Black Holes as Lumps of Fluid

Óscar Dias

DAMTP

University of Cambridge

MARIE CURIE **ACTIONS**

Based on:

Marco Caldarelli, OD, Roberto Emparan, Dietmar Klemm, 0811.2381

Marco Caldarelli, OD, Dietmar Klemm, 0812.0801

Vitor Cardoso, OD, 0902.3560

Workshop on the Fluid-Gravity Correspondence

Arnold Sommerfeld Center , September 2009

Plan of the talk

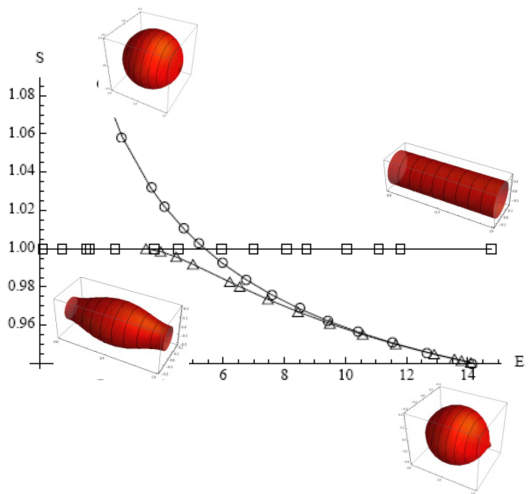
- I. Motivation:

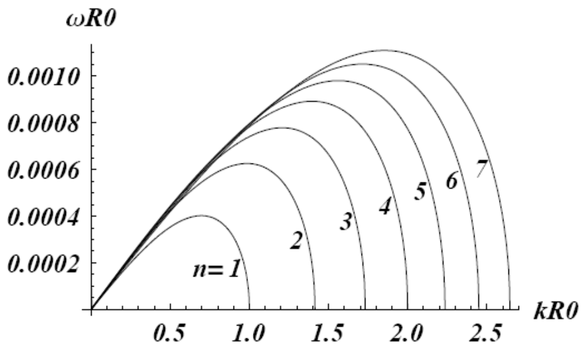
  - Gravity / Gauge theory duality
  - For high energy density, QFT has a hydrodynamic description

  → Gravity / Hydrodynamics duality

- II. Gravity / Hydrodynamics duality ...

... So what ?





Fluid dynamics from gravity

(Battacharrya, Hubeny, Minwalla, Rangamani, 0712.2456)
("previous", Mandal, Morita, Reall, 0803.2526)

Start with (planar, *k*=0) black brane solution of Einstein-AdS (EF coord.):

$$ds^2 = -2dv\,dr - r^2 f(br)dv^2 + r^2 dx^i dx^i \qquad f(r) = 1 - \frac{1}{r^4}$$

$$T = \frac{1}{\pi b}$$

Boost the black brane : β^i

$$u^v = \frac{1}{\sqrt{1-\beta_i^2}} \qquad u^i = \frac{\beta^i}{\sqrt{1-\beta_i^2}} \qquad P_{\mu\nu} = u_\mu u_\nu + \eta_{\mu\nu} \quad \perp u^\mu$$

$$ds^2 = -2u_\mu dx^\mu\,dr - r^2 f(br) u_\mu u_\nu\,dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu$$

Allow boost and temperature to vary slowly with x^μ (bdry coords):

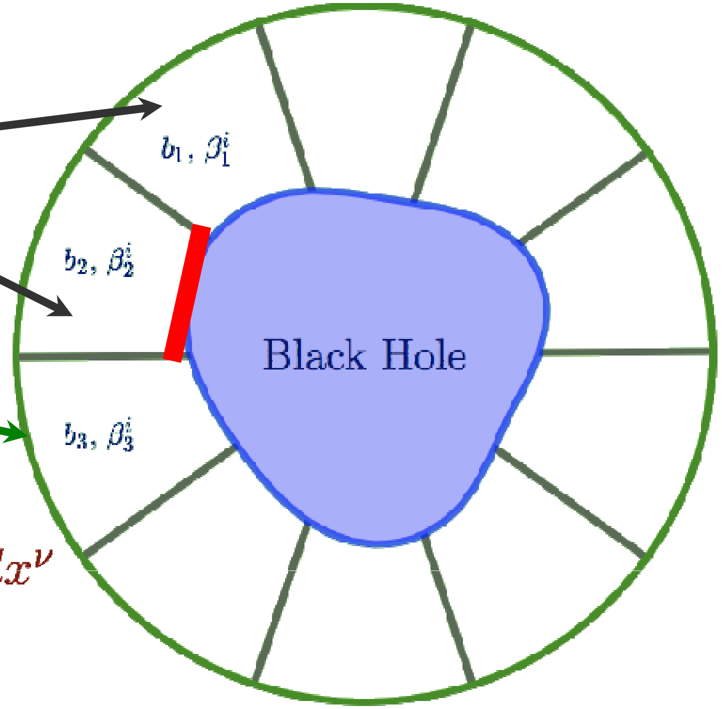
$$b = b(x^\mu) \qquad \beta^i = \beta^i(x^\mu)$$

Question:

Can we have BHs that **tubewise** *approximate* black branes in AdS ?

Patches

AdS boundary



$$ds^2 = -2u_\mu dx^\mu\, dr - r^2 f(br) u_\mu u_\nu\, dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu$$

$$b = b(x^\mu) \qquad\qquad \beta^i = \beta^i(x^\mu)$$

- Generically, such a metric , g(0), is ***not*** a solution to Einstein-AdS equations.
- Nevertheless , for slowly varying functions $b(x^\mu),\ \beta_i(x^\mu)$

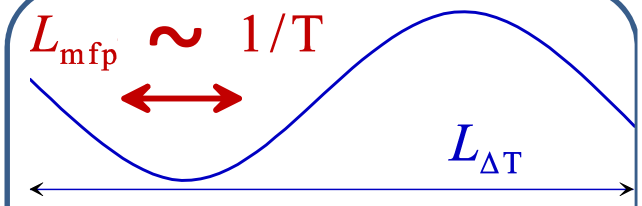
  it is a good approximation to a true solution provided the functions $b(x^\mu), \beta_i(x^\mu)$

  obey a set of eqs which turn out to be the equations of boundary fluid dynamics.

Solve perturbatively Einstein-AdS's eqs order by order in a boundary derivative expansion:

$$g = g^{(0)}(\beta_i, b) + \varepsilon\, g^{(1)}(\beta_i, b) + \varepsilon^2\, g^{(2)}(\beta_i, b) + \mathcal{O}\left(\varepsilon^3\right)$$

$$\beta_i = \beta_i^{(0)} + \varepsilon\, \beta_i^{(1)} + \mathcal{O}\left(\varepsilon^2\right), \qquad b = b^{(0)} + \varepsilon\, b^{(1)} + \mathcal{O}\left(\varepsilon^2\right)$$



$$L_{\Delta T, \Delta\beta_i} \gg L_{\mathrm{mfp}}$$

Given the solution at order ***n***, use AdS/CFT dictionary to construct the **boundary stress tensor**:

compute **extrinsic curvature** tensor of surface at fixed *r* , and use *def.*:

$$T^{\mu}_{\nu} \sim -2 \lim_{r\to\infty} r^4 \left(K^{\mu}_{\nu} - \delta^{\mu}_{\nu}\right)$$

Perturbative solutions to the gravitational eqs **exist only** when the velocity and temperature fields **obey** certain equations of motion: $\nabla_\mu T^{\mu\nu}_{(n)} = 0$

$$T^{\mu\nu} = \underbrace{(\pi T)^4 \left(\eta^{\mu\nu} + u^\mu u^\nu\right)}_{\text{Perfect fluid: } T^{\mu\nu}_{(0)}} \underbrace{-2(\pi T)^3 \sigma^{\mu\nu}}_{T^{\mu\nu}_{(1)}} + \mathcal{O}(\partial^2 u) + \cdots$$

Shear viscosity: $\eta = \pi^3 T^3$

Policastro, Son, Starinets

Einstein-AdS gravity is dual to Fluid dynamics **!**

Scherk-Schwarz (SS) AdS/ SS QFT

(Witten 1998)
(Nice review: Mateos 2007)

- AdS/CFT (IIB ST on $AdS_5 \times S^5$ / $\underbrace{4d \text{ N=4 SYM}}$) $\neq$ QCD
  - Non-Conformal
  - Non-SUSY
  - Has Confinement
- Would like to have: String Theory / QCD duality
- To approach QCD: Non-AdS / Non-Conformal gauge theory duality

- **Start with *N* D4-branes.**
  Gauge theory on it: 5*d* max SUSY SU(*N*) SYM theory (gluons + scalars + fermions)

- **To get 4*d* theory, compactify along y-direction w/ antiperiodic BC (SS) for fermions:**
  - **breaks SUSY**
  - **Non-Conformal**
  - **Confined & Deconfined phases**

  Confinement/Deconfinement phase transition

  Key Features of QCD

- **Gravity solution dual to gauge theory on D4-branes** ?

  Start with near-extremal D4-branes
  and take appropriate decoupling (low-energy) limit.

- **Black hole** (from decoupling limit of D4-branes):

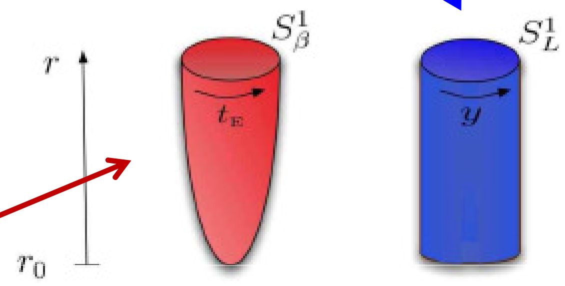
$$f(r) = 1 - \frac{r_0^3}{r^3}$$

$$ds^2_{\mathrm{BH}} = \left(\frac{r}{R}\right)^{3/2}\left(-f\,dt^2 + dx^2_{(3)} + dy^2\right) + \left(\frac{R}{r}\right)^{3/2}\frac{dr^2}{f} + \cdots$$

- $\{t, x_{(3)}, y\}$ span worldvolume of the D4-branes $\equiv$ gauge theory coords: $y \sim y + L$
- Hawking temperature *T* (identify w/ gauge theory *T*): $t \longrightarrow it_{\mathrm{E}}$

$$ds^2_{\mathrm{E}} = \left(\frac{r}{R}\right)^{3/2}\left(f\,dt^2_{\mathrm{E}} + dx^2_{(3)} + dy^2\right) + \left(\frac{R}{r}\right)^{3/2}\frac{dr^2}{f}$$

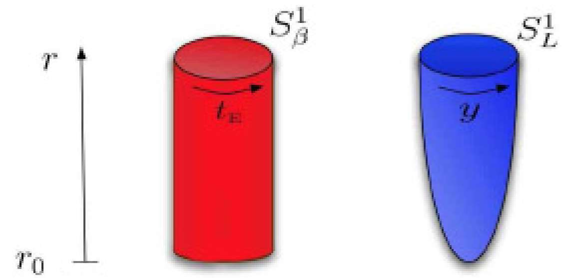
Regularity at $r = r_0$,
compactify t_{E} on S^1_β w/ length $\beta = 1/T$



- Trivial observation: Relabelling $t_{\mathrm{E}} \leftrightarrow y$ we still have solution (Euclidean) gravity,

$$ds^2_{\mathrm{E}} = \left(\frac{r}{R}\right)^{3/2}\left(\underline{dt^2_{\mathrm{E}}} + dx^2_{(3)} + \underline{f\,dy^2}\right) + \left(\frac{R}{r}\right)^{3/2}\frac{dr^2}{f} + \cdots$$

But now it's S^1_L that shrinks to zero size at $r = r_0$



Lorentzian sector:
AdS soliton
(Horowitz-Myers)

$$ds^2_{\mathrm{soliton}} = \left(\frac{r}{R}\right)^{3/2}\left(\underline{-dt^2} + dx^2_{(3)} + \underline{f\,dy^2}\right) + \left(\frac{R}{r}\right)^{3/2}\frac{dr^2}{f}$$

- # Consequences ?

