# Aspects of non-geometric compactifications

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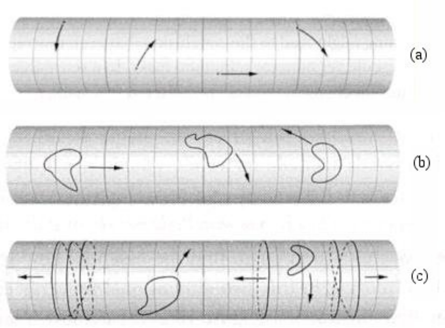
D. Andriot, M. L., D. Lüst, P. Patalong (1106.4015, 1211.XXXX),
D. Andriot, O. Hohm, M. L., D. Lüst, P. Patalong (1202.3060, 1204.1979),

# Motivation and summary: Why non-geometry?

Because it's there.




(universe-review.ca)

Strings see geometry differently from particles.

- T-duality
- Non-geometric configurations

# Motivation and summary: Why non-geometry?

Because it's interesting.

## String compactifications to 4D

Standard fluxes don't give all 4D supergravities.
Many no-go's for 4D de Sitter solutions.

"Non-geometric fluxes" $Q, R$: more gaugings; avoids no-go's.

Can 4D non-geometric solutions be given a 10D description?

**Proposal:** 10D field redefinition $\Rightarrow$ non-geometric flux in 10D.
New field basis: dimensional reduction possible.

Focus: NSNS-sector $(g, \phi, B)$.

## Non-commutativity

$Q$-flux $\Longrightarrow$ Closed string non-commutativity?

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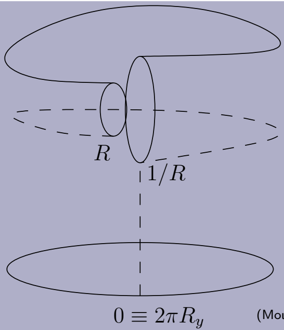
# Outline

# Background: Non-geometry in 10D

## T-folds etc. Hull:04, Hellerman et. al:02, Flourney et. al:04, ...

- Locally geometric
- Global non-geometry: stringy transition functions
- T-fold: $g, B$ patched by T-duality




(Moutsopoulos: PhD thesis 2008)

# Background: Non-geometry in 4D

Shelton–Taylor–Wecht:05

Flux compactification $\rightsquigarrow$ 4D gauged supergravity.
Many more 4D gaugings than NSNS fluxes!

Require T-duality covariance

$$\rightsquigarrow \boxed{\text{non-geometric fluxes } Q, R.}$$

## T-duality chain

$$H_{abc} \xrightarrow{T_a} f^a{}_{bc} \xrightarrow{T_b} Q_c{}^{ab} \xrightarrow{T_c} R^{abc}$$

$f^a{}_{bc}$ metric flux: $\mathrm{d}e^a = -\frac{1}{2} f^a{}_{bc} e^b \wedge e^c$.

10D origin for $Q, R$?

# Field redefinition

ALLP:11, AHLLP:12

See also: Blumenhagen et al:12 (talk by E. Plauschinn)

Find 10D description for $Q, R$ through a field redefinition.

## Starting point

$$e^{2d}\mathcal{L}_{NSNS} = \mathcal{R}(g) + 4(\partial\phi)^2 - \frac{1}{12}H_{ijk}H^{ijk}$$

## Field redefinition (using GG/DFT)

$$(\tilde{g}^{-1} + \beta)^{-1} \equiv \tilde{\mathcal{E}}^{-1} = \mathcal{E} = g + b$$
$$\sqrt{|g|}e^{-2\phi} = e^{-2d} = \sqrt{|\tilde{g}|}e^{-2\tilde{\phi}}$$

$\beta$: antisymmetric bi-vector $\beta^{mn}$

## Goal

$$e^{2d}\mathcal{L}_{\text{final}} = \mathcal{R}(\tilde{g}) + 4(D\tilde{\phi})^2 - \frac{1}{12}R^{ijk}R_{ijk} + (Q \text{ terms})$$

# Field redefinition

Why would this work?

