right)d\sigma^{a}d\sigma^{b} + \left(1 - \frac{r_{0}^{n}}{r^{n}}\right)^{-1}dr^{2} + r^{2}d\Omega_{n+1}^{2}$$

introduce slowly varying collective coordinates (derivative expansion):

$$ds_{(\text{short})}^2 = \left(\gamma_{ab}(\sigma) + \frac{r_0^n(\sigma)}{r^n} u_a(\sigma) u_b(\sigma)\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n(\sigma)}{r^n}} + r^2 d\Omega_{n+1}^2 + \dots$$

induced metric $\gamma_{ab} = g_{\mu\nu} u^{\mu}_{a} u^{\nu}_{b}$ $u^{\mu}_{a} \equiv \partial_{a} X^{\mu}$

stress tensor:

$$T^{ab}(\sigma) = \frac{\Omega_{(n+1)}}{16\pi G} r_0^n(\sigma) \left(n u^a(\sigma) u^b(\sigma) - \gamma^{ab}(\sigma) \right) + \dots$$

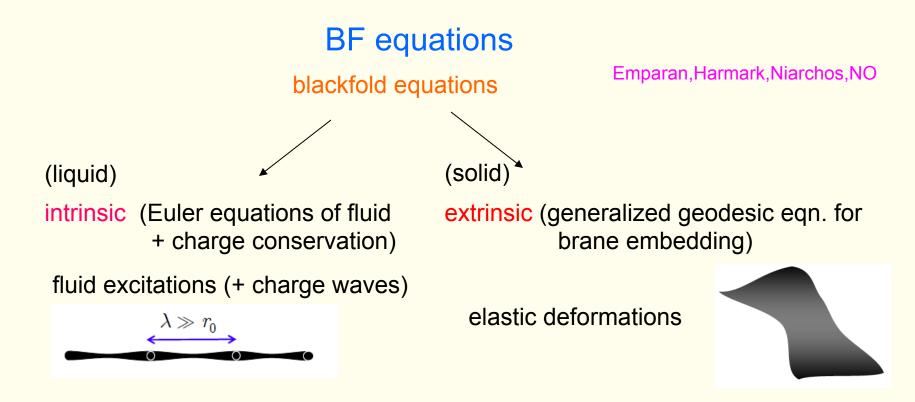
$$\varepsilon = \frac{\Omega_{(n+1)}}{16\pi G}(n+1)r_0^n\,,\qquad P = -\frac{1}{n+1}\varepsilon$$

Blackfold dynamics

consistent coupling of wv. to long-wave length gravitational field

 $\nabla_{\mu}T^{\mu\rho} = 0$ (stress tensor supported on worldvolume) (using embedding tensors, extrinsic $T^{\mu\nu}K_{\mu\nu}{}^{\rho} + \partial_b X^{\rho}D_a T^{ab}$ (Using embedding tensors curvature tensor etc. etc.) (for particle worldline: $ma^{\nu} + (D_{\tau}m)u^{\mu} = 0$) stress tensor conservation. $T^{\mu\nu}K_{\mu\nu}{}^{\rho} = 0$ $D_a T^{ab} = 0$ extrinsic equations (D-p-1) intrinsic equations (p+1) determine the collective brane coords. generalized $T^{ab}\left(D_a\partial_b X^{\rho} + \Gamma^{\rho}_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu}\right) = 0$ geodesic equation

• charge conservation: $D_a J^{a_1 \cdots a_p + 1} = 0$



- gives novel stationary black holes (metric/thermo) + allows study of time evolution
- generalizes (for charged branes) DBI/NG to non-extremal solns. (thermal)
- possible in principle to incorporate higher-derivative corrections (self-gravitation + internal structure/multipole) recently: BF equations have been derived from Einstein equations !

