

# **Hawking radiation at strong coupling**

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September 3, 2009

# Hawking radiation at strong coupling

- Veronika Hubeny, Don Marolf, MR: 0908.2270
- Veronika Hubeny, Don Marolf, MR: 0909.0005
- Veronika Hubeny, Don Marolf, MR: to appear

# Outline

- Motivation
- CFTs on black hole backgrounds
- Black funnels & black droplets
- Examples of black funnels
- Novel dynamics of plasma in black hole bgs
- Hawking radiation in AdS
- Summary and discussion

# **Motivation**

## Motivation

How do black holes radiate strongly coupled matter?

- Hawking radiation is conventionally discussed for perturbative fields.
- Quantify differences when matter is strongly coupled?
- Precise nature of the quantum stress tensor due to the radiation (matter)?
- How does the nature of the radiation change if the matter can be in distinct phases?
- Dynamical behaviour during collapse: how does the field theory state evolve. Relatedly, Unruh vacuum.

# Motivation

## Plasma dynamics in black hole backgrounds

- How does a fluid plasma react to the gravitational field of a black hole?
- Does the fluid generically drain away into the black hole or is it possible for a stationary equilibrium solution to exist?
- Do fluid plasmas exhibit qualitative differences as function of the background parameters, with regard to say the phase structure?
- Does a deconfined plasma always behave universally in a black hole background?

# Motivation

## Strong coupling dynamics in curved spacetime

- Characteristic behaviour of a strongly coupled field theory in a generic curved spacetime background.
- How does the matter react to the background curvature: differences in vacuum polarization effects, etc..
- Phase structure?

## The basic framework:

- Attempt to use AdS / CFT to address some of these issues.

# Motivation

## Brane-world black holes

- Does one have a complete understanding of the potential set of brane-world black holes in induced gravity models?
- Are there new solutions apart from localized black holes and black strings?
- Why don't large localized black holes seem to exist?

Emparan, Fabbri, Kaloper

Fitzpatrick, Randall, Wiseman

- Induced gravity models will be treated as AdS/CFT with a UV cut-off.

**CFTs on black hole  
backgrounds**

# CFTs on black hole backgrounds

## Perturbative fields

- Consider a conformally coupled scalar field in a black hole background such as Schwarzschild.

$$ds_{\partial}^2 = \gamma_{\mu\nu} dx^{\mu} dx^{\nu} = -{}^d f(r) dt^2 + \frac{dr^2}{{}^d f(r)} + r^2 d\Omega_{d-2}^2 .$$

$${}^d f_{\text{Schw}}(r) = 1 - \frac{r_+^{d-3}}{r^{d-3}} .$$

- One can also consider the Schwarzschild-AdS black hole:

$${}^d f_{\text{SAdS}}(r) = \frac{r^2}{\ell_d^2} + 1 - \frac{r_+^{d-3}}{r^{d-3}} \left( 1 + \frac{r_+^2}{\ell_d^2} \right) .$$

# CFTs on black hole backgrounds

## Perturbative fields

- On these black hole backgrounds we want to study the dynamics of a conformally coupled scalar field with action

$$\mathcal{S}_{\text{scalar}} = \int d^d x \sqrt{-\gamma} \left( \frac{1}{2} \gamma^{\mu\nu} \nabla_{\mu} \phi \nabla_{\nu} \phi + \frac{d-2}{8(d-1)} R^{(\gamma)} \phi^2 \right)$$

- Of particular interest to us will be the nature of the quantum stress tensor, which encapsulates the physics of vacuum polarization, Hawking radiation etc..

# CFTs on black hole backgrounds

## Perturbative fields: Expectations

- ◎ Scalar radiation via the Hawking process at a temperature set by the black hole size.
- ◎ Stress tensor is thermal asymptotically ( $r \rightarrow \infty$ ).
- ◎ Thermal stress tensor is not regular on the event horizon.
- ◎ Correct quantum stress tensor should be regular at the horizon.
- ◎ Nothing singles out the event horizon: were the quantum stress tensor not regular there, we would be able to tell where the horizon was!