## Doubling the geometry

Transition functions $\in$ structure group of tangent bundle.

$O(d,d)$ transition function $\to$ $2d$-dim gen. tangent bundle.

- Doubled geometry Hull:04,...
- Generalized geometry Hitchin:02, Gualtieri:04, ...

Generalized metric:

$$\mathcal{H}^{MN} = \begin{pmatrix} g_{ij} - b_{ik} g^{kl} b_{lj} & b_{ik} g^{kj} \\ -g^{ik} b_{kj} & g^{ij} \end{pmatrix} = \begin{pmatrix} \tilde{g}_{ij} & -\tilde{g}_{ik}\beta^{kj} \\ \beta^{ik}\tilde{g}_{kj} & \tilde{g}^{ij} - \beta^{ik}\tilde{g}_{kl}\beta^{lj} \end{pmatrix}$$

$$\mathcal{H} = E^T \mathbb{I}_{2d} E \ .$$

Field redefinition: choice of generalized vielbeine.

# GG: $\beta$ and non-geometry Graña–Minasian–Petrini–Waldram:08

See also: Grange–Schafer-Nameki:06,07, Ellwood:06, Halmagyi:08,09

Globally consistent field basis:

- Standard geometric description: $g\ ,\quad B\ ,\quad \phi$.
- Locally, can always transform $E \to KE$; $K^T \mathbb{I}_{2d} K = \mathbb{I}_{2d}$
- Transformation not globally defined — non-geometric set-up.

# Field redefinition

Use DFT: immense technical simplification

$$\begin{array}{ccc}
\mathcal{L}_{\mathrm{DFT}}(\mathcal{E}, d) & \overset{\text{field redef.}}{=\!=\!=} \overset{\text{formal T-d.}}{=\!=\!=} & \mathcal{L}_{\mathrm{DFT}}(\tilde{\mathcal{E}}, d) \\
\Big\| {\scriptstyle \tilde\partial=0} & & \Big\| {\scriptstyle \tilde\partial=0} \\
\mathcal{L}_{\mathrm{NSNS}}(g, b, d)(x) + \partial(..) & \overset{\text{field redef.}}{=\!=\!=} & \mathcal{L}_{\mathrm{final}}(\tilde{g}, \beta, d)(x) + \partial(..)
\end{array}$$

## Double Field Theory

Hull, Zwiebach:08,Hohm, H, Z:10,...

- Complement coordinates $x^i$ with their duals $\tilde{x}_i$.
- Strong constraint: $\partial_i A\, \tilde{\partial}^i B + \tilde{\partial}^i A\, \partial_i B = 0$
- $\mathcal{L}_{\mathrm{DFT}}\Big|_{\tilde{\partial}=0} = e^{-2\phi}\sqrt{|g|}\left(\mathcal{R} + 4(\partial\phi)^2 - \frac{1}{12} H_{ijk} H^{ijk}\right) + \partial(\ldots)$
- $S_{DFT}[\mathcal{E}, d]$ is $O(d, d)$ invariant.

# Field redefinition

## Result of DFT rewriting

$$
\begin{aligned}
e^{2d}\mathcal{L}_{\mathrm{DFT}}(\tilde{g},\beta,d) = {} & \mathcal{R}(\tilde{g}) + 4(\partial\tilde{\phi})^2 + 4(\tilde{D}d)^2 - \frac{1}{12}R^{ijk}R_{ijk} \\
& - \frac{1}{4}\tilde{g}_{ik}\tilde{g}_{jl}\tilde{g}^{rs}\,Q_r{}^{kl}\,Q_s{}^{ij} - \frac{1}{2}\tilde{g}_{pq}Q_k{}^{lp}Q_l{}^{kq} - \tilde{g}_{ij}\,Q_p{}^{pi}\,Q_q{}^{qj} \\
& - 2\tilde{g}_{ij}\,\tilde{D}^i Q_k{}^{kj} - 2\tilde{D}^i\tilde{g}_{ij}\,Q_k{}^{kj} - \tilde{g}_{jl}\tilde{g}_{pq}\,Q_k{}^{lp}\tilde{D}^j\tilde{g}^{kq} \\
& - \tilde{D}^i\tilde{D}^j\tilde{g}_{ij} - \frac{1}{4}\tilde{g}_{ik}\tilde{g}_{jl}\tilde{g}_{pq}\left(\tilde{D}^p\tilde{g}^{kl}\tilde{D}^q\tilde{g}^{ij} - 2\tilde{D}^i\tilde{g}^{lp}\tilde{D}^j\tilde{g}^{kq}\right) .
\end{aligned}
$$