Camps, Emparan

- cf. closely related precedents of mappings black holes to fluid dynamics
 - membrane paradigm
 - fluid/AdS-gravity correspondence

Stationary solutions

equilibrium configurations stationary in time = stationary black holes

$$u = \frac{k}{|k|}$$
, $\nabla_{(\mu}k_{\nu)} = 0$, $\mathcal{T}(\sigma^a) = \frac{T}{|k|}$ $k = \xi + \Omega\chi$

can solve intrinsic blackfold equations explicitly (e.g. for thickness and velocity)
 only need to solve extrinsic equations for the embedding

$$r_0 = \frac{n\sqrt{1-V^2}}{2\kappa}$$
 $V^2 = \sum_i \Omega_i^2 R_i^2(\sigma)$ velocity field

• blackfolds with boundaries: fluid approaches speed of light at bdry. (horizon closes off !)

extrinsic equations: $K^{\rho} = \perp^{\rho\mu} \partial_{\mu} \ln(-P)$

derivable from action
$$\tilde{I} =$$

$$\tilde{I} = \int_{\mathcal{W}_{p+1}} d^{p+1} \sigma \sqrt{-\gamma} P$$

- thermodynamics: all global quantities: mass, charge, entropy, chemical potentials by integrating suitable densities over the worldvolume

Action principle for stationary blackfolds and 1st law

for any embedding (not nec. solution) the "mechanical" action is proportional to Gibbs free energy:

$$\beta^{-1}I = G = M - \sum_{i} \Omega_i J_i - TS$$

varying G -> 1st law of thermodynamics

 $dM = TdS + \Omega dJ \qquad \text{(fixed } Q_p\text{)}$

1st law of thermo = blackfold equations for stationary configurations

 can also use Smarr relation to show that: total tension vanishes for stationary blackfolds

$$(D-3)M - (D-2)(TS + \Omega J) - n\Phi_H^{(p)}Q_p = \mathcal{T}_{\text{tot}}$$

Neutral stationary blackfolds

Emparan, Harmark, Niarchos, NO

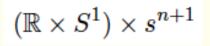
using neutral and black branes of higher dim gravity:

Quick overview of results:

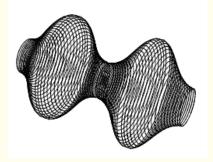
-new helical black strings and rings

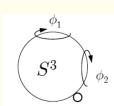
 odd-branes wrapped on odd-spheres (generalizes 5D black ring) $(\prod_{p_a = \text{odd}} S^{p_a})$

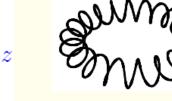
- non-uniform black cylinders
- static minimal blackfolds (non-compact)

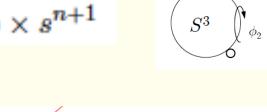












Blackfolds in supergravity and string theory

Emparan, Harmark, Niarchos, NO Caldarelli, Emparan, v. Pol Grignani, Harmark, Marini, NO, Orselli

- BF method originally developed for neutral BHs, but even richer dynamics when considering charged branes
- simplest case: curving the fundamental black branes of string/M-theory into black holes with with compact horizon topologies (F1, Dp, NS5, M2, M5)

more generally: consider dilatonic black branes that solve action

$$I = \frac{1}{16\pi G} \int d^D x \sqrt{g} \left(R - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2(p+2)!} e^{a\phi} F_{(p+2)}^2 \right)$$

(p=0: particle charge, p=1: string dipole charge, etc.)

+ important generalizations when having furthermore dissolved charges (anistropic charged fluids), e.g. using multi-charge black branes

Also many interesting cases with charged q=0 or 1 branes dissolved in neutral p-brane - electric rotating black holes (near-extr, w. slow rotation, new instabilities) - new classes of black holes with string dipoles