# CFTs on black hole backgrounds

## Perturbative fields: Results

- The general expectation is borne out by an explicit computation of quantum stress tensor.
- The computation was done by Page using the so called **optical metric** for static spacetimes:

Page '82

$$\overline{ds}_\partial^2 = \overline{\gamma}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + \frac{dr^2}{f(r)^2} + \frac{r^2}{f(r)} d\Omega_{d-2}^2 .$$

- One finds a regular stress tensor using the optical metric using heat kernel techniques. The result is given as:

$$T_\nu^\mu = \frac{\pi^2 T^4}{90} \left[ A \left( \frac{r_+}{r} \right) (\delta_\nu^\mu - 4 \delta_0^\mu \delta_\nu^0) + B \left( \frac{r_+}{r} \right) (3 \delta_0^\mu \delta_\nu^0 + \delta_1^\mu \delta_\nu^1) \right]$$

$$A(x) = \frac{1 - (4 - 3x)^2 x^6}{(1 - x)^2} , \quad B(x) = 24 x^6$$

# CFTs on black hole backgrounds

## Perturbative fields: Results

- At large distances  $r \gg r_+$  one has thermal radiation at

$$T_{\text{local}} = \frac{1}{\sqrt{d f(r)}} T_{\text{BH}}$$

- On the black hole horizon the quantum stress tensor evaluates to:

$$T_{\mu}^{\nu} = \frac{2\pi^4}{15} T_{\text{BH}}^4 \text{diag}\{3, 3, 1, 1\}$$

- Note that vacuum polarization renders local energy density negative at the horizon.

# Fluids on black hole backgrounds

## Stationary fluids

- In a static black hole background one can try to solve the relativistic Navier-Stokes equations:

$$\nabla_{\mu} T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (P + \rho) u^{\mu} u^{\nu} + P g^{\mu\nu}$$

- It is a-priori clear that the only stationary solution should have the fluid velocity aligned with the asymptotic time-translational Killing field.
- However, noting that a black hole solution is not globally static, we realize that the fluid is classically going to drain away down the black hole.

# CFTs on black hole backgrounds

## Conformal fluids in Schwarzschild

- ◎ The naive solution to viscous, conformal fluids is

$$u^\mu = \frac{1}{\sqrt{d\mathfrak{f}(r)}} (\partial_t)^\mu, \quad T = T_H \frac{1}{\sqrt{d\mathfrak{f}(r)}}$$

- ◎ This data can be used to attempt to find a solution to the fluid-gravity equations for a bulk solution.
- ◎ Use second order fluid dynamics in curved background to ask where the bulk horizon would be.
- ◎ See corrections away from thermal behaviour at  $\mathcal{O}(r^{-1})$ .
- ◎ However, the fluid approximation breaks down well before the horizon.

# CFTs on black hole backgrounds

## Beyond perturbation theory: Expectations

- Even when the matter fields are strongly coupled one should obtain a stress tensor that is regular on the horizon and is thermal asymptotically.
- The interpolation between the two regimes will be sensitive to the details of the system.
- Also take into account the phase structure of the QFT.
  
- To understand these issues we are going to embed the problem within the AdS/CFT correspondence, which is the cleanest framework to discuss these issues.

**The AdS/CFT story:  
black funnels and droplets**

## The AdS / CFT story

QFT + Gravity  $\rightsquigarrow$  Classical gravity

- Consider a situation in the AdS / CFT framework where the boundary field theory is on a non-dynamical black hole background  $\mathcal{B}_d$  with metric  $\gamma_{\mu\nu}$ .
- Dynamics of these field theories is governed at strong coupling by “asymptotically locally AdS” geometries which asymptote to  $\mathcal{B}_d$ .
- This involves finding bulk spacetimes  $\mathcal{M}_{d+1}$  which have as their conformal boundary the preferred manifold  $\mathcal{B}_d$ .
- One has to therefore find solutions to Einstein’s equations with a negative cosmological constant which has the correct boundary conditions.

## The AdS / CFT story

### Known solutions for boundary black holes

- Consider the case where the boundary is the asymptotically flat Schwarzschild black hole.
- One solution which satisfies the field equations is the AdS black string

$$ds^2 = z^2 ds_{\partial, \text{Schw}}^2 + \frac{dz^2}{z^2} .$$

- This solution is singular on the Poincare horizon (due to the black hole singularity which extends along  $z$ ).
- Ignoring this issue we can ask what the induced stress tensor is on the boundary.
- The result is simply:

$$T_{\mu}^{\nu} = 0$$

## The AdS / CFT story

### Schwarzschild boundary conundrums:

- The vanishing stress tensor for the AdS black string is at odds with expectations of thermal Hawking radiation.
- Anticipate strong vacuum polarization effects due to the strongly interacting matter.
- Rather curious that the conventional Hawking thermal spectrum is not reproduced.
- A similar result can be derived for the boundary Schwarzschild-AdS black hole.
- Note that this result in particular implies that if we make the boundary gravity dynamical then it would see no back-reaction from the strongly coupled matter sector!

