Dynamical Black Holes & Expanding Plasmas

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based on work w/ P. Figueras, M. Rangamani, & S. Ross (arXiv:0902.4696)

OUTLINE

- Motivation & Background
- Conformal Soliton flow
- Boost-invariant (Bjorken) flow
- Open issues: entropy dual?

Motivation

- Time-dependence in AdS/CFT
- Fluid/Gravity correspondence far from (local) equilibrium
- Holographic description of entropy in dynamical setting

Fluid/gravity correspondence

- Long-wavelength dynamics of interacting QFT (fluid dynamics) = gravitational dynamics of asymp. AdS black hole [Bhattacharyya, VH, Minwalla, Rangamani]
- If geometry settles down, $\exists$ regular event horizon; & its area $\rightsquigarrow$ entropy current in CFT [Bhattacharyya, VH, Loganayagam, Mandal, Minwalla, Morita, Rangamani, Reall]
- What if the geometry does not settle down?

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- Motivation & Background
- Conformal Soliton flow (=`CS)
  - CS construction & geometry
  - CS event horizon
  - CS apparent horizon
- Boost-invariant (Bjorken) flow
- Open issues: entropy dual?

Conformal Soliton flow

- CS geometry = 'Poincare patch' of Schw-AdS black hole [Friess, Gubser, Michalogiorgakis, Pufu]

- e.g. in 3D, constructed by applying the coord. transf. (global AdS $\rightsquigarrow$ Poincare AdS)

$$ds^2 = -(r^2+1)\,d\tau^2 + \frac{dr^2}{r^2+1} + r^2\,d\varphi^2 \quad \rightsquigarrow \quad ds^2 = \frac{-dt^2+dz^2+dx^2}{z^2}$$

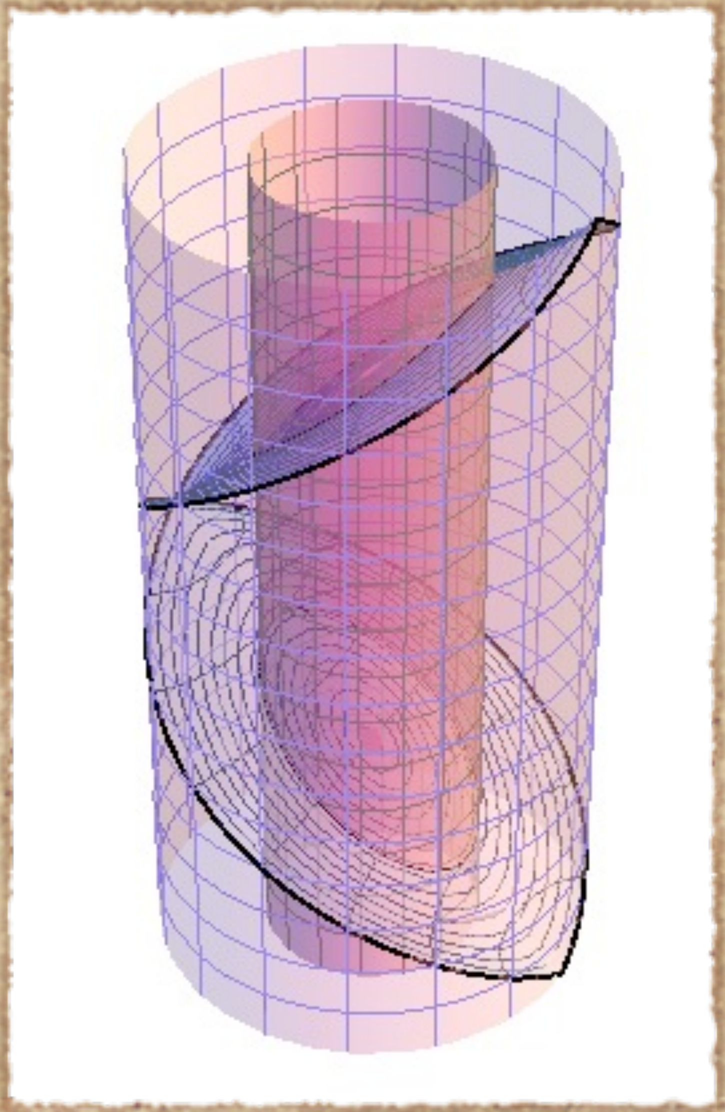
to BTZ : $ds^2 = -(r^2 - r_+^2)\,d\tau^2 + \frac{dr^2}{r^2 - r_+^2} + r^2\,d\varphi^2$ details

- bulk coord transf $\rightsquigarrow$ conformal transf. on bdy

- resultant metric $g_{\mu\nu}(t,x,z)$ looks dynamical.