$$
\begin{aligned}
R^{ijk} &= 3\tilde{D}^{[i}\beta^{jk]} &\longrightarrow\quad& 3\beta^{p[i}\partial_p\beta^{jk]} \\
Q_m{}^{nk} &= \partial_m\beta^{nk} &\longrightarrow\quad& \partial_m\beta^{nk} \\
\tilde{D}^i &\equiv \tilde{\partial}^i - \beta^{ij}\partial_j &\longrightarrow\quad& -\beta^{ij}\partial_j
\end{aligned}
$$

$$
e^{2d}\mathcal{L}_{\mathrm{DFT}}(\tilde{g},\beta,d) \longrightarrow \mathcal{R}(\tilde{g}) + 4(\partial\tilde{\phi})^2 - \tfrac{1}{4}\tilde{g}_{ik}\tilde{g}_{jl}\tilde{g}^{rs}\,Q_r{}^{kl}\,Q_s{}^{ij}
$$

when $\tilde{\partial} = 0$ , $\beta^{ij}\partial_j = 0$ , $\partial_j\beta^{ij} = 0$

# Geometry of $Q, R$

Geometric interpretation of new fluxes?

### Geometric fluxes

- $H_{ijk}$ tensor
- ${f^i}_{jk} \sim$ connection

### Non-geometric fluxes

- $R^{ijk}$ tensor?
- ${Q_k}^{ij} \sim$ dual connection?

### Strategy

Make $x$ diffeomorphism invariance manifest in $S_{DFT}$

# Geometry of $Q, R$

$\tilde{g}$ covariant: $\Delta_\xi \tilde{g}_{ij} \equiv (\delta_\xi - \mathcal{L}_\xi)\tilde{g}_{ij} = 0$

$\beta$ non-cov.: $\Delta_\xi \beta^{ij} = \tilde{\partial}^i \xi^j - \tilde{\partial}^j \xi^i$

$\tilde{\phi}$ scalar: $\Delta_\xi \tilde{\phi} = 0$

$(\tilde{\partial}^i \tilde{\phi})$ non-cov.: $\Delta_\xi (\tilde{\partial}^i \tilde{\phi}) = \tilde{\partial}^i \xi^j - \tilde{\partial}^j \xi^i \Longrightarrow \boxed{\Delta_\xi (\tilde{D}^i \tilde{\phi}) = 0}$

Similarly $\boxed{\Delta_\xi R^{ijk} = 0}$

$\tilde{D}^i$: covariant derivative for scalar

Construct covariant derivative for other tensors!

$\tilde{\nabla}^i V^j = \tilde{D}^i V^j - \check{\Gamma}_k{}^{ij} V^k$ metricity $+ [\tilde{\nabla}^i, \tilde{\nabla}^j]\tilde{\phi} = -R^{ijk}\partial_k \tilde{\phi}$

$$\Longrightarrow \boxed{\check{\Gamma}_k{}^{ij} = \tilde{\Gamma}_k{}^{ij} + \tilde{g}_{kl}\tilde{g}^{p(i} Q_p{}^{j)l} - \tfrac{1}{2} Q_k{}^{ij}}$$