# Phase structure of the CFT

## Resolution: Confinement vs. Deconfinement

- The vanishing stress tensor for the AdS black string seems to suggest that the dual field theory is actually in the confined phase, i.e.,

$$0 \sim \mathcal{O}(1)$$

- Typically in the AdS/CFT context, the boundary field theories on Minkowski space doesn't undergo a confinement-deconfinement transition.
- At any non-zero temperature, the preferred phase is the deconfined phase with  $\mathcal{O}(N^2)$  free energy.
- It is somewhat reassuring that the black string geometry actually is classically unstable, indicating that it is not a local minimum of the free energy.

# Phase structure of the CFT

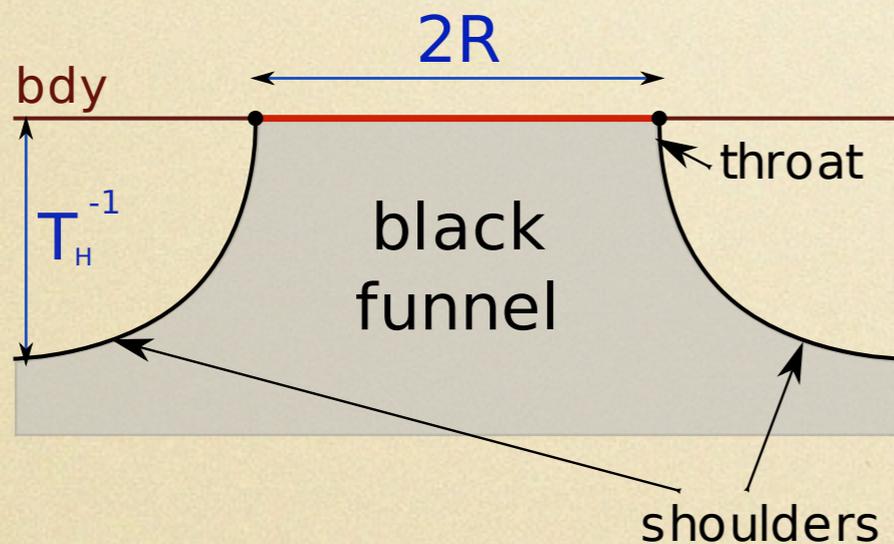
## Whither the Deconfined phase?

- The stable phase at any finite temperature ought to be the deconfined phase.
- This phase has a thermal stress tensor asymptotically and should give rise to a strongly coupled version of the Page stress tensor.
- In AdS/CFT this should just amount to a multiplicative renormalization of the free field result (the famous  $3/4$ ).
- Question then remains: what is the bulk solution which encapsulates the physics of a deconfined field theory plasma in equilibrium with the thermal Hawking radiation of the boundary black hole?
- Relatedly, is there a unique candidate bulk geometry or can there be phase transitions?

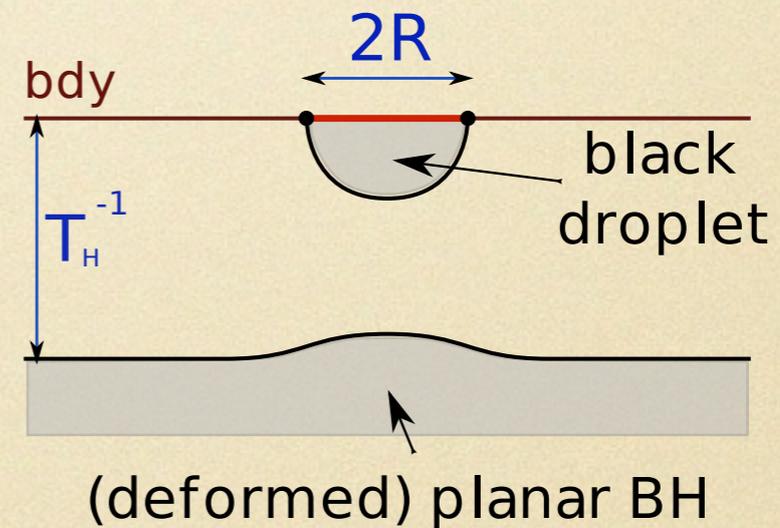
# Black Funnels, Black Droplets in AdS

## Motivating new classes of solutions

- ◎ The deconfined phase is dual to either
  - a **Black Funnel** geometry.
  - a **Black Droplet** together with a bulk black hole (disconnected horizons).