Conformal Soliton geometry

- Black hole enters through past Poincare edge
- and leaves through future Poincare edge
- but geometry is time-reversal invariant.



Conformal Soliton on bdy

- Static Schwarzschild-AdS black hole corresponds a static ideal fluid in the bdy CFT;
- whereas the CS flow describes dynamically contracting & expanding plasma.
- Ideal fluid, no entropy production
  - Total entropy is invariant under conformal transformation (and given by the area of the event horizon of the BH in global coordinates)

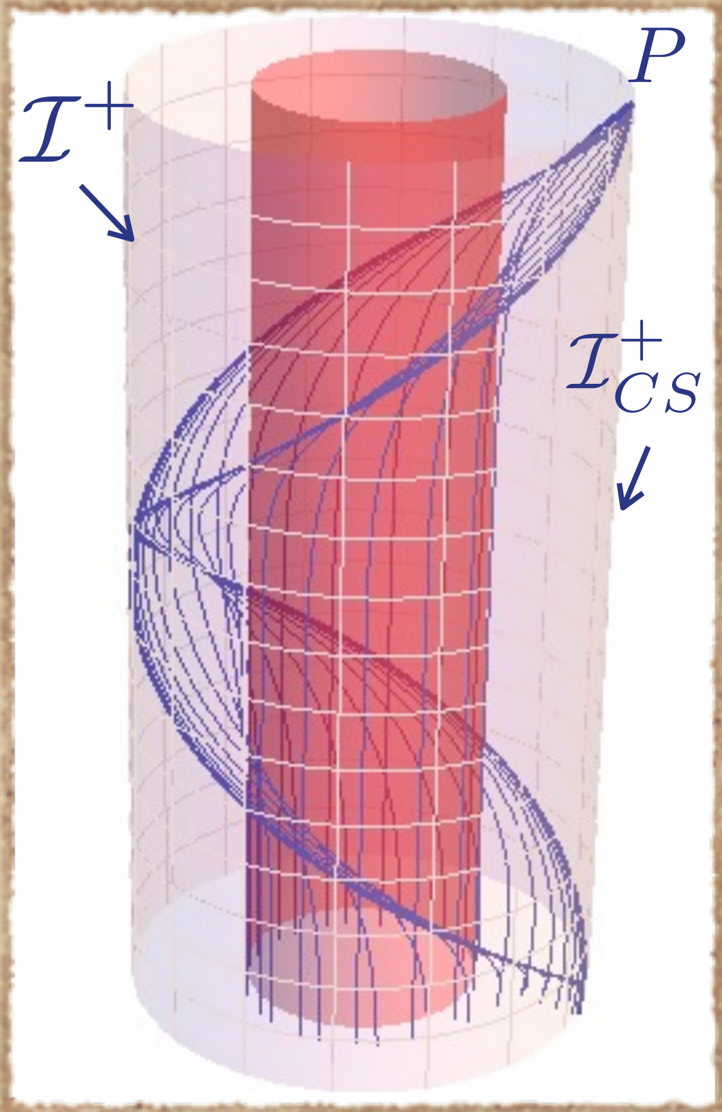
CS event horizon

- Recall: event horizon $\mathcal{H}^+$ is defined as the boundary of the past of the future null infinity, $\mathcal{H}^+ \equiv \partial I^-[\mathcal{I}^+]$
- hence it is generated by null geodesics.
- For the CS spacetime, $\mathcal{I}^+_{CS}$ is a subset of $\mathcal{I}^+$.
- Hence the CS event horizon $\mathcal{H}^+_{CS}$ does NOT coincide with the global event horizon $\mathcal{H}^+$.
- It is easy to find $\mathcal{H}^+_{CS}$ explicitly...