Intermezzo: blackfolds in art





contact

h30 45x40cm 07

Black branes as fluids and solids

Goal: show that (charged) black branes have both fluid and solid properties

Method: perturb -> consider derivative corrections

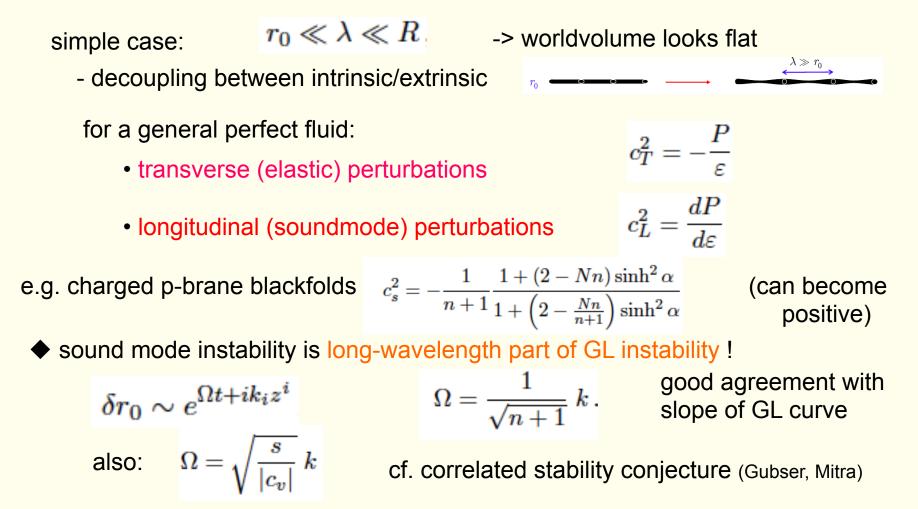
two ways

- intrinsic perturbations along worldvolume (wiggle)
- extrinsic perturbations transverse to the worldvolume (bend)

Punchline:a new parallel between (electro)elasticity theory and gravitational physics

Gregory-Laflamme instability

- blackfold approach captures perturbative dynamics of BH when $\lambda \gg r_0$
- can be intrinsic variations (thickness, local velocity) or extrinsic (embedding)
 -> generally coupled



Intrinsic transport coefficients: viscosities

Camps, Emparan, Haddad

- can study in derivative expansion the intrinsic perturbations for flat branes
- to first order in derivative expansion

$$T^{ab} = \left(\epsilon u^a u^b + PP^{ab} - 2\eta\sigma_{ab} - \zeta\vartheta P^{ab}\right)\delta^{n+2}(x^\rho - X^\rho)$$

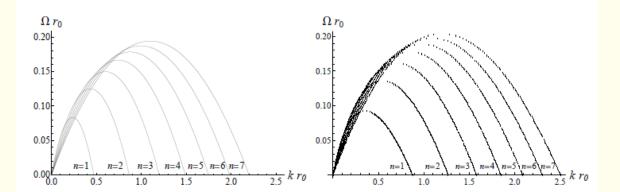
$$\eta = \frac{s}{4\pi}$$
, $\zeta = 2\eta \left(\frac{1}{p} - c_s^2\right)$

shear and bulk viscosity

can apply e.g. to GL instability: include viscous damping of the soundmode perturbations in dispersion relation

$$\Omega = \frac{k}{n+1} \left(1 - \frac{n+1}{n\sqrt{n+1}} k r_0 \right)$$

fits very well with GL curve



recent work: Skenderis et al

Fine structure corrections to blackfolds

Armas, Camps, Harmark, NO

can explore corrections in BF approach that probe the fine structure: go beyond approximation where they are approximately thin

$$T^{\mu\nu}(x^{\alpha}) = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma\sqrt{-\gamma} \left[B^{\mu\nu}(\sigma^a) \frac{\delta^{(D)}(x^{\alpha} - X^{\alpha})}{\sqrt{-g}} - \nabla_{\rho} \left(B^{\mu\nu\rho}(\sigma^a) \frac{\delta^{(D)}(x^{\alpha} - X^{\alpha})}{\sqrt{-g}} \right) + \dots \right]$$

acounts for:

- dipole moment of wv stress energy
- internal spin degrees of freedom (conserved angular momentum density)

corrected pole/dipole BF equations:

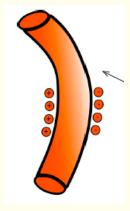
$$D_a T^{ab} = K_c{}^b{}_\mu \nabla_a d^{ac\mu}$$

$$T^{ab}K^{\rho}_{ab} = -K_{ac}{}^{\rho}K^{(c}{}_{b\lambda}d^{a)b\lambda} - \bot^{\rho}{}_{\sigma}\nabla_{b}\nabla_{a}d^{abc}$$

(including internal spin: generalizes Papapetrou equations of spinning point particle in GR to extended objects)

$$d^{ab\rho} = u^a_\mu u^b_\nu \bot^\rho{}_\lambda B^{\mu\nu\lambda} = B^{\rho ab}_\bot$$

$$\begin{split} j^{a\mu\nu} &= 2u^a_\rho \bot^\mu{}_\sigma \bot^\nu{}_\lambda B^{\rho[\sigma\lambda]} = 2B^{a\mu\nu}_\bot \ , \\ u^\mu_a &\equiv \partial_a X^\mu \end{split}$$