(a)



(b)

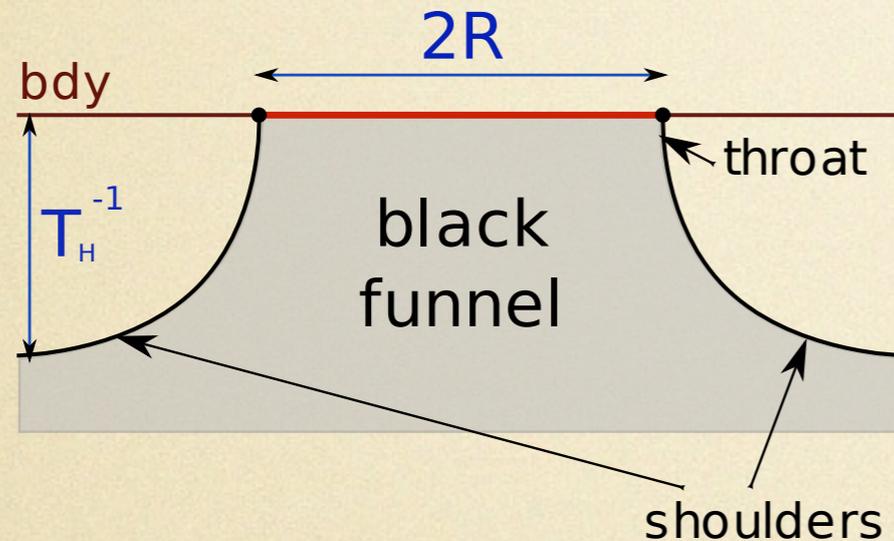
# Black Funnels

## Motivating the solutions

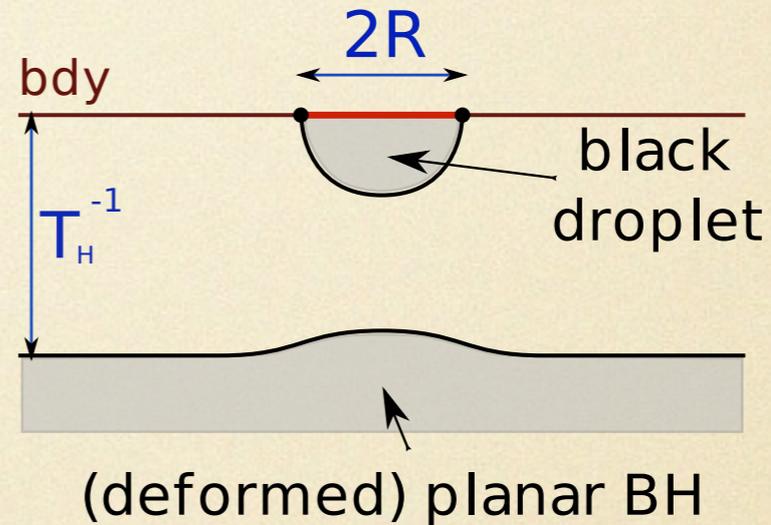
- Without the boundary black hole the spacetime dual of the deconfined phase is the Schwarzschild AdS black hole.
- Induces the correct thermal stress tensor on the boundary.
- Expect the bulk spacetime to behave similarly when we are asymptotically far away from the black hole on the boundary.
- Denote boundary radial coordinate as  $r$  and the bulk direction as  $z$ .
- Then as  $r \rightarrow \infty$  expect that the bulk solution for all  $z$  behaves like the Schwarzschild AdS black hole.
- In addition the bulk horizon could smoothly connect to the horizon on the boundary (black funnel) or there could be two disconnected horizons (a droplet and a planar bh).