CS event horizon

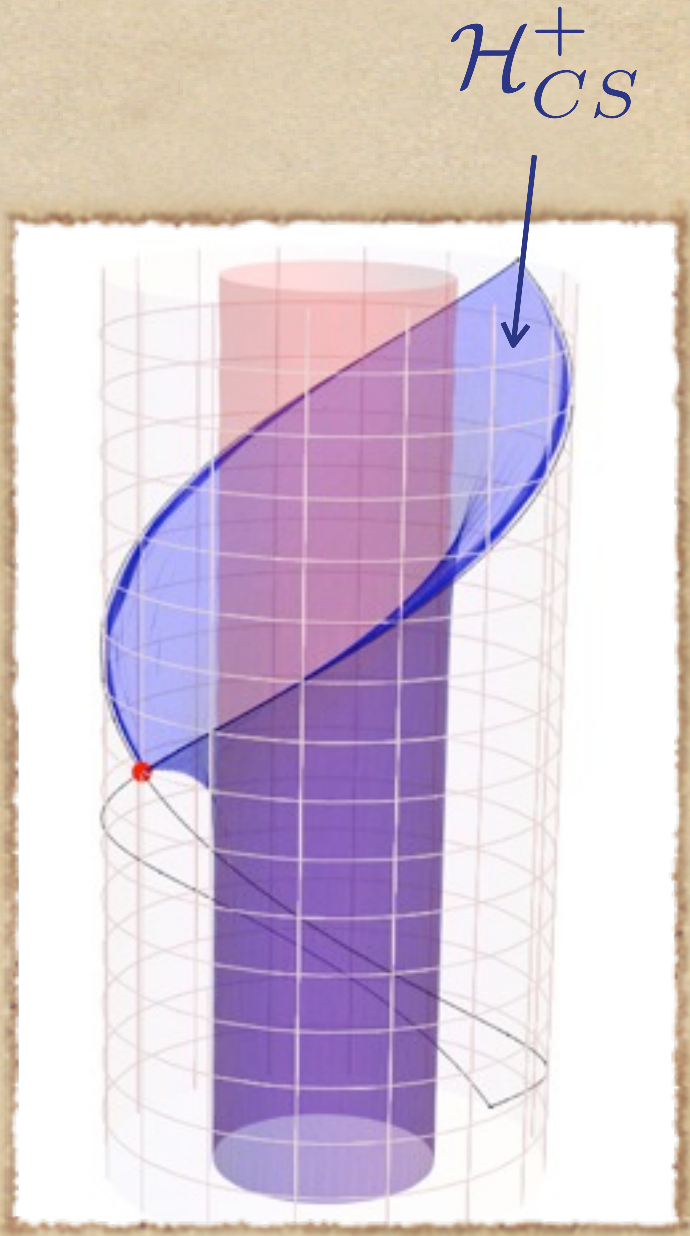
- $\mathcal{H}^+_{CS} \equiv \partial I^-[\mathcal{I}^+_{CS}] = \partial I^-[P]$
- generated by null geodesics ending at P

details



CS event horizon

- $\mathcal{H}^+_{CS} \equiv \partial I^-[\mathcal{I}^+_{CS}] = \partial I^-[P]$
- generated by null geodesics ending at P
- cut off at curve of caustics at $\varphi = \pi$



Note: $\mathcal{H}^+_{CS}$ interpolates between global event horizon at early times and Poincare horizon at late times.

Area of CS event horizon

- CS event horizon $\mathcal{H}^+_{CS}$ grows in time.
- $\mathcal{H}^+_{CS}$ touches the boundary at $t = 0$.
- Proper area at constant Poincare time slice diverges at $t = 0$.

details

- Hence event horizon area does NOT correctly reproduce the CFT entropy!

Apparent horizon

- Recall: given a foliation of a spacetime, a trapped surface S is a closed surface with negative divergence θ for both outgoing and ingoing null congruence normal to S.

details

- The apparent horizon is defined as the boundary of the trapped surfaces, or outermost marginally trapped surface.
- We will consider this as co-dim. 1 tube evolving in time
- Technically speaking, this tube is called isolated horizon if it is null, and dynamical horizon if spacelike.

Foliation-dependence of AH

- In general dynamical spacetime admitting trapped surfaces, the location of apparent horizon depends on the choice of foliations. [cf. Wald & Iyer]
- On the other hand, if the spacetime admits a Killing horizon, then this Killing horizon is an apparent horizon for any foliation which contains full slices of the horizon.