Extrinsic response coefficients: Young modulus

 bending of black string (or brane) induces dipole moments of stress can be measured from approximate analytic solution (obtained using MAE)

$$g_{\mu\nu} = \eta_{\mu\nu} + h^{(M)}_{\mu\nu} + h^{(D)}_{\mu\nu} + \mathcal{O}\left(r^{-n-2}\right) \qquad \nabla_{\perp}^2 \bar{h}^{(D)}_{\mu\nu} = 16\pi G d_{\mu\nu}{}^r{}_{\perp} \partial_{r_{\perp}} \delta^{(n+2)}(r)$$

dipole correction is induced elastically by bending of black brane
 long wave length linear response coefficient capturing this effect

$$d_{ab}{}^{\hat{
ho}} = \mathcal{C}(n) Y_{ab}{}^{cd} K_{cd}{}^{\hat{
ho}}$$

relativistic generalization of Young modulus of elastic materials

linear relation between stress and strain (Hookean regime)

new response coefficient for black objects !

computed for neutral black strings (and p-branes)

$$Y^{ttzz} = Y^{zztt} = -\frac{\Omega_{(n)}(n+2)}{16\pi G r_0^2} (n^2 + 3n + 4)\xi(n) \qquad \qquad Y^{tttt} = Y^{zzzz} = \frac{\Omega_{(n)}(n+2)(n+4)}{16\pi G r_0^2} (3n+4)\xi(n)$$

Fine structure: Charged black branes

- branes charged under Maxwell fields: multipole expansion of current

(resulting corrected BF equations generalize those of the charged spinning point particle)

- for piezo electric materials: dipole moment proportional to strain $p^{a\rho} = \tilde{\kappa}^{abc} K_{cd}{}^{\rho}$
- can be measured in gravity by computing the first order correction to bent charged black branes

example: charged black branes of EMD theory obtained by uplift-boost-reduce from neutral charged branes (other examples with higher p-form charge also possible: To appear)

Extrinsic response coefficients: Piezo-electric moduli

Armas,Gath,,NO

bending of charged black string (or brane) induces dipole moments of charge can be measured from approximate analytic solution (obtained using MAE)

 $A_{\mu} = A_{\mu}^{(M)} + A_{\mu}^{(D)} + \mathcal{O}\left(r^{-n-2}\right) \qquad \nabla_{\perp}^{2} A_{\nu}^{(D)} = 16\pi G p_{\nu}{}^{r_{\perp}} \partial_{r_{\perp}} \delta^{(n+2)}(r)$

 electric dipole correction is induced elastically by bending of charged black brane

$$p^{a\rho} = \tilde{\kappa}^{abc} K_{cd}{}^{\rho}$$

relativistic generalization of piezo-electric modulus found in electro-elasticity

new response coefficient for charged black objects !

- can also compute the relativistic Young moduli (which depend on charge param.)

Full 1st order corrected stress tensor and current

corrected blackfold equations take simple form:

$$D_a T^{ab} = 0, \ T^{ab} K_{ab}{}^{
ho} = 0, \ D_a J^a = 0$$

 $T^{ab} = T^{ab}_{(0)} + \tau^{ab}_{(1)}, \qquad \qquad J^a = J^a_{(0)} + \Upsilon^a_{(1)}$

$$\tau^{ab}_{(1)} = \tilde{Y}^{abcd} K_{cd}{}^{\rho} K_{\rho} , \quad \Upsilon^a_{(1)} = \tilde{\kappa}^{abc} K_{bc}{}^{\rho} K_{\rho}$$

full first-order corrected stress tensor:

$$T_{ab} = \left(\varepsilon u_a u_b + PP_{ab} - 2\eta\sigma_{ab} - \zeta\vartheta P_{ab} + \tilde{Y}_{ab}{}^{cd}K_{cd}{}^i\partial_i\right)\delta_{\perp}^{(n+2)}\left(x^i - X^i(\sigma^a)\right)$$

Relativistic Elasticity of Stationary Fluid Branes

Armas,NO

Question: can we formulate the extrinsic dynamics more generally in terms of relativistic elasticity theory ?