# Black Funnels, Black Droplets in AdS

## Funnel vs droplets



(a)



(b)

- Note that the boundary black hole can have two scales
  - characteristic size  $R$ .
  - Hawking temperature  $T_H$ .
- A-priori these can be unrelated and the dominant saddle point depends on  $RT_H$ .

# Black Funnel & the phase structure of the CFT

## A Conjecture: field theory on Schwarzschild bg

- The stable deconfined phase is dual to a Black Funnel geometry.
- Corresponds to the phase of the CFT in equilibrium with the thermal Hawking radiation.
- The unstable deconfined phase is dual to the AdS black string.

## Extensions:

- Can be extended to situations where the field theory has a confining phase and to boundary metrics which have different asymptotics.

# Examples

# Black Funnel: Examples

## Black funnels in $AdS_3$

- The simplest explicit example of black funnels can be obtained when the bulk is three dimensional.
- The non-dynamical boundary metric can be taken to be the two dimensional black hole with metric

$$ds^2 = -\tanh^2 r dt^2 + dr^2$$

- Einstein's equations (with -ve cc) can be easily solved.
- Work in Fefferman-Graham coordinates for simplicity and take the metric ansatz to be

$$ds^2 = \frac{1}{z^2} \left( -f(r, z) dt^2 + g(r, z) dr^2 + dz^2 \right)$$

# Black Funnel: Examples

## Black funnels in $\text{AdS}_3$

● The metric functions turn out to be:

$$f(r, z) = \frac{1}{16} \left( 4 \tanh r + z^2 \frac{1 - 2r_+ \cosh^4 r}{\sinh r \cosh^3 r} \right)^2$$

$$g(r, z) = \left( 1 + z^2 \frac{2r_+ \cosh^4 r - 1 - 4 \sinh^2 r}{4 \sinh^2 r \cosh^2 r} \right)^2$$

● Naively we have a one-parameter family of solutions, but the parameter  $r_+$  is fixed by demanding regularity:

$$T_{\text{bdy}} = \frac{1}{2\pi} \implies r_+ = \frac{1}{2}$$

# Black Funnel: Examples

## Black funnels in $\text{AdS}_3$

● There is a bulk event horizon located at

$$z_H(r) = \frac{2 \cosh r}{\sqrt{\cosh^2 r + 1}}, \quad \& \quad r = 0$$

# Black Funnel: Examples

## Black funnels / droplets in $AdS_4$

- One can also find black funnel like solutions within the AdS C-metric family.
- These turn out to correspond to boundary black holes which are asymptotically  $\mathbf{R} \times \mathbf{H}^2$ .
- Once again the expectations regarding the stress tensor are borne out by explicit computation.
- There are also black droplets within the C-metric family.
- Curiously, except for a degenerate case the solutions tend not to represent equilibrium states (temperatures don't match between the droplet and the planar black hole).
- There also exist funnels and droplets for spatially compact boundaries.

**Novel dynamics of the  
deconfined plasma**

# Funnel vs droplets

## A novel phase transition

- ◎ Consider CFT on  $\mathcal{B}_d$  and imagine varying the parameters so that  $R T_H$  goes from large to small values.
- ◎ We expect that as  $R T_H$  passes through unity to see an exchanged in dominance of the two solutions.
- The black funnel throat could become thin and be subject to a Gregory-Laflamme type instability.
- As envisaged in the early days of the brane-world black holes perhaps this GL instability could end up in the droplet solution.

# Funnel vs droplets

## Novel dynamics in the deconfined phase

- ◎ Black funnels represent deconfined plasma in equilibrium with the bdy black hole.
- ◎ Perturbations of this system relax as expected in the deconfined regime.
  
- ◎ Black droplets + planar bhs are more exotic.
- ◎ While the planar bh ensures that the system is in deconfined phase, perturbations of the system relax as if it were confined!
- ◎ This is because any change on one horizon has to be transferred to the other through bulk Hawking radiation which is a very slow process.

## Funnel vs droplets

### Black holes and finite size excitations

- Slow equilibration of droplets is curious from the field theory.
- Suggests that the intrinsic excitations of the field theory are very weakly coupled to the boundary black hole.
- Natural interpretation: field theory excitations have finite size (cf. size of hadrons in string theory).
- If  $R_e \gg R$  then the field theory quanta just fly past the bdy black hole without much ado (modulo tidal effects).
- In the bulk one again sees this by the small distortion on the droplet caused by a gravitational perturbation deep inside the bulk.