Proof:

- Null normals to any cross-sectional slice $\mathcal{S}$ of the horizon coincide with horizon generators.
- Expansion vanishes $\rightsquigarrow$ $\mathcal{S}$ is marginally trapped surface
- Since no trapped surfaces outside event horizon, $\mathcal{S}$ is the outer-most marginally trapped surface $\Longrightarrow$ apparent horizon.

CS apparent horizon

- CS geometry admits a Killing horizon: requisite Killing field is simply $\left(\frac{\partial}{\partial\tau}\right)^a$ with τ = global time (hence same as global Schw-AdS event horizon)
- $\Longrightarrow$ for any foliation this Killing horizon at $r = r_+$ is an apparent horizon
- Hence CS apparent horizon has area which stays constant in time, and does correctly reproduce CFT entropy.

Summary for CS geometry

- Poincare patch of Schw-AdS: looks highly dynamical (even though secretly static).
- Event horizon of CS does not coincide with global event horizon, and its area diverges at finite time.
- Apparent horizon of CS does coincide with global event horizon, and has constant area.

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- Boost-invariant (Bjorken) flow (=`BF')
- Open issues: entropy dual?

Bjorken flow geometry

- Proposed to describe QGP at RHIC [Bjorken]

  fluid on $\mathbb{R}^4$ $ds^2 = -d\tau^2 + \tau^2\, dy^2 + dx_\perp^2$

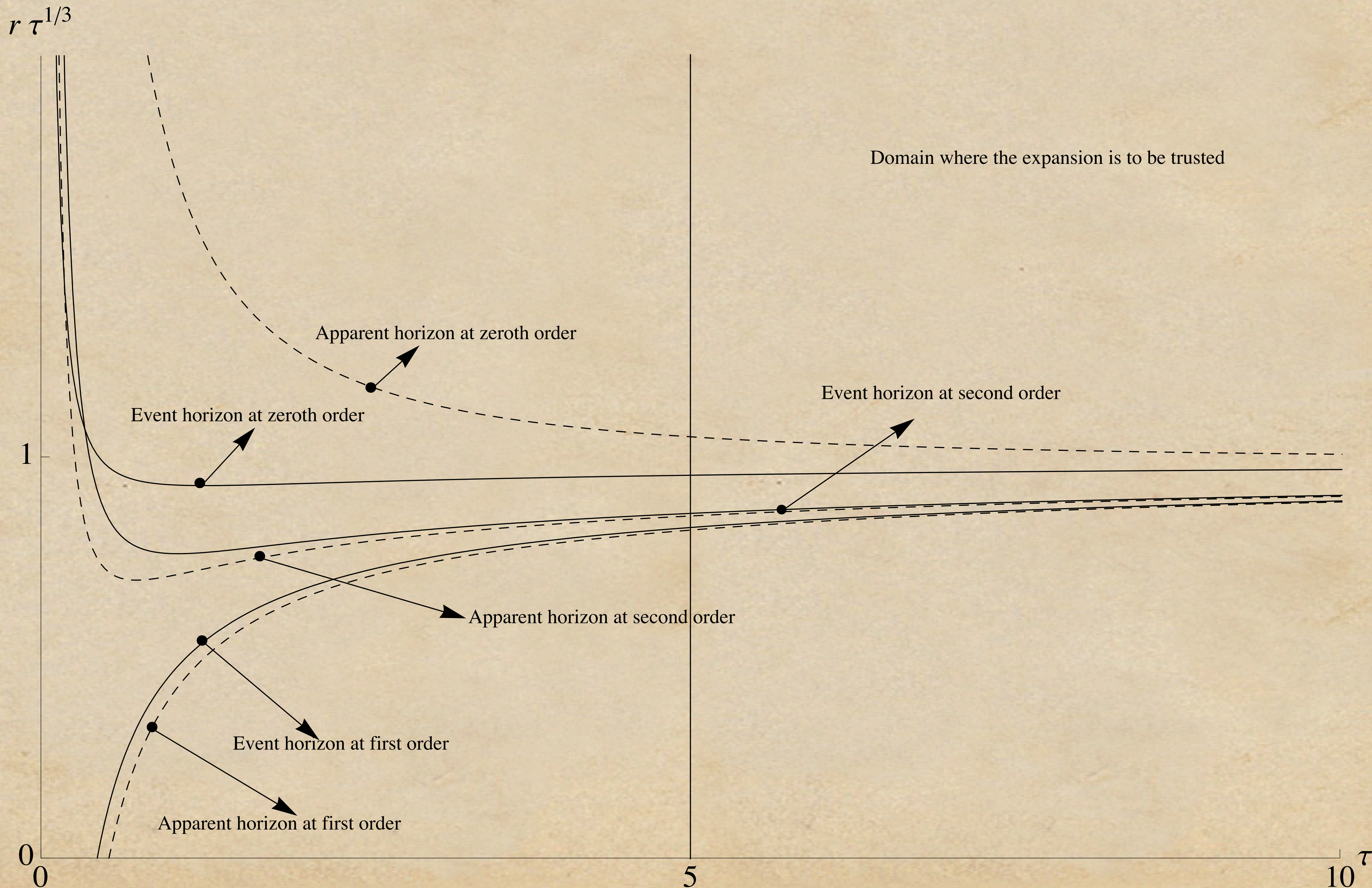
- Boost invariance along collision direction all physical quantities depend only on τ.
- Exact dual geometry not known; but can expand around late proper times (metric functions of τ & radial coord r) [Janik & Peschanski]
- Describes a dynamically receding black hole in bulk, whose temperature $T \sim \tau^{-1/3}$