Two candidates for the geometry dual to gauge theory on D-branes.

Compute Free Energy to find which one dominates the partition function.

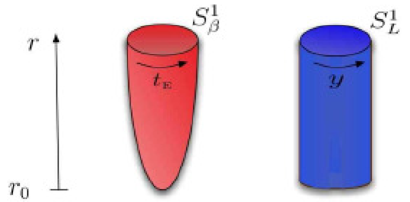
T	Gravity	Gauge Theory
	Black Hole phase (minimizes Free Energy)	**Deconfined phase:** gluon plasma
T_c		
	AdS-soliton phase (minimizes Free Energy)	**Confined phase:** " $Q - \overline{Q}$ "

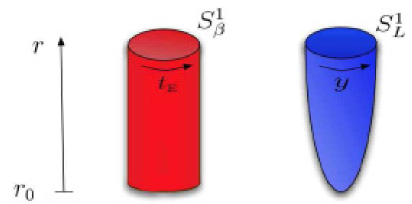
$$T_c = 1/L$$

Confinement/Deconfinement
(" Hawking-Page ")
phase transition

- **At (& in vicinity of)** $T_c = 1/L$

the two phases can co-exist



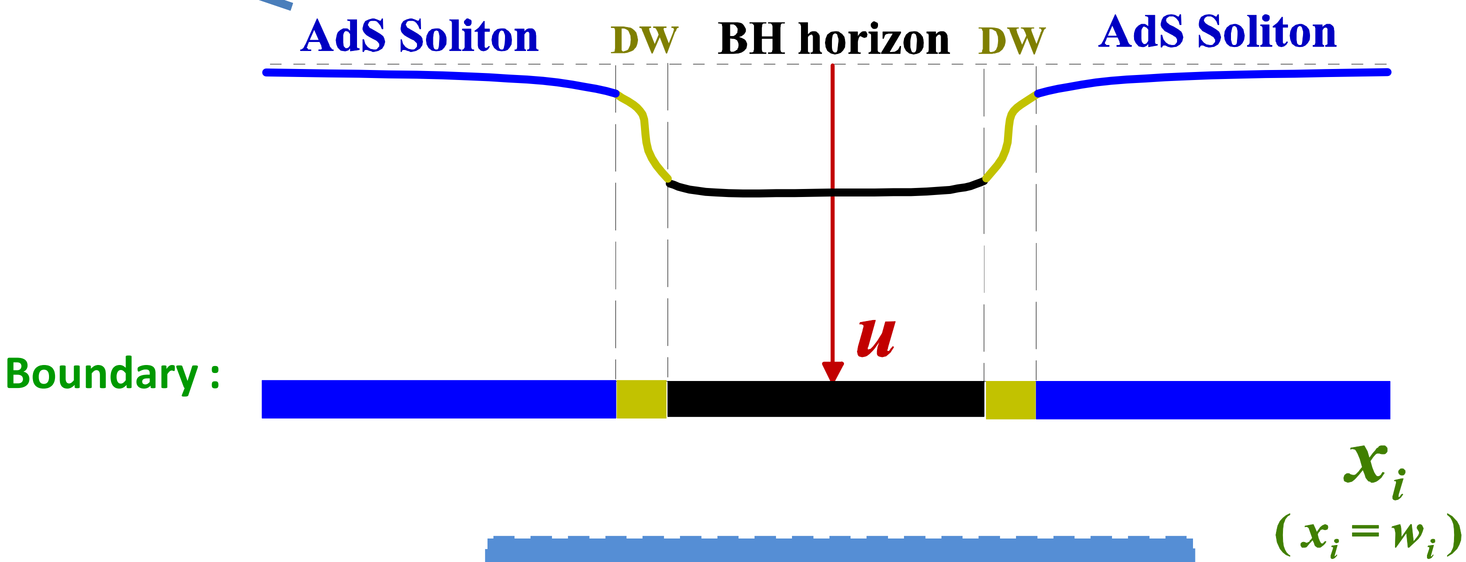


- **At (& in vicinity of)** $T_c = 1/L$ **the two phases can co-exist separated by domain wall**

$$T_x(u) = 1 - \left[\frac{x}{4\pi}(d+1)e^u\right]^{-(d+1)}$$

$$ds^2_{d+2} = \ell^2\left(e^{2u}[-T_\beta(u)dt^2 + d\theta^2 + dw_i^2] + \frac{1}{T_\beta(u)}du^2\right)$$

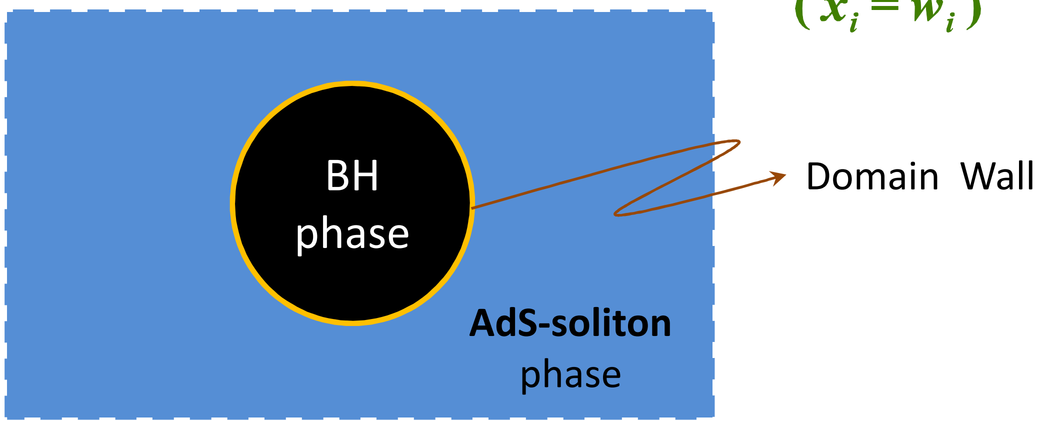
$$ds^2_{d+2} = \ell^2\left(e^{2u}[-dt^2 + T_{2\pi}(u)d\theta^2 + dw_i^2] + \frac{1}{T_{2\pi}(u)}du^2\right)$$



On the AdS Boundary:

(Aharony, Minwalla, Wiseman)

(Lahiri, Minwalla)



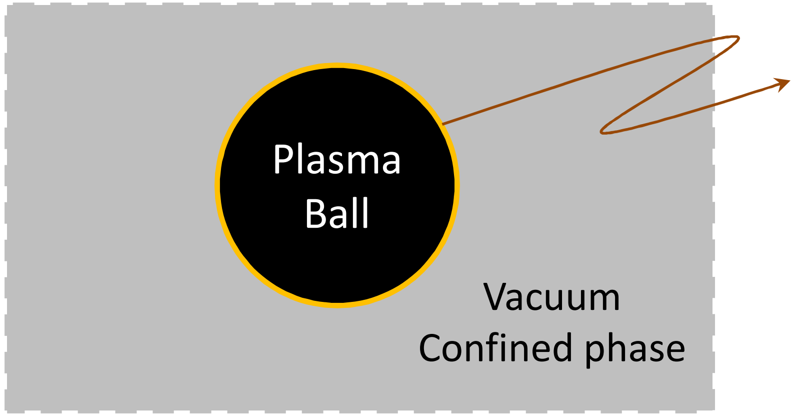
- **SS AdS / gauge theory duality**
- **BUT, for high energy density, $\ell_{\mathrm{mfp}} \ll 1$, expect QFT to have an effective hydrodynamic description**

Key Step

→ SS AdS / hydrodynamic duality

(previously, we have shown this is indeed the case for AdS/CFT)

→ Hydrodynamic description of SS QFT



Domain Wall with surface tension

(Aharony, Minwalla, Wiseman)

(Lahiri, Minwalla)

- **For large plasma lumps (neglect thickness of wall), & neglecting Dissipation, Diffusion:**

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{perf}} + T^{\mu\nu}_{\mathrm{bdry}} \,,$$

$$T^{\mu\nu}_{\mathrm{perf}} = [(\rho + P)\, u^\mu u^\nu + P g^{\mu\nu}]\, \Theta(-f)\,, \qquad T^{\mu\nu}_{\mathrm{bdry}} = -\sigma h^{\mu\nu} |\partial f|\, \delta(f)$$

➞ II. Gravity / Hydrodynamics duality ...

... So what ?

Henceforth,
we do hydrodynamics with a fluid whose equation of state describes the *d*-dim (non-conformal) SS plasma that the gauge theory is "made of"

.... to get information on Black holes !