# Summary

# Summary & Discussion

## New exotic AdS black holes

- Holographic techniques allow us for the first time to investigate the Hawking radiation of strongly coupled quanta.
- Analysis of the dual gravitational solutions leads to a new class of solutions which we've dubbed black funnels.
- Explicit solutions exist when the boundary is 2 dimensional, or BTZ or an exotic hyperbolic black hole.
- Not only can such solutions be constructed in a limited class of examples, but also they confirm our intuition regarding the quantum stress tensor due to curved spacetime effects.

# Summary & Discussion

## Realistic Black funnels?

- ◎ Black funnels with Schwarzschild boundary should exist (numerical relativists please step up)!
- ◎ Would be interesting to explore the explicit behaviour of these solutions and figure out the quantum stress tensor induced by strongly coupled  $\mathcal{N} = 4$  SYM in this background.

There are important implications for:

- ❖ Plasma balls in confining theories
- ❖ Brane-world and induced gravity models.

**Hawking radiation and AdS  
black holes**

# Hawking radiation in AdS

## Black funnels in $\text{AdS}_4$ with BTZ boundary

- The simplest non-trivial example where we can find explicit solution and in fact examine the full phase structure is when the boundary is a BTZ black hole.

$$ds^2 = -(r^2 - r_+^2) dt^2 + \frac{dr^2}{r^2 - r_+^2} + r^2 d\varphi^2$$

- Obtain all the requisite solutions via a set of solution generating tricks.

# Black Funnel: Examples

## Black funnels in $\text{AdS}_3$

- Given the solution we can use holographic techniques to find the induced stress tensor on the boundary:

$$T_{tt} = \alpha \tanh^2 r \left( \frac{\cosh^2 r - 3}{2 \cosh^2 r} \right), \quad T_{rr} = \alpha \frac{\cosh^2 r + 1}{2 \cosh^2 r}$$

- $\alpha$  is a normalization constant measuring the effective number of degrees of freedom.

$$\alpha \sim \frac{\ell}{\ell_p}$$

- This stress tensor is thermal asymptotically far from the black hole and regular on the event horizon as expected.

# Black Funnel in AdS

## Black funnels in $AdS_4$ with BTZ boundary

- ◎ Recall that for the global  $AdS_4$  spacetime with the standard Einstein Static Universe  $\mathbb{R} \times S^2$  boundary one has the following solutions dual the various phases:
  - ❖ Confined phase: Thermal AdS spacetime
  - ❖ Deconfined phase: Large Schwarzschild AdS black hole
  - ❖ Unstable phase: Small Schwarzschild AdS black hole
- ◎ By an analytic continuation + conformal transformation one can map the ESU to BTZ.
- ◎ Applying the same trick to the bulk solutions, we recover the desired black funnel geometries.

## Black Funnel in AdS

### Black funnels in $\text{AdS}_4$ with BTZ boundary

In particular, starting with the known class of solutions

$$ds^2 = -f(\rho) d\tilde{t}^2 + \frac{d\rho^2}{f(\rho)} + \rho^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

the following set of transformations

$$\tilde{t} = i\chi, \quad \varphi = it, \quad \tilde{r} = \frac{1}{w}$$

takes us to the full set of solutions dual to BTZ on the boundary:

$$ds^2 = \frac{\rho^2}{w^2} \left[ -(w^2 - 1) dt^2 + \frac{dw^2}{w^2 - 1} + w^2 \frac{f(\rho)}{\rho^2} d\chi^2 \right] + \frac{d\rho^2}{f(\rho)}$$

## Black Funnel in AdS

### Black funnels in $AdS_4$ with BTZ boundary

While the “funnelness” of the solution is not manifest, one can compute the boundary stress tensor having fixed the non-normalizable part to coincide with BTZ metric.

$$f(\rho) = 1 + \rho^2 - \frac{M}{\rho}, \quad M = \rho_+ (1 + \rho_+^2)$$

$$T_{\mu}^{\nu} = \frac{M}{3r^3} \text{diag}\{1, 1, -2\}$$

- The stress tensor is regular on the horizon, but seems to behave somewhat strangely as  $r \rightarrow \infty$ .
- But this in fact turns out to be the precise form computed for perturbative fields.