Horizons in BF spacetime

- Apparent horizon (for constant τ foliation) found previously. [Kinoshita, Mukohyama, Nakamura, Oda]
- Event horizon can be obtained by analysing radial null geodesics (find the outer-most one which reaches $\mathcal{I}^+$ at late time $\tau \to \infty$)
- This is carried out by systematic expansion:

$$r_+^{(2)}(\tau) = \tau^{-\frac{1}{3}} - \frac{1}{2}\tau^{-1} + \frac{12 + 3\pi - 4\ln 2}{72}\tau^{-\frac{5}{3}} + \mathcal{O}(\tau^{-7/3})$$

Horizons in BF spacetime



Summary for BF geometry

- At 2nd order, EH lies (just) outside AH
- metric is regular everywhere on & outside EH
- Curiosity @ 0th order metric: AH is outside EH (But does not solve Einstein eqns at higher orders and violates energy conditions.)

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- Summary & open issues: entropy dual?

Dual of CFT entropy?

- We've seen:
  - in original fluid/gravity framework, and for the BF geom, CFT entropy is given by area of EH $\approx$ AH
  - in CS geom, entropy given by area of AH $\neq$ EH
    CFT entropy is NOT always reproduced by EH area.
- In retrospect, EH is too teleological to give entropy (since S is defined indep. of future evolution)
- Natural guess: entropy given by AH area ??

Dual of CFT entropy?

- BUT: this cannot hold universally!
  - AH is not always well-defined: it can jump discontinuously
  - AH is foliation-dependent for general dynamical backgrounds
- Possible resolution: entropy is well-defined only when temporal evolution is sufficiently slow.
  - Note: contrast with entanglement entropy...
- Under such conditions AH evolves smoothly.

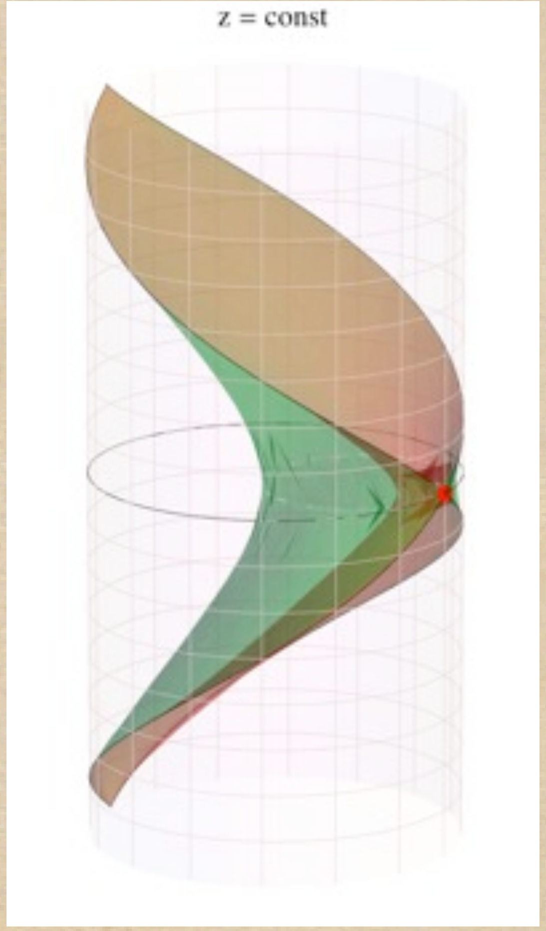
Dual of CFT entropy?