# Result

New torsion: $\mathcal{T}^i = \check{\Gamma}_k{}^{ki}$

Dual Riemann tensor: $\left[\tilde{\nabla}^i, \tilde{\nabla}^j\right] V_k = -R^{ijp} \nabla_p V_k + \check{\mathcal{R}}^{ij}{}_k{}^l V_l$

## Manifest diffeomorphism invariant action:

$$\int dx d\tilde{x} \sqrt{|\tilde{g}|}\, e^{-2\tilde{\phi}} \Big[ \mathcal{R} + \check{\mathcal{R}} - \frac{1}{12} R^{ijk} R_{ijk} + 4(\partial \tilde{\phi})^2 + 4\big(\tilde{D}^i \tilde{\phi} + \mathcal{T}^i\big)^2 \Big]$$

# Result

New torsion: $\mathcal{T}^i = \check{\Gamma}_k{}^{ki}$

Dual Riemann tensor: $[\tilde{\nabla}^i, \tilde{\nabla}^j] V_k = -R^{ijp} \nabla_p V_k + \check{\mathcal{R}}^{ij}{}_k{}^l V_l$

**Manifest diffeomorphism invariant action:**

$$\int dx d\tilde{x} \sqrt{|\tilde{g}|}\, e^{-2\tilde{\phi}} \left[ \mathcal{R} + \check{\mathcal{R}} - \frac{1}{12} R^{ijk} R_{ijk} + 4(\partial\tilde{\phi})^2 + 4\left(\tilde{D}^i \tilde{\phi} + \mathcal{T}^i\right)^2 \right]$$

$Q$

# Result

## Geometric fluxes

- $H_{ijk}$ tensor
- $f^i{}_{jk} \sim$ connection

## Non-geometric fluxes

- $R^{ijk}$ tensor
- $Q_k{}^{ij} \sim$ dual connection

In general case: DFT needed for geometric interpretation.

## Comparison with 4D

Global non-geometry (T-fold) $\longrightarrow$ local $Q$, $R$ terms.
$\implies$ Standard flux compactification possible:
expected 4D gaugings reproduced $\checkmark$

# Non-commutativity

## Open string    Seiberg, Witten:99; also Chu, Ho:98, Schomerus:99,...

String ending on fluxed $D$-brane:

$$[X^m(\tau,\sigma^*), X^n(\tau,\sigma^*)]_{\text{open}} = i\theta^{mn}$$

open string metric: $G^{mn} = \left(\frac{1}{g+2\pi\alpha' B} g \frac{1}{g-2\pi\alpha' B}\right)^{mn}$

non-com. parameter: $\theta^{mn} = -(2\pi\alpha')^2 \left(\frac{1}{g+2\pi\alpha' B} B \frac{1}{g-2\pi\alpha' B}\right)^{mn}$

## Non-geometric closed string

redefined metric: $\tilde{g}^{mn} = \left(\frac{1}{g+b} g \frac{1}{g-b}\right)^{mn}$

$\beta$ field: $\beta^{mn} = \left(\frac{1}{g+b} b \frac{1}{g-b}\right)^{mn}$

# Non-commutativity

## Closed string non-associativity

$$[[X^m(\tau,\sigma), X^n(\tau,\sigma)], X^k(\tau,\sigma)]_{\text{closed}} + (\text{cyclic}) \sim R^{mnk}$$

Blumenhagen et al:10,11, Mylonas et al:12

## Closed string non-commutativity

Examples found — Lüst:10, Condeescu et al:12, Saemann, Szabo:12

also Mathai et al:00, 04, 07, Mylonas et al:12

(talk by P. Schupp)

Conjecture

$$[X^m(\tau,\sigma), X^n(\tau,\sigma')]_{\text{closed}} \xrightarrow{\sigma'\to\sigma} i \oint Q_k{}^{mn}(X) dX^k$$

AHLLP:12

# Non-commutativity

## Closed string non-commutativity

Conjecture

$$[X^m(\tau,\sigma), X^n(\tau,\sigma')]_{\rm closed} \xrightarrow{\sigma'\to\sigma} i\oint Q_k{}^{mn}(X)\,dX^k \qquad \text{AHLLP:12}$$