## Black Funnel in AdS

### Black funnels in $AdS_4$ with BTZ boundary

For the BTZ geometry

$$ds^2 = -(r^2 - r_+^2) dt^2 + \frac{dr^2}{r^2 - r_+^2} + r^2 d\varphi^2$$

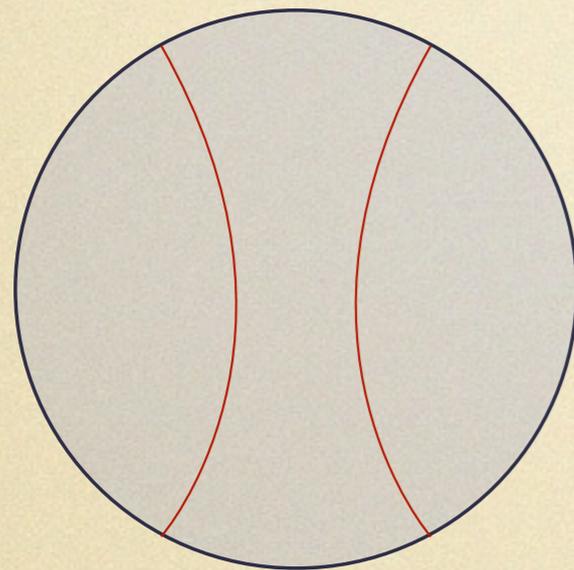
the calculation of the quantum stress tensor for scalar fields leads to the result:

$$T_{\mu}^{\nu} = \frac{A(r_+)}{3 r^3} \text{diag}\{1, 1, -2\} ,$$
$$A(r_+) = \frac{\sqrt{2}}{32\pi} \sum_{n=1}^{\infty} \frac{\cosh(2\pi n r_+) + 3}{(\cosh(2\pi n r_+) - 1)^{\frac{3}{2}}}$$

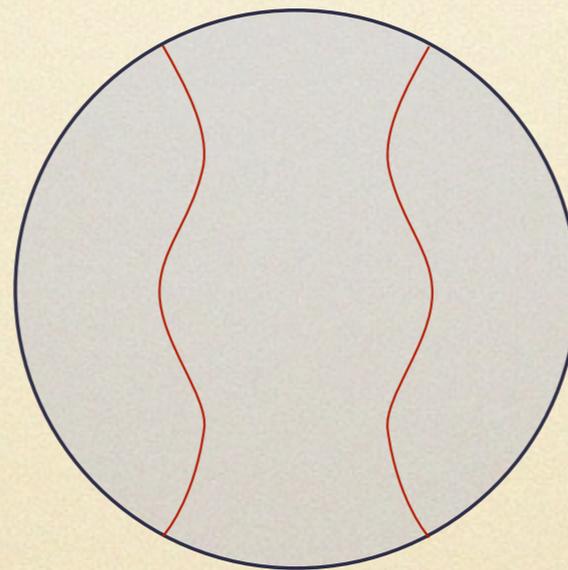
whose form coincides with the holographic computation.

# Black Funnels in AdS

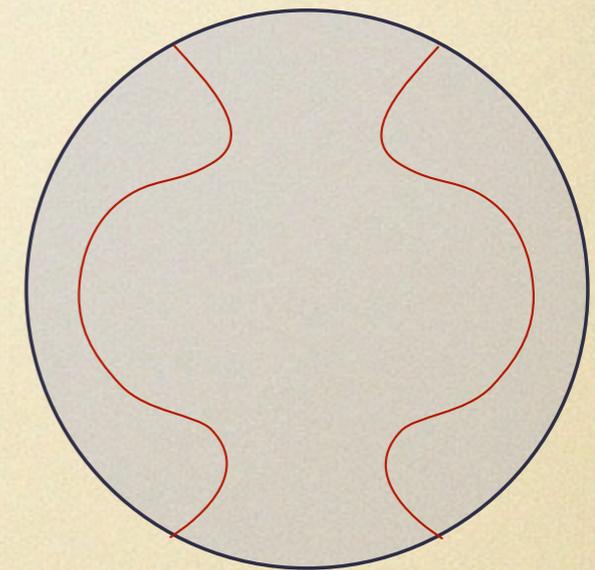
## Black funnels in $AdS_4$ with BTZ boundary



AdS black string  
Confined phase



Small Black Funnel  
Unstable phase



Large Black Funnel  
Deconfined phase