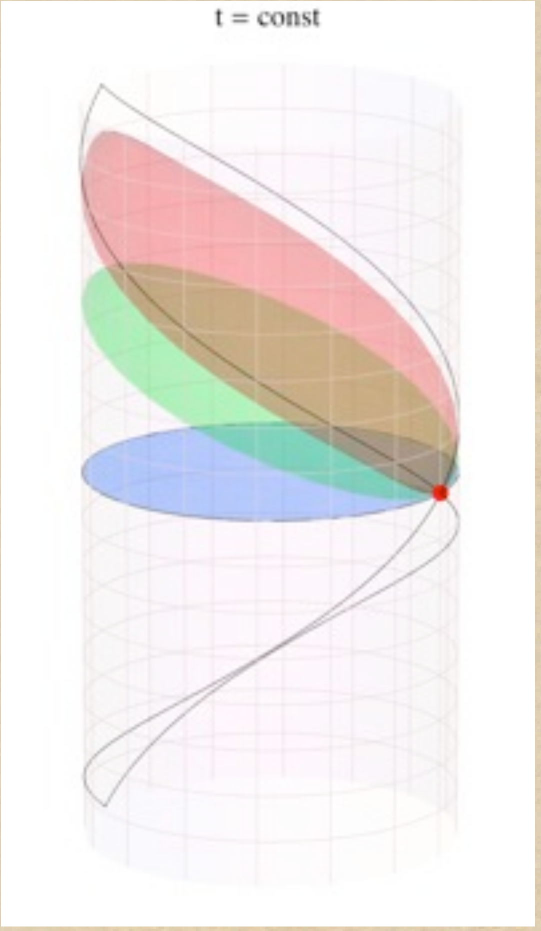
- BUT: there is still a problem with foliation dep.:
  - Consider collapsed star in AdS which settles down at late times.
  - Naively: CFT entropy @ late times given by late time EH area.
  - BUT: $\exists$ bulk foliations admitting no trapped surfaces even at arbitrarily late times!

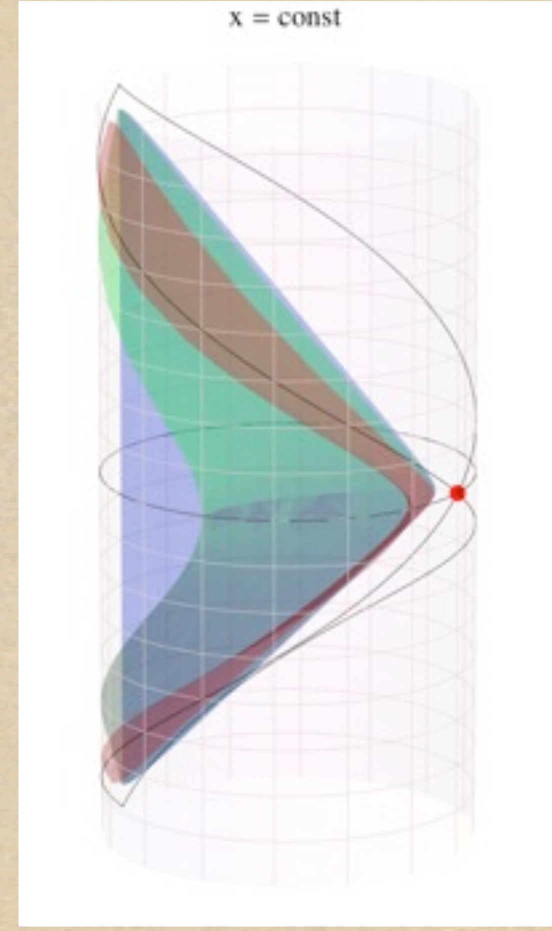
- Other possible resolutions:
  - CFT prescribes a natural/preferred foliation (but no real evidence)
  - Only in static (equilibrium) configurations does the bulk dual of CFT entropy correspond to a geometrical object in the bulk.