SS reduced AdS (d+2) gravity $\longleftrightarrow$ (*d +1)*-dim SS gauge theory

d-dim Hydrodynamics
with (non-conformal) plasma

Hydrodynamic description of deconfined plasma lumps

➡ Navier-Stokes, Continuity and Young-Laplace eqs

- **For large plasma lumps (neglect thickness of wall), & neglecting Dissipation, Diffusion:**

$$T^{\mu\nu} = T^{\mu\nu}_{\text{perf}} + T^{\mu\nu}_{\text{bdry}} \,,$$

$$T^{\mu\nu}_{\text{perf}} = \left[(\rho + P)\, u^\mu u^\nu + P g^{\mu\nu}\right] \Theta(-f)\,, \qquad T^{\mu\nu}_{\text{bdry}} = -\sigma h^{\mu\nu} |\partial f|\, \delta(f)$$

- **Eqs describing dynamics of the fluid follow from the conservation of stress tensor :**

$$\nabla_\mu T^{\mu\nu} = 0$$

Volume contribution

- **Continuity eq.:** $u^\mu \nabla_\mu \rho + (\rho + P) \nabla_\mu u^\mu = 0$
- **Navier-Stokes eq.:** $(\rho + P) u^\mu \nabla_\mu u^\nu = -\left(g^{\mu\nu} + u^\mu u^\nu\right) \nabla_\mu P$

Boundary contribution

- **Young-Laplace eq.:** $P_< - P_> = \sigma K \,, \qquad K \equiv h_\mu{}^\nu \nabla_\nu n^\mu$

Boundary condition: $u^\mu n_\mu = 0$

Hydrodynamic description of deconfined plasma lumps

Stationary Plasma Configurations

$$T^{\mu\nu}_{\text{diss}} = \underbrace{-\zeta\vartheta P^{\mu\nu}}_{\text{Bulk visc.}} \underbrace{- 2\eta\sigma^{\mu\nu}}_{\text{Shear visc.}} + \underbrace{q^\mu u^\nu + u^\mu q^\nu}_{\text{Heat diffusion}}$$

- **Stationarity** $\Longrightarrow$ **NO Dissipation** $\Longrightarrow \begin{cases} \vartheta = 0 \\ \sigma_{\mu\nu} = 0 \end{cases}$
- **Plug in vorticity def.:** $\nabla^\mu u^\nu = \omega^{\mu\nu} - u^\mu a^\nu$
- **Euler relation:** $\rho + P = \mathcal{T} s \xrightarrow[\text{+ 1st law}]{\text{Differentiate}}$ **Gibbs-Duhem:** $dP = s d\mathcal{T}$
- **Mechanical equilibrium** $\Longrightarrow u^\mu \nabla_\mu P = 0$

Local entropy density

Local fluid temperature

Euler Corrections @ Cardoso, Dall'Agata, Grasso

Navier-Stokes

$$a_\mu = -(\rho + P)^{-1}\nabla_\mu P = -\nabla_\mu \ln \mathcal{T}$$

$\Longrightarrow$ Heat flux vanishes : $q^\mu = -\kappa(g^{\mu\nu} + u^\mu u^\nu)(\nabla_\nu \mathcal{T} + a_\nu \mathcal{T}) = 0$

Conc: Stationary plasmas are both at hydrodynamical & thermal equilibrium

$$u^\mu \nabla_\mu \mathcal{T} = 0 \quad \Rightarrow u^\mu a_\mu = 0$$

- Plug $a_\mu = -(\rho + P)^{-1}\nabla_\mu P = -\nabla_\mu \ln \mathcal{T}$

  in vorticity eq: $\nabla^\mu u^\nu = \omega^{\mu\nu} - u^\mu a^\nu$

$$\Rightarrow \nabla_{(\mu}(\alpha u_{\nu)}) = \alpha u_{(\mu}\nabla_{\nu)} \ln(\alpha \mathcal{T})$$

- **If** $\alpha \equiv \frac{T}{\mathcal{T}} \;\Rightarrow\; \nabla_{(\mu}[\alpha u_{\nu)}] = 0$

αu^μ **solves Killing eqs** $\Rightarrow$ **Must** be linear combination of background **Killing vectors:**

$$u = \frac{\mathcal{T}}{T}(\xi - \Omega_I \chi_I)$$

$$u^2 = -1 \;\Rightarrow\; \mathcal{T} = \frac{T}{\gamma}$$

stationary fluid must be in **rigid** roto-translational motion

$\mathcal{T}$ is the **equilibrium plasma temperature**

Redshift factor relating LAB and comoving observers

- **Euler relation + Young-Laplace:**

$$T = \frac{\sigma K + \rho}{\gamma s}$$

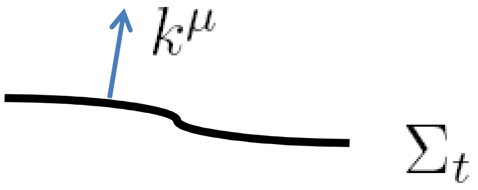
- **Plasma *T*** is **dual** to the **Hawking temperature** of the horizon.
  T is **not** proportional to the surface tension or to the mean curvature, but grows with both.

- For a **static** fluid ***K*** is **constant** over the surface,
  <u>but</u> in a **stationary** fluid ***K*** **adjusts** to variations of fluid velocity near boundary.

Variational principles for equilibrium plasma configurations

I. Maximization of the entropy at constant energy and momenta

Stationary background: $\xi = \partial_t$



- Given any Killing vector ψ^μ,

  define associated conserved charges (***E, J***):

$$\mathcal{Q}[\psi] = \int_{\Sigma_t} dv \; T_{\mu\nu} k^\mu \psi^\nu$$

- Conserved charge associated w/ $J_S^\mu = s u^\mu$

$$S[\mathcal{P}] = -\int_{\mathcal{P}} (k \cdot u) s \, dv$$

Action : **Maximization of the entropy at constant energy and momenta**

$$I[\mathcal{P}] = S[\mathcal{P}] - \beta E[\mathcal{P}] + \tilde{\omega}_I J_I[\mathcal{P}]$$

Euler-Lagrange eqs with Lagrange multipliers, $\beta = 1/T$, $\tilde{\omega}_I = \beta \Omega_I$

Young-Laplace eq.: $P_< - P_> = \sigma K$

Derivation assumes:

- only stationarity of the background geometry and fluid; independent of fluid equation of state.
- does *not* assume any condition on the shape of bdry: it covers *non*-axisymmetric cases

2. Minimization of potential energy for fixed volume

Action : **Minimization of potential energy for fixed volume**

$$\widehat{I}[\mathcal{P}] = U_\sigma[\mathcal{P}] + U_{\rm cf}[\mathcal{P}] - \eta V[\mathcal{P}]$$

surface tension + centrifugal contribution

Euler-Lagrange with Lagrange multiplier, $\eta = P_>$

Young-Laplace eq.: $P_< - P_> = \sigma K$

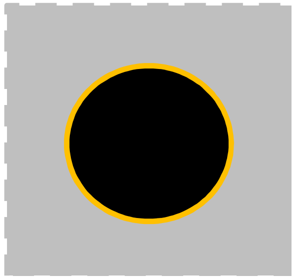
Static case : **fluids pick configurations that minimize area for fixed volume** (well known result

- Actions for the two extremization problems are the same up to a negative constant :

Use
- Definition conserved charges
- Euler relation
- Expression for Lagrange multipliers

$$I[\mathcal{P}] = -\beta \widehat{I}[\mathcal{P}]$$

$\Longrightarrow$ Maximizing entropy for fixed conserved charges $\Longleftrightarrow$ Minimizing potential energy for fixed volume



- Fluid area **minimization** $\Longleftrightarrow$ BH area **maximization**, because in the duality: the BH horizon is **not** mapped to the fluid boundary, but to the entire fluid **bulk**.

Static equilibrium Plasmas (with a compact BDRY direction)

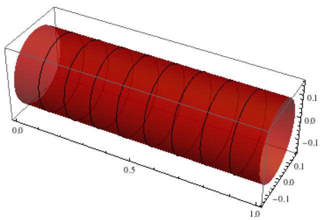
- **Young-Laplace:** **+** **Equation of state:** (*d*-dim SS plasma)

$$P_< - P_> = \sigma K \qquad\qquad P = \frac{\rho_*}{n+3}\,\gamma^{n+4} - \rho_0$$