## Commutators through T-duality

ALLP work in progress

$$S = -\frac{1}{4\pi\alpha'}\int_\Sigma \mathrm{d}^2\sigma\left(G_{mn}(\mathcal{X})\ \eta^{\alpha\beta} + B_{mn}(\mathcal{X})\ \varepsilon^{\alpha\beta}\right)\partial_\alpha\mathcal{X}^m\partial_\beta\mathcal{X}^n$$

| Geometric background | Non-geometric background |
|---|---|
| $\mathcal{X}^m = Y^m$ | $\mathcal{X}^m = Z^m$ |
| Canonical commutators | — |
| $[Y^m(\tau,\sigma), Y^n(\tau,\sigma')] = 0$ | $[Z^m(\tau,\sigma), Z^n(\tau,\sigma')] =?$ |

# Non-commutativity: Example

Kachru et. al:02, Lowe et. al:03

String duality can relate geometric and non-geometric set-ups.

## T-dualities of a three-torus with flux

Kachru et. al:02

Frame A: Square three-torus with B-field:

$$g_A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, B_A = \begin{pmatrix} 0 & -z & 0 \\ z & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \phi_A = \phi(z)$$

Frame B: Twisted three-torus, zero B-field:

$$g_B = \begin{pmatrix} 1 & z & 0 \\ z & 1+z^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, B_B = 0 \,, \phi_B = \phi(z)$$

Frame C: Non-geometric set-up (T-fold):

$$g_C = \begin{pmatrix} \frac{1}{1+z^2} & 0 & 0 \\ 0 & \frac{1}{1+z^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, B_C = \frac{1}{1+z^2} \begin{pmatrix} 0 & z & 0 \\ -z & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\phi_C = \phi(z) - \frac{1}{2} \ln(1+z^2) \,.$$

# Non-commutativity

## Closed string non-commutativity

Conjecture

$$[X^m(\tau,\sigma), X^n(\tau,\sigma')]_{\text{closed}} \xrightarrow{\sigma'\to\sigma} i \oint Q_k{}^{mn}(X) dX^k$$ AHLLP:12

## Commutators through T-duality

ALLP work in progress

- Find solutions $Y^m(\tau,\sigma)$ for geometric background $G, B$.
- Canonical quantization: commutators for modes.
- Determine $Z^m(\tau,\sigma)$ through T-duality (on $k$):

$$\partial_\tau Z^k = G_{kk}\partial_\sigma Y^k + G_{km}\partial_\sigma Y^m + B_{km}\partial_\tau Y^m$$
$$\partial_\sigma Z^k = G_{kk}\partial_\tau Y^k + G_{km}\partial_\tau Y^m + B_{km}\partial_\sigma Y^m$$
$$\partial_\tau Z^m = \partial_\tau Y^m\,, \quad \partial_\sigma Z^m = \partial_\sigma Y^m\,.$$

- Compute $[Z^m(\tau,\sigma), Z^n(\tau,\sigma')]$

# Non-commutativity

## Closed string non-commutativity

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## Commutators through T-duality

ALLP work in progress

$$\boxed{[Z^1(\tau,\sigma), Z^2(\tau,\sigma')] \xrightarrow{\sigma'\to\sigma} -\frac{i}{2}\frac{\pi^2}{3} N^3 Q}$$

# Conclusions

## Non-geometry

Field redefinition: non-geometric fluxes in 10D

$$
\begin{array}{lclcl}
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- $R$ tensor, $Q$ part of connection (cf. $H$ and $f$)
- Dimensional reduction: matches 4D

## Non-geometry and non-commutativity

Use T-duality:

$$[X^m(\tau,\sigma), X^n(\tau,\sigma')]_{\mathrm{closed}} \xrightarrow{\sigma'\to\sigma} i\oint Q_k{}^{mn}(X)dX^k$$

# Conclusions

## Non-geometry

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