Yes:

given fluid configuration satisfying local thermodynamic laws living on thin space-time surface, assuming diffeomorphism invariance and ignoring backreaction:

extrinsic dynamics in transverse directions to the surface correspond to that of elastic brane

new interpretation:

expected to lead in the future to formal development of general relativistic elasticity theory of branes from gravity

cf. rigorous development of fluid and superfluid dynamics using fluid/gravity correspondence

Elasticity tensor

strain tensor $U_{ab} = -\frac{1}{2} \left(\gamma_{ab} - \bar{\gamma}_{ab} \right) \qquad dU_{ab} = N_{\rho} K_{ab}{}^{\rho}$

stress

$$T^{ab} = \rho u^a u^b + \sigma^{ab}$$

 $ho = \epsilon + P$, $\sigma^{ab} = P \gamma^{ab}$

intrinsic fluid dynamics implies extrinsic elastic dynamics

$$d
ho = \mathcal{T} ds + \sigma^{ab} dU_{ab}$$

Fluid dynamics fixes the properties:

T , Ω_i

Elastic dynamics fixes the properties:

K , E^{abcd}

Modulus of hydrostatic compression: Elasticity tensor:

$$K = \left(\frac{\partial P}{\partial \mathcal{V}}\right)_T = -P$$

$$E^{abcd} = 2\left(P\gamma^{a(c}\gamma^{d)b} - \left(\frac{\partial P}{\partial\gamma_{ab}}\right)\gamma^{cd} - 2\left(\frac{\partial^2 P}{\partial\gamma_{ab}\partial\gamma_{cd}}\right)\right)$$

$$dT^{ab} = E^{abcd} dU_{cd}$$

can compute these quantities for neutral black branes

Relevance of BF method

- new stationary BH solutions: approximate analytic construction of BH metrics in higher D gravity/ supergravities (cf. String Theory)
 EHONR/EHON/ Caldarelli,Emparan,Rodriguez Armas,NO/Camps,Emparan,Giusto,Saxena/..
 - possible horizon topologies, thermodynamics, phase structure, ...
 - new non-extremal and extremal BH solutions
 - useful for insights/checks on exact analytic/numeric solutions
- BH instabilities and response coefficients: Camps,Emparan,Haddad Armas,Camps,Harmark,NC understand GL instabilities in long wavelength regime, dispersion relation, elastic (in) stabilities, new long wavelength response coefficients for BHs, Young modulus (hydro + material science)
- Thermal probe branes/strings:

new method to probe finite T backgrounds with probes that are in thermal equilibrium with the background (e.g. hot flat space, BHs)

• AdS/CFT:

Grignani,Harmark,Marini,NO,Orsell Armas,Harmark,NO,Orselli,Vigand Pedersen

many potential applications (new black objects in AdS, connection with fluid/gravity, thermal probes thermal giant gravitons, BHs on branes, ...)

+ interrelations between the four items above

Outlook

• new response coefficients for higher p-form charged branes

Armas, Gath, NO (to appear)

AdS backgrounds ? Caldarelli, Empran, Rodriguez Armas, NO

- response coefficients in other backgrounds with non-zero fluxes (susceptibility, polarizability)
- interpretation of new response coefficients in AdS/CFT context
- relation to fluid/AdS-gravity correspondence
- further elucidate relation of BFs with DBI/NG
- use of BF as new method for thermal probe branes in AdS/CFT

The end

Outlook (con'td)

- blackfold gives a new method for D-brane (and other) probes in thermal backgrounds
- + applied to simplest case: Bion in hot flat space
 - contrary to previous work:
 - takes into account that the probe itself is a thermal object
- apply new perspective to AdS probes (thermal AdS or AdS BH)
 may resolve discrepancies between gravity and gauge theory found for Polaykov loops based on D3/D5
- revisit other previously studied cases
- -> Polyakov loops using thermal fundamental string probes in AdS/CFT.

thermal giant gravitons

Grignani, Harmark, Marini,NO,Orsell Armas,Harmark,NO,Orselli,Vigand Pedersen

reveals new qualitative/quantitative effects