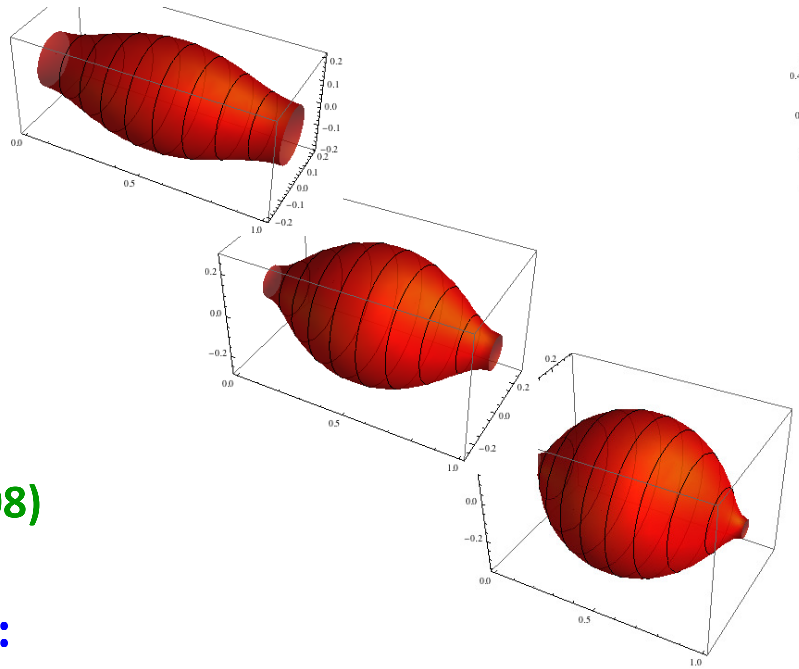
$$\rho_* = (n+3)\alpha T^{n+4}$$

$$r_z'' - \frac{n}{r}(1 + r_z'^2) + K(1 + r_z'^2)^{3/2} = 0$$

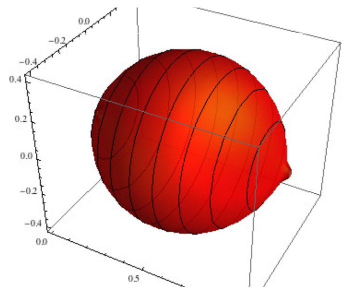
- Uniform Plasma Tubes



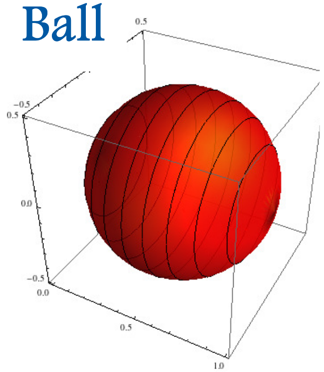
- Non-Uniform Plasma Tubes



- Merging Configuration



- Plasma Ball

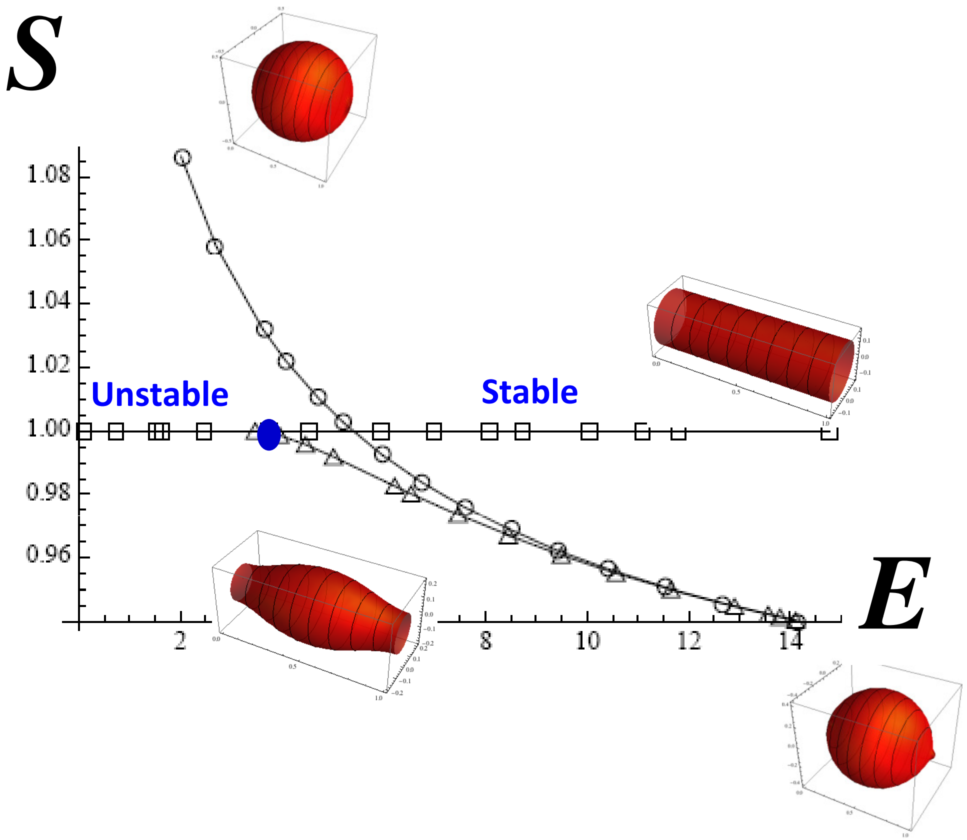


(See also: Maeda-Miyamoto, 2008)

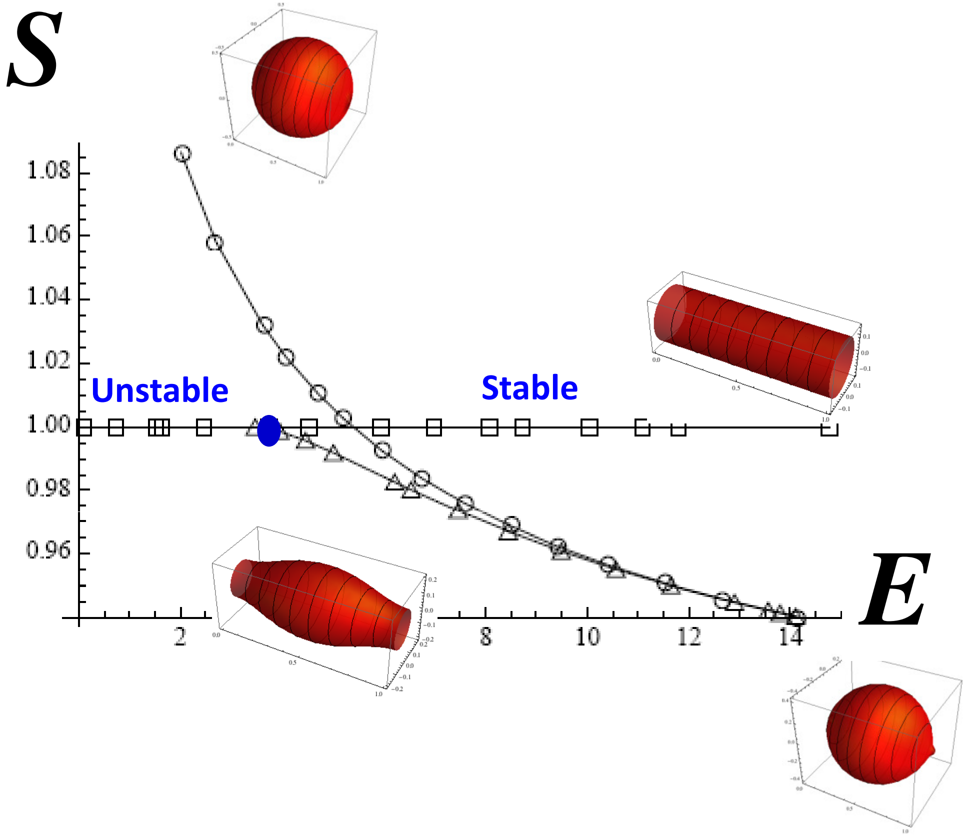
- **Gravitational dual:**

  Possible topologies of static BHs

Static plasmas in the phase diagram:



Static plasmas in the phase diagram:

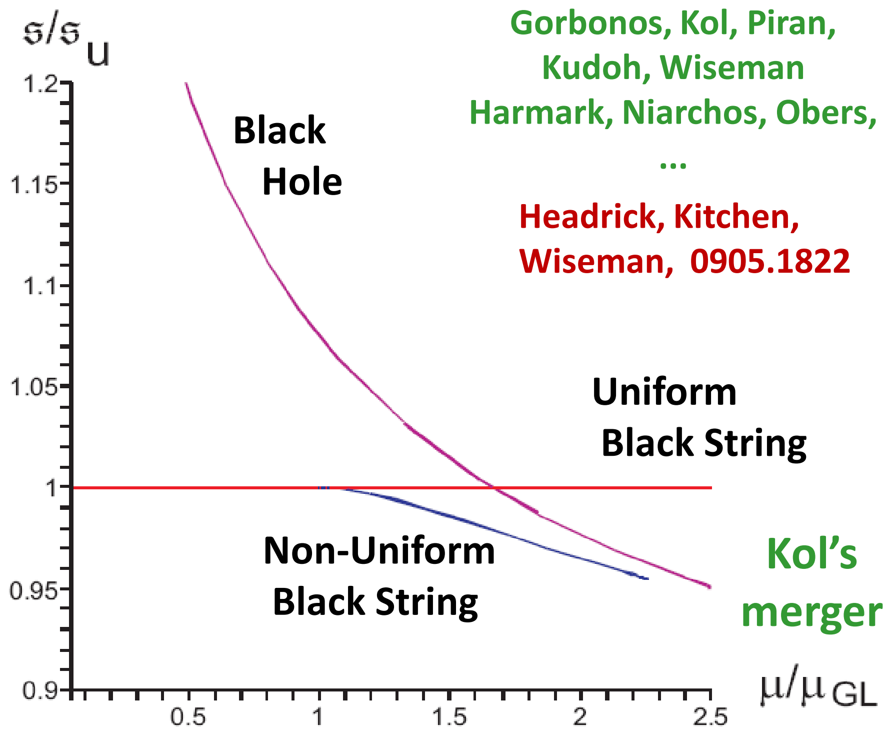


Compare with

Gravitational phase diagram:

Gubser, Gorbonos, Kol, Piran, Kudoh, Wiseman Harmark, Niarchos, Obers, ...

Headrick, Kitchen, Wiseman, 0905.1822



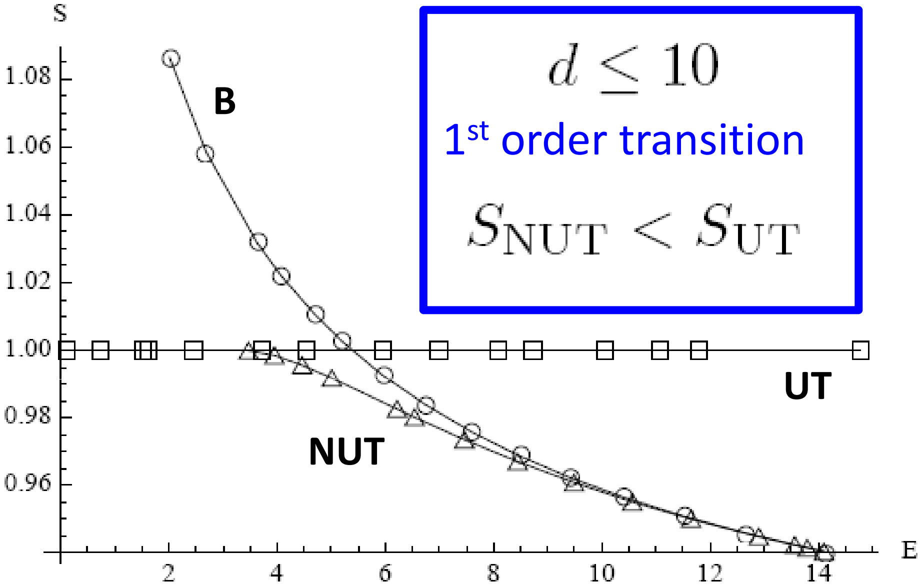
NOTE:

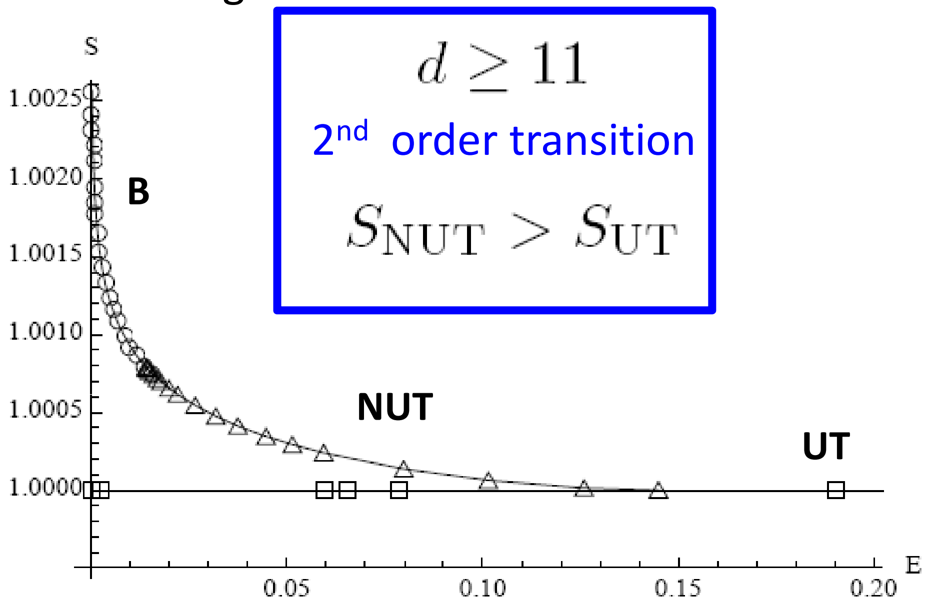
- These are available results for **Vacuum** BHs
- **Predictions** for **SS AdS** BHs

Static equilibrium Plasmas

Critical dimension in phase diagram

Critical dimension of spacetime where
transition between UT and NUT phases changes from 1st to 2nd order.