Coordinate tranformation

$$z = \frac{1}{\sqrt{r^2+1}\,\cos\tau + r\,\cos\varphi}\,, \quad t = \frac{\sqrt{r^2+1}\,\sin\tau}{\sqrt{r^2+1}\,\cos\tau + r\,\cos\varphi}\,, \quad x = \frac{r\,\sin\varphi}{\sqrt{r^2+1}\,\cos\tau + r\,\cos\varphi}$$







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CS event horizon - details

- geodesic equations (ℓ = ang.mom./energy, λ = affine parameter):

$$\left(\frac{dr}{d\lambda}\right)^2 = 1 - \ell^2 + \frac{\ell^2\, r_+^2}{r^2}\,, \qquad \frac{d\tau}{d\lambda} = \frac{1}{r^2 - r_+^2}\,, \qquad \frac{d\varphi}{d\lambda} = \frac{\ell}{r^2}$$

- surface of event horizon $\mathcal{H}^+_{CS}$ is parameterized by:

$$\begin{aligned} r(\lambda,\ell)^2 &= (1-\ell^2)\ \lambda^2 - \frac{\ell^2\, r_+^2}{1-\ell^2} \\ \tau(\lambda,\ell) &= \pi - \frac{1}{r_+}\,\mathrm{arccoth}\left(\frac{1-\ell^2}{r_+}\,\lambda\right) \\ \varphi(\lambda,\ell) &= -\frac{1}{r_+}\,\mathrm{arccoth}\left(\frac{1-\ell^2}{\ell\, r_+}\,\lambda\right) \end{aligned}$$

- induced metric on horizon:

$$ds^2_{ind} = \frac{d\ell^2}{(1-\ell^2)^2}$$

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event horizon area - details

- CS event horizon area in terms of ang.mom.

$$\mathcal{A} = 2\int_0^{\ell_{\max}} \frac{d\ell}{1-\ell^2} = 2\,\mathrm{arctanh}\,\ell_{\max}$$

- $\ell_{\max}$ is determined by position of caustics

$$\ell_{\max} = \frac{r\sinh(\pi r_+)}{\sqrt{r_+^2 + r^2\,\sinh^2(\pi r_+)}}$$

- Area diverges when horizon touches bdy $r \to \infty$

$$\mathcal{A} = 2\,\mathrm{arctanh}\left(\frac{r\sinh(\pi r_+)}{\sqrt{r_+^2 + r^2\,\sinh^2(\pi r_+)}}\right)$$

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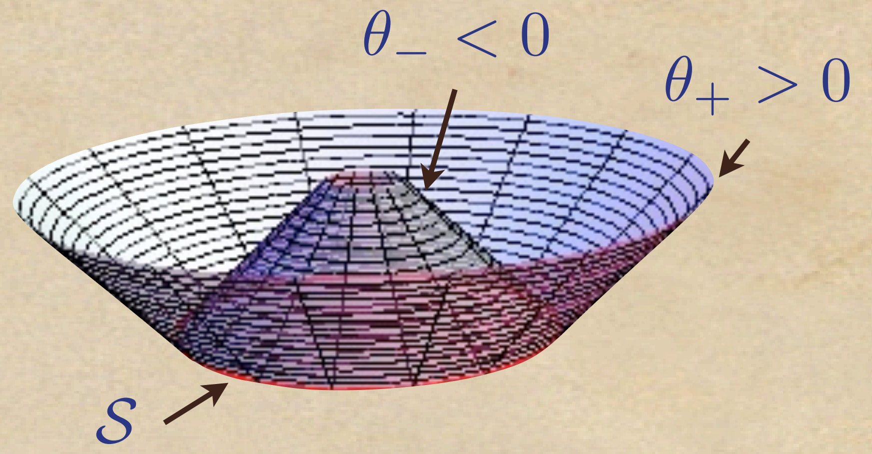
Trapped surface

- For a closed surface $\mathcal{S}$ the divergence $\theta_\pm$ of outgoing / ingoing null congruence is defined as the fractional change in area along wavefronts of outgoing /ingoing null geodesics emanating perpendicularly to $\mathcal{S}$:

For area A along 'wavefronts' at constant λ , expansion θ is given by

$$\theta = \frac{1}{A}\frac{dA}{d\lambda}$$



- Trapped surface has both $\theta_- < 0$ and $\theta_+ < 0$.

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