Static equilibrium Plasmas

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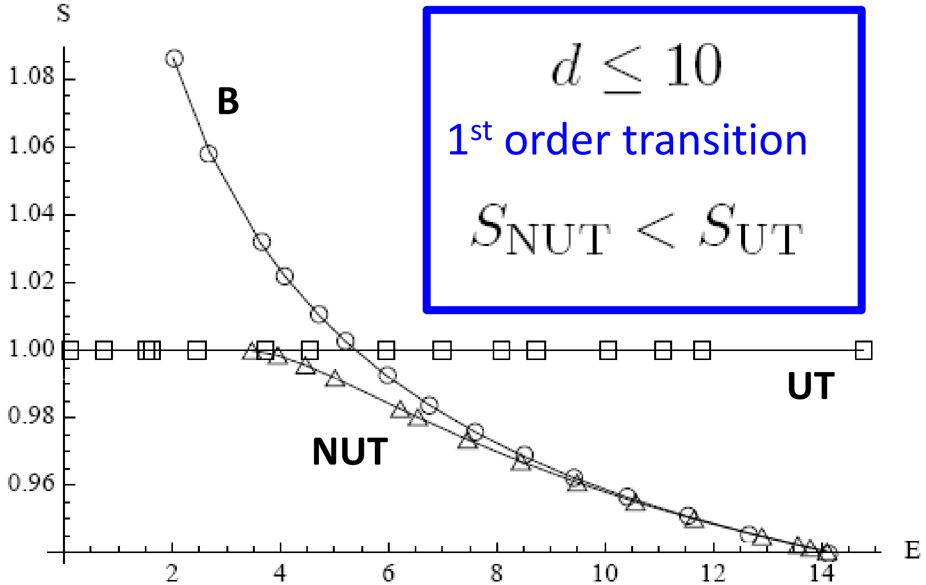
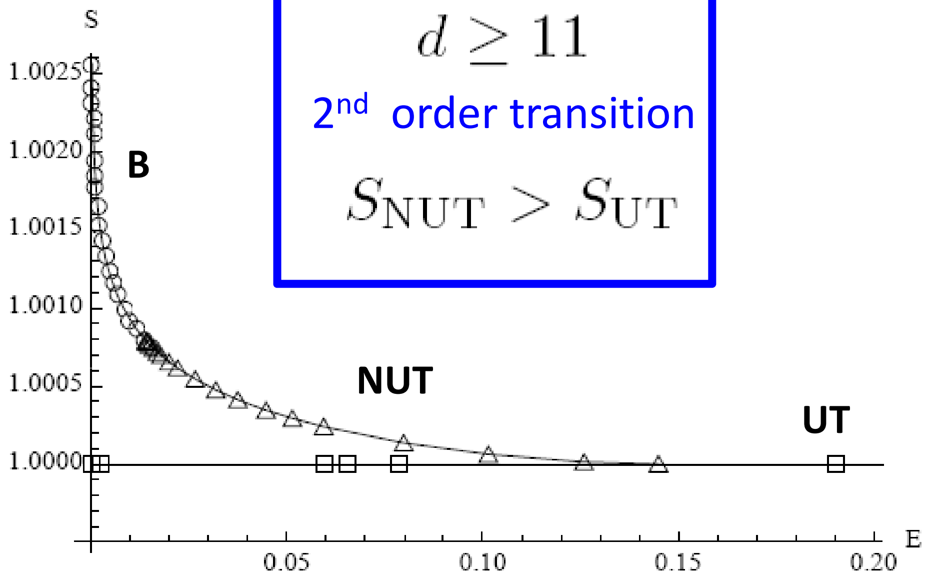


$d \le 10$

1st order transition

$S_{\rm NUT} < S_{\rm UT}$



$d \ge 11$

2nd order transition

$S_{\rm NUT} > S_{\rm UT}$

- **Compare with <u>Gravitational</u> phase diagram:**

$D \le 12$

1st order transition

$S_{\rm NUBS} < S_{\rm UBS}$

$D \ge 13$

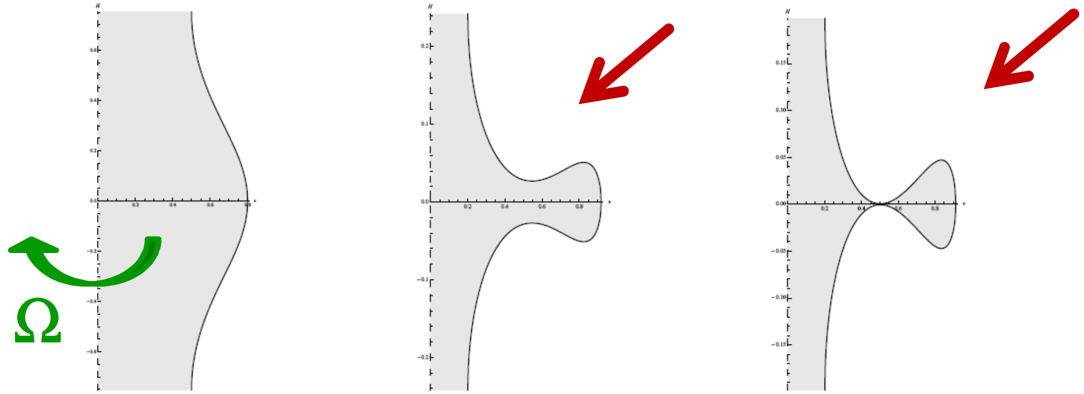
2nd order transition

$S_{\rm NUBS} > S_{\rm UBS}$

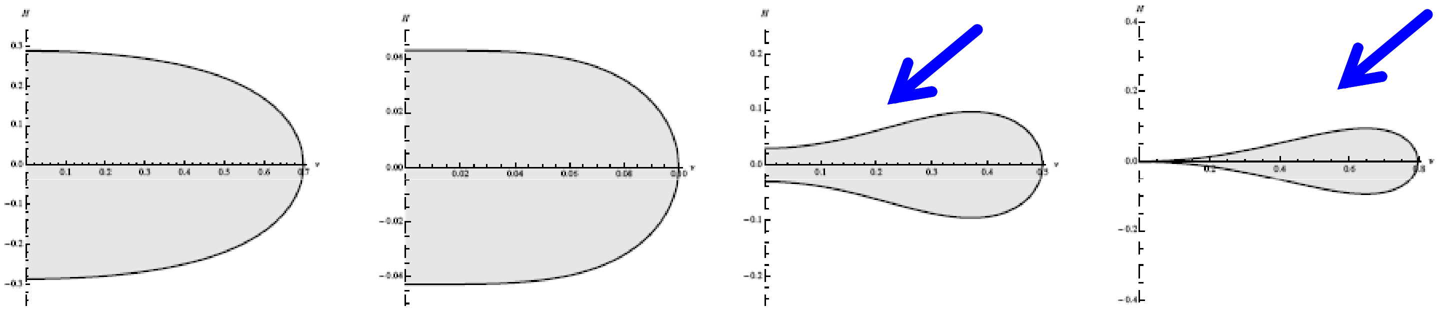
- In the AdS context, d-dim fluids are related to $D = d + 2$-dim BHs : relation between the two critical dimensions is truly startling

Rotating equilibrium Plasmas

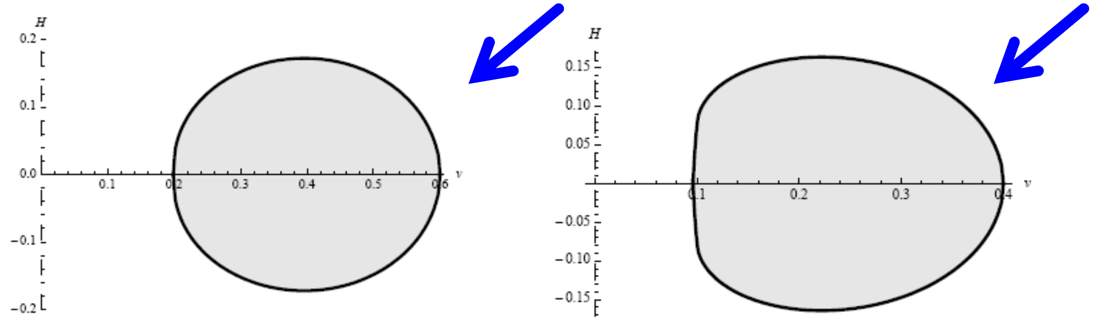
Rotating Non-Uniform Plasma Tubes



Rotating Plasma Balls



Rotating Plasma Rings

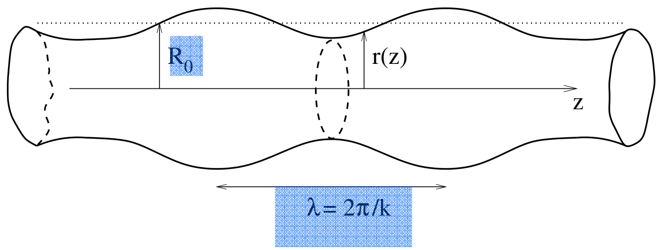


Lahiri, Minwalla

- **Gravitational dual**:
  - BHs, ultra-spinning BHs, (Non-) Uniform Black strings, Black Rings &
  - Predictions for new BH phases in SS AdS and even in asymptotically flat BHs !

Uniform plasma tubes are unstable (Rayleigh-Plateau)

Plateau, 1849



Perturbation:

$$r(z) = R_o + \epsilon R_1 \cos(kz) + \epsilon^2 R_2$$

Tube length = (possible) instability λ : $L = \lambda = 2\pi/k$

Volume to 2nd order:

$$V = \lambda \frac{\pi^{(n+1)/2}}{\Gamma\left(\frac{n+3}{2}\right)} R_o^{n-1} \left[R_o^2 + \frac{n+1}{2} \left(\frac{n}{2} R_1^2 + 2 R_o R_2 \right) \epsilon^2 \right]$$

Fix Volume:

$$R_2 = -\frac{n}{4} \frac{R_1^2}{R_o}$$

Difference between the perturbed and unperturbed surface tension potential energy:

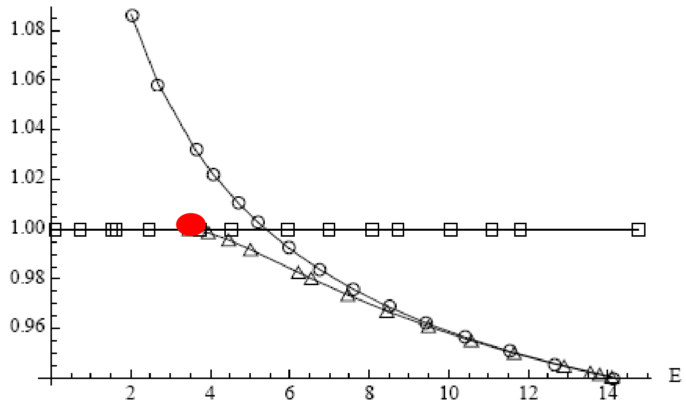
$$\Delta U_\sigma = \sigma \frac{2\pi^{(n+1)/2}}{4\Gamma\left(\frac{n+1}{2}\right)} \left(k^2 R_o^2 - n\right) R_1^2 \epsilon^2$$

Unstable modes decrease the potential energy for fixed volume (or increase S for fixed E):

$$\Delta U_\sigma \leqslant 0 \text{ for fixed volume}$$

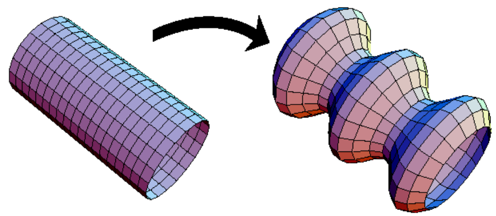
$$\Longrightarrow \qquad k R_o \leqslant \sqrt{n}$$

- **Marginal unstable mode signals Bifurcation point in phase diagram**



Rayleigh-Plateau instability on a plasma tube

(Previous work on *analogue* model: Cardoso, OD, 2006)



- Boundary Perturbation:

$$r = R(t,z,\phi)\,, \qquad R(t,z,\phi) = R_o + \epsilon\, e^{\omega t} e^{ikz + im\varphi}\,, \qquad \epsilon \ll R_o$$

- Fluid Perturbations:

$$u^\mu = u^\mu_{(0)} + \delta u^\mu\,, \qquad P = P_{(0)} + \delta P\,, \qquad \rho = \rho_{(0)} + \delta\rho$$

$$\delta\rho = (n+3)\delta P$$

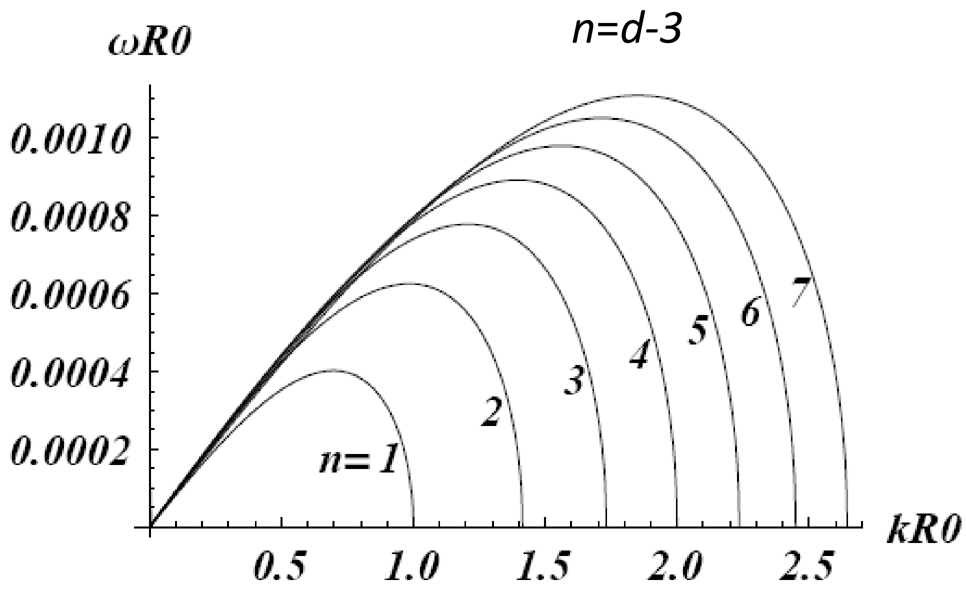
$$\delta Q(t,r,z,\phi) = \delta Q(r) e^{\omega t} e^{ikz+im\phi}\,, \qquad \delta Q \equiv \{\delta u^\mu, \delta P, \delta\rho\}$$

- **Perturbed Continuity & Navier-Stokes**
- **BCs: Perturbed Young-Laplace &** $u^\mu n_\mu = 0$

Dispersion Relation :

$$\omega^2 = \frac{n+3}{n+4}\,\frac{\sigma}{\rho_* R_o^3}\,\frac{pR_o\, I_{\frac{n+1}{2}}(pR_o)}{I_{\frac{n-1}{2}}(pR_o)}\left(n - k^2R_o^2 - \omega^2 R_o^2\right)$$

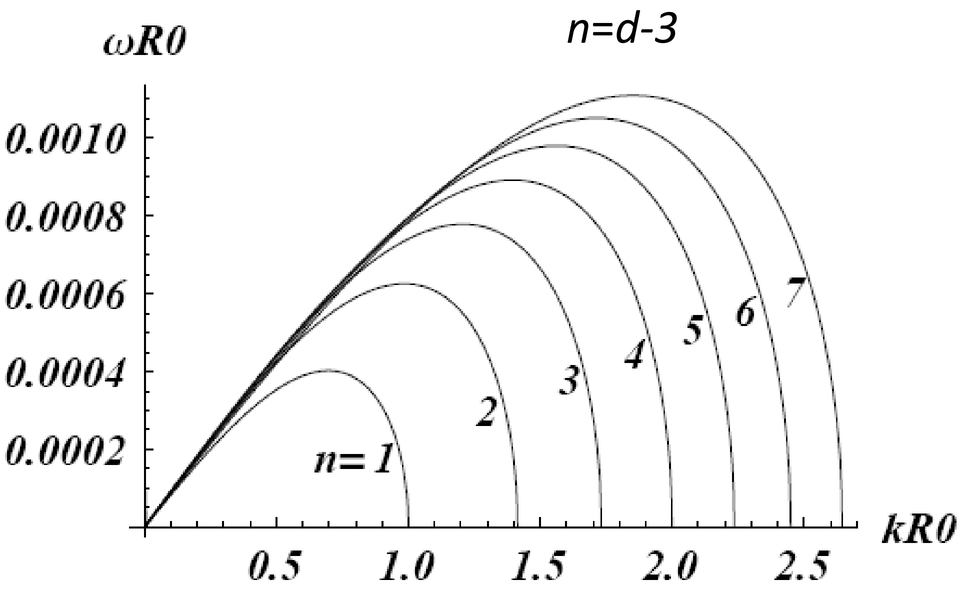
$$p = k\left(1 + (n+3)\frac{\omega^2}{k^2}\right)^{\frac{1}{2}}$$



- **Instability strength increases with the dimension.**

- **Threshold (marginal) wavenumber also:**

$$kR_o \leqslant \sqrt{n}$$

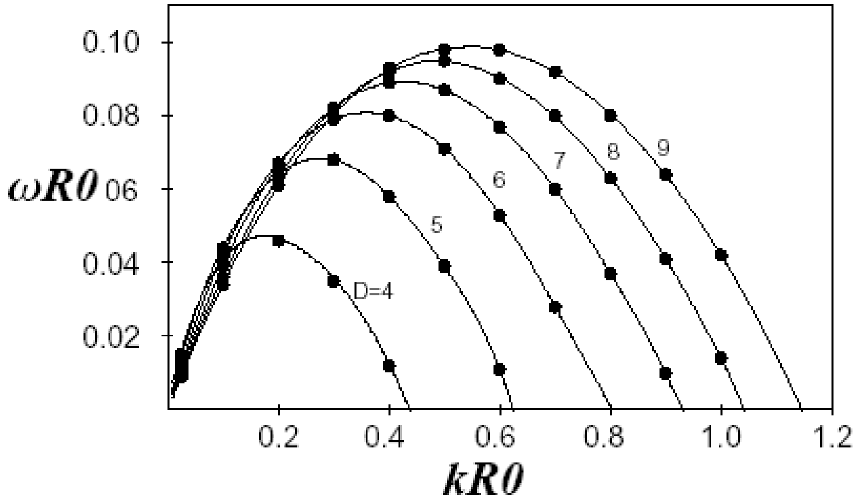


- **Instability strength increases with the dimension.**

- **Threshold (marginal) wavenumber also:**

$$kR_o \leqslant \sqrt{n}$$

Compare with <u>Gravitational</u> Gregory-Laflamme dispersion relation:



Again, NOTE:

- These are available results for **<u>Vacuum</u>** BHs
- **Predictions** for **SS AdS** BHs

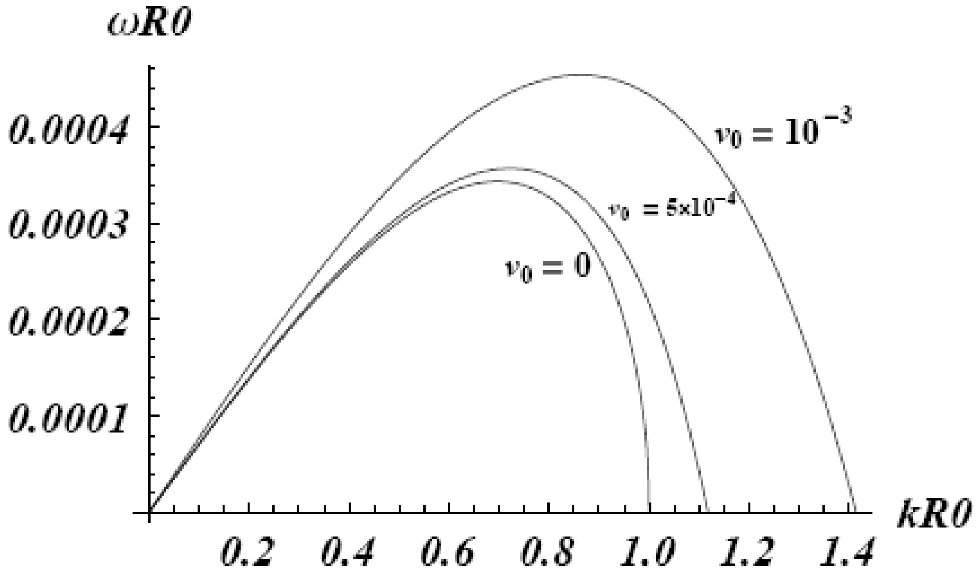
- **Threshold wavenumber :**

$$k_c R_0 \sim \sqrt{D} \qquad (\text{large } D)$$

Kol, Sorkin, 2004

Rayleigh-Plateau instability on a plasma tube ⟷ Gregory-Laflamme instability on a black string

• Addition of rotation increases the RP instability strength:

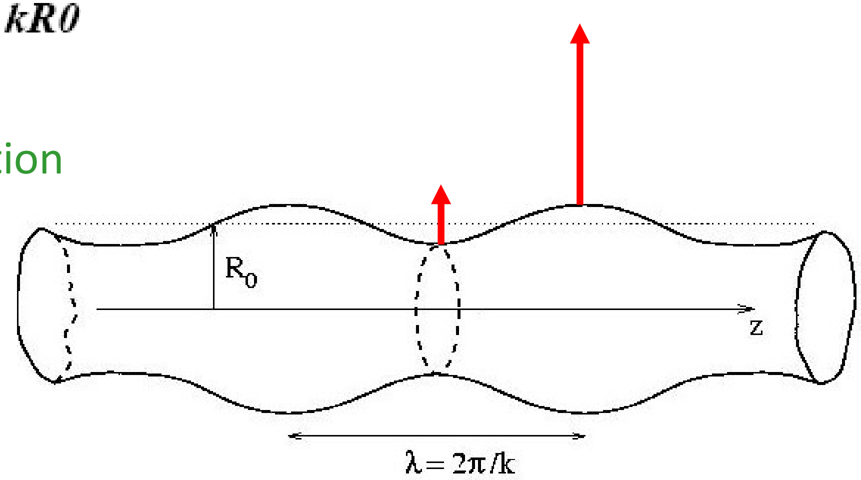


Moreover,
Non-axisymmetric modes with $m \neq 0$
can now be also **unstable** (not only s-wave)

$$R(t, z, \phi) = R_o + \epsilon\, e^{\omega t} e^{ikz + im\phi}$$

Competition: Inertia + Surface Tension + Rotation

$$F_{\text{centrif}} \sim \omega^2 r$$



• **Gravitational dual**:

Addition of rotation increases the GL instability strength
and threshold k

(Kleihaus, Kunz, Radu 2007) **(Monteiro, Perry, Santos, 2009)**

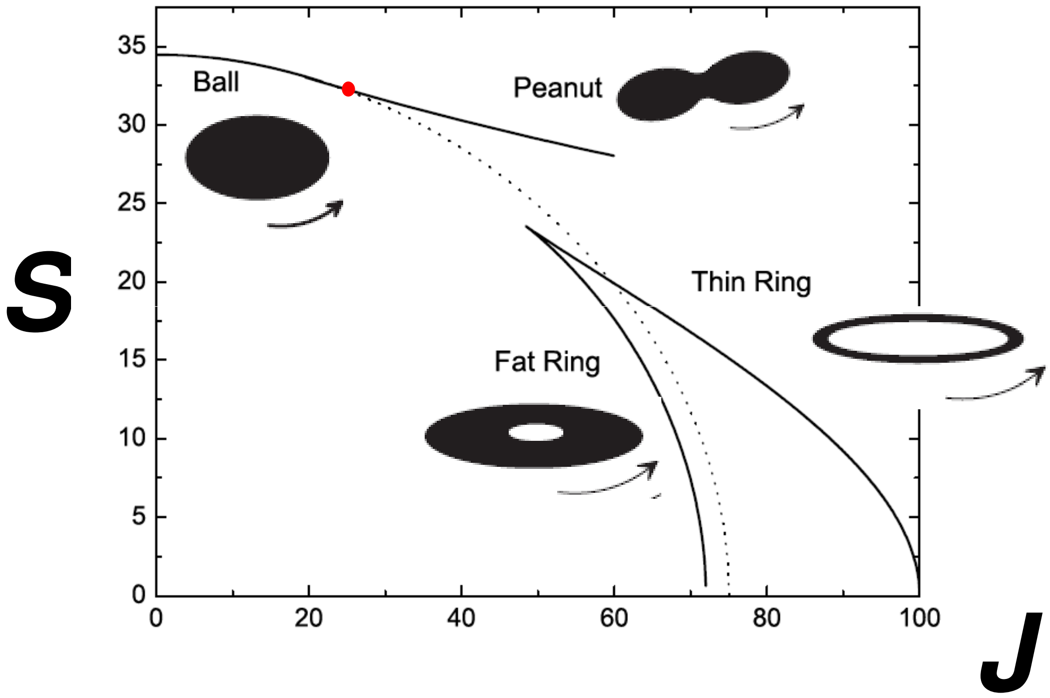
Plasma peanut instability of rotating plasma balls

V. Cardoso, OD, 0902.3560 V. Cardoso, OD, J. Rocha, (ongoing)

Rotating plasma **balls** are unstable against ***m-lobed*** **perturbations** for high rotation

signals a **bifurcation** to a new branch of **non-axisymmetric stationary** solutions:

plasma peanuts



For experiments on rotating (non-relativistic) drops and "peanut" configurations see:

Hill and Eaves, PRL (2008)

Decays through emission of Gravitational Waves

Gravitational dual :

BHs are unstable and possibly decay to a **non-axisymmetric long-lived** SS AdS BH

This instability **bounds dynamically the rotation** of SS AdS BHs

Regime of validity of hydrodynamic description

- Thermodynamic quantities must vary slowly over the mean free path of the fluid, which is of the order of the deconfinement temperature:

$$L_{\Delta T} \gg L_{\rm mfp} \sim \frac{1}{T_c} \longrightarrow \lambda \gg T_c^{-1} \sim \frac{\sigma}{\rho_0}$$

Rayleigh-Plateau unstable frequencies and wavenumbers satisfy:

$$\{\omega R_o, k R_o\} \gg \frac{\sigma}{\rho_0 R_o}$$

- Fluid **surface**, has a finite **thickness** of the order $1/T_c$.

We want the curvature of the surface to be small with respect to $1/T_c$:

$$R_{\rm bdry} \gg \Delta R_{\rm bdry} \sim \frac{1}{T_c} \longrightarrow \{R_{\rm o} > R_{\rm i}, R_{\rm o} - R_{\rm i}\} \gg \frac{\sigma}{\rho_0}$$

- We have **neglected** the dependence of the **surface tension** on the **temperature**.

Demand that on the boundary the temperature of the plasma remains close to T_c

- **In short :** $\frac{\sigma}{\rho_0 R} \ll 1 \longrightarrow$ **Large** plasma balls

Fluid dynamics : a guide for unknown dynamics of *vacuum* BHs ?

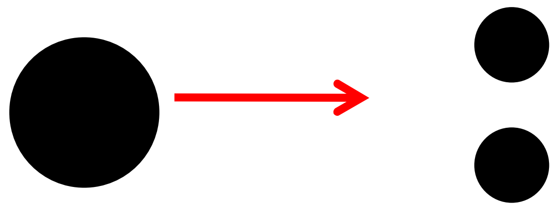
- Cannot take $R_{AdS} \rightarrow \infty$ because fluid description of AdS BHs requires $\frac{r_h}{R_{ads}} \gg 1$
- We have to find "large" vacuum BHs.... But what is large in vacuum?
- **Vacuum GR** is **scale invariant**: all properties of a black hole scale uniformly with mass
  (all Schw. BHs are equal; all Kerr BHs with same J/GM^2 are equal,...)
- In the **fluid**, **NO** scale invariance: σ sets a scale that distinguishes lumps of different size.
  (a fluid ball with radius *2R* can break up more easily than a ball with radius *R*).

- **Fluid**: relative **entropy cost** of breaking into two pieces becomes
  arbitrarily **small** as the radius R of the **ball** gets arbitrarily **large** ($\Delta S < 0$, supressed):

$$\left.\frac{\Delta S}{S}\right|_{\text{fluid}} = \left.\frac{2S_1 - S_0}{S_0}\right|_{\text{fluid}} \propto \frac{-\sigma}{\rho_0 R}$$

- **Black hole**, in contrast, it remains **constant** independently of the black hole size:

$$\left.\frac{\Delta S}{S}\right|_{\text{bh}} = \left.\frac{2S_1 - S_0}{S_0}\right|_{\text{bh}} = 2^{-\frac{1}{D-3}} - 1$$



Fluid dynamics : a guide for unknown dynamics of *vacuum* BHs ?

.... So, what is large BH in *vacuum*?

• Important **difference** between fluids and BHs:

two disconnected lumps of fluid do **not** attract each other

• Limit in which vacuum black holes behave as fluids must be a limit

in which **gravitational attraction** is **suppressed**

• Gravitational attraction gets suppressed as number of dimensions ***D* grows**

(Grav. potential becomes steeper & + localized near source, and flatter & weaker at large distances)

• **Our proposal:**

- **a BH is large or small depending on the number of spacetime dimensions it lives in.**
- **a large BH (*ie* living in high *D*) should have a fluid description**
- **identify the "new vacuum GR scale" with the fluid scale:** $\frac{\sigma}{\rho_0 R} = \frac{1}{D}$

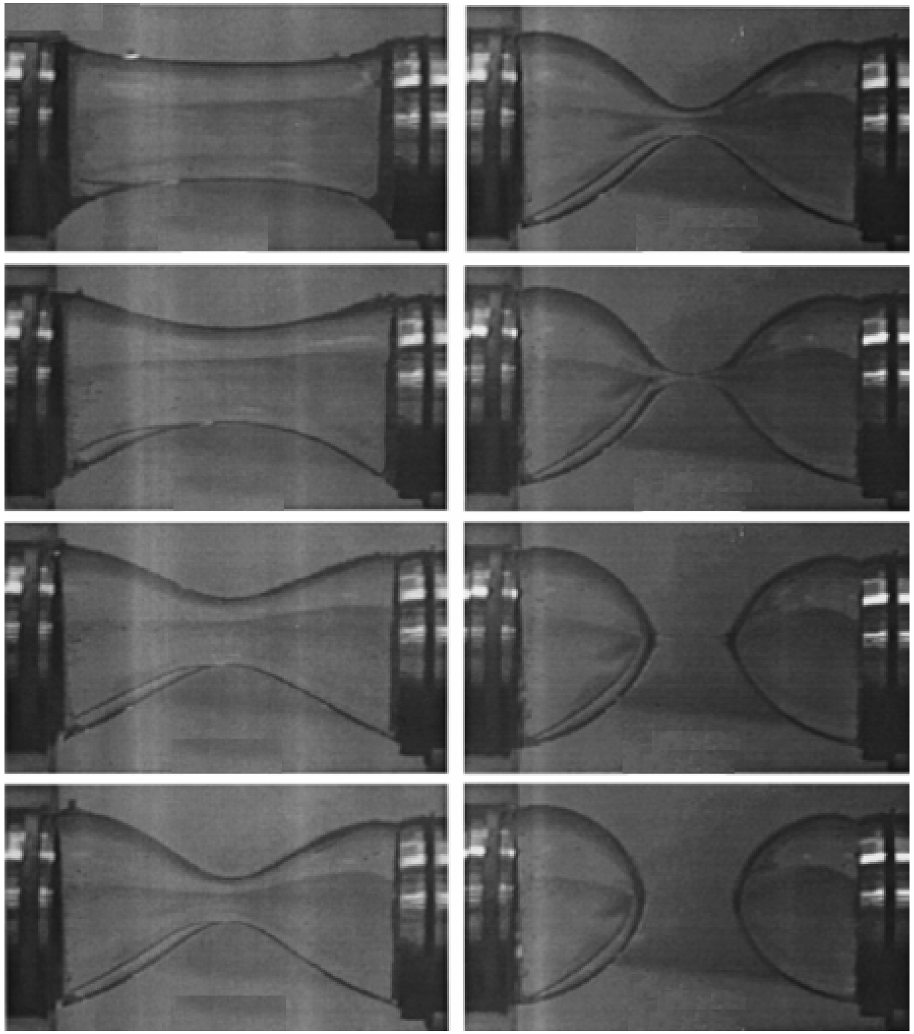
• In large *D* limit:

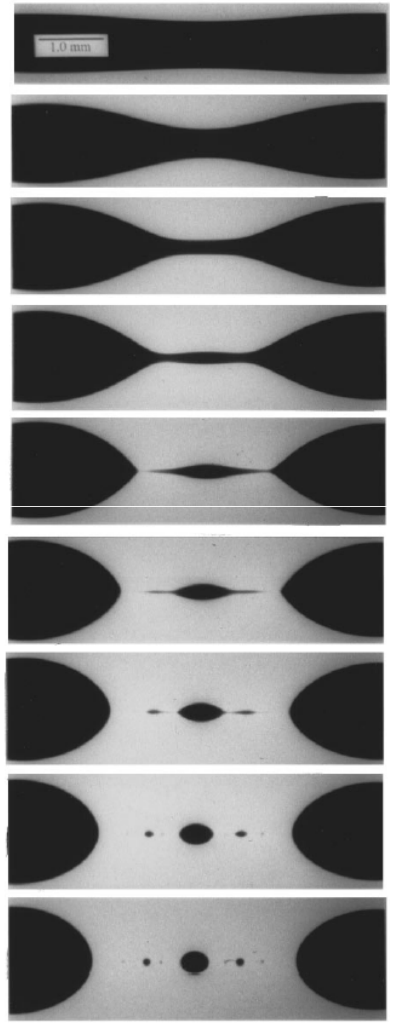
• **No** entropy **cost** in splitting a BH: $\Delta S/S \to 0$ (as for fluids)

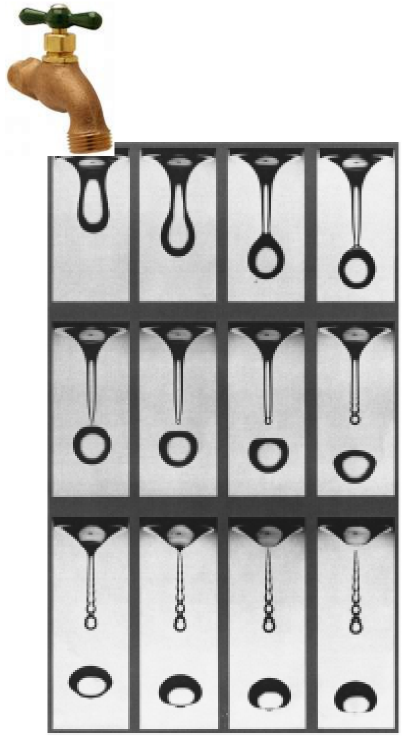
• Gregory-Laflamme wave equation and threshold mode reduce to the fluid form:

$$\chi''(r) + \frac{D}{r}\chi'(r) - k^2\chi(r) = 0 \qquad\qquad k_c R_0 \sim \sqrt{D}$$

Rayleigh-Plateau Time evolution on a plasma tube

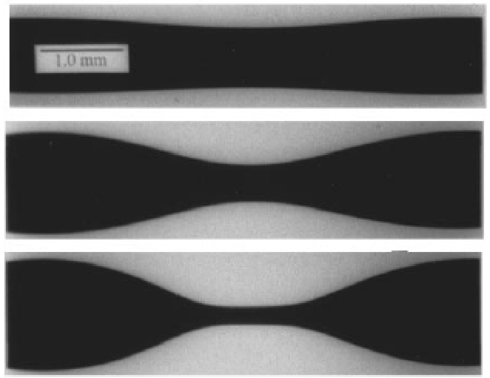




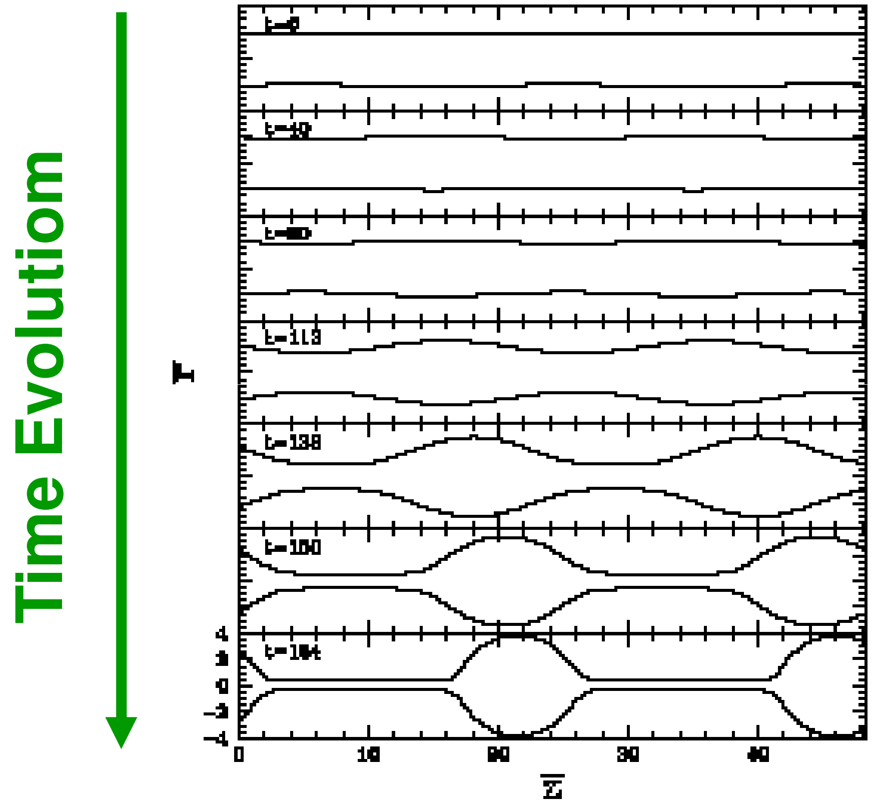


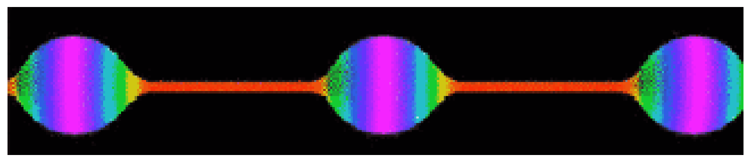
(Tjahjadi, Stone, Ottino, 1992)

Rayleigh-Plateau Time Evolution



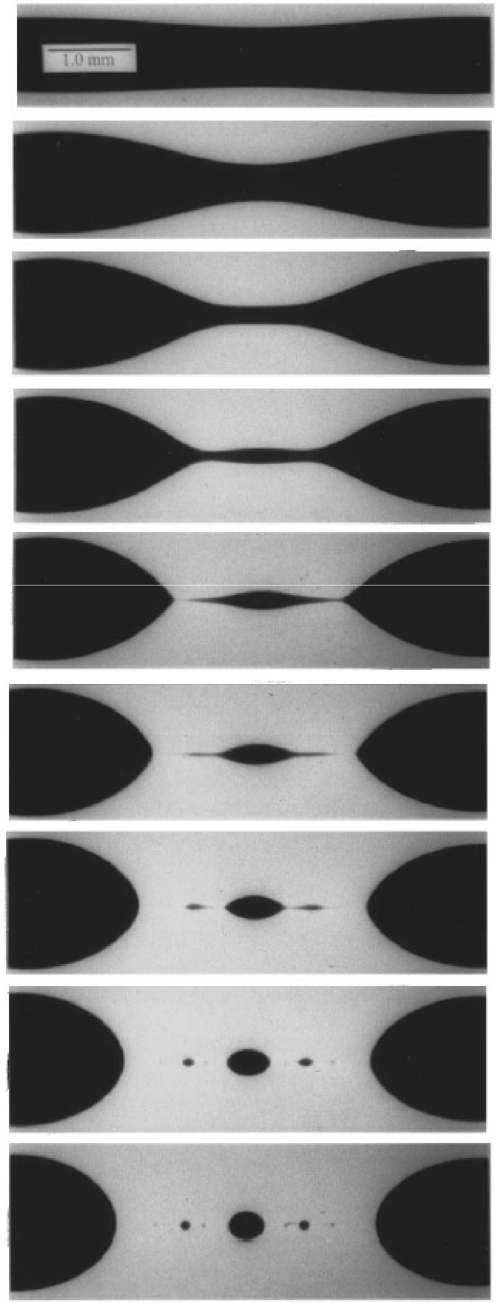
Gregory-Laflamme Time evolution





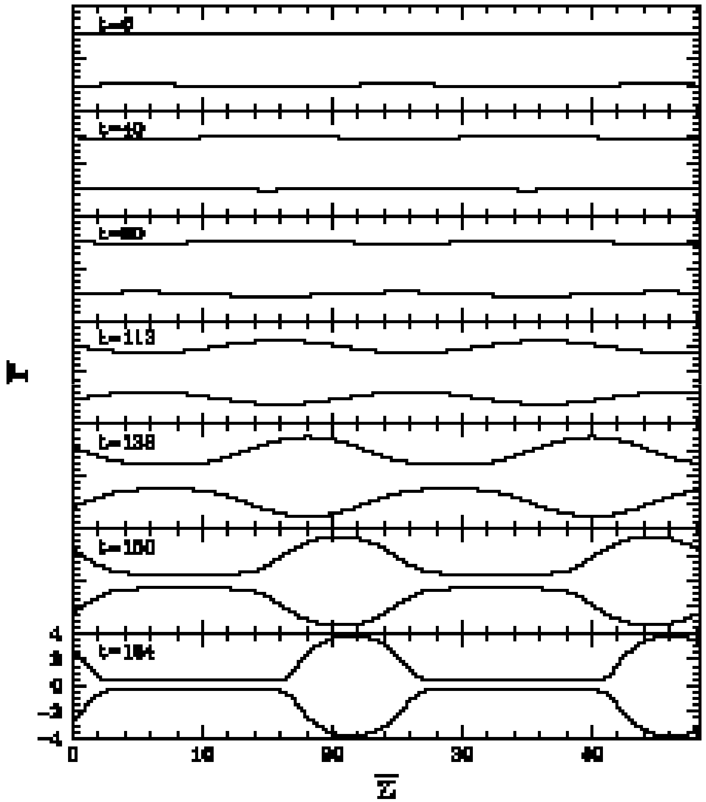
Choptuik, Lehner, Olabarrieta, Petryk, Pretorius, Villegas, gr-qc/0304085

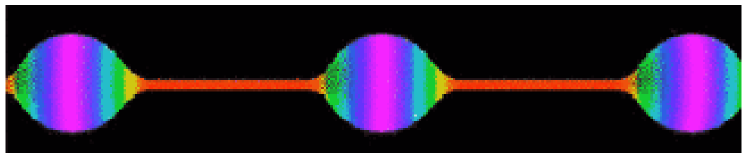
Rayleigh-Plateau Time Evolution



Gregory-Laflamme Time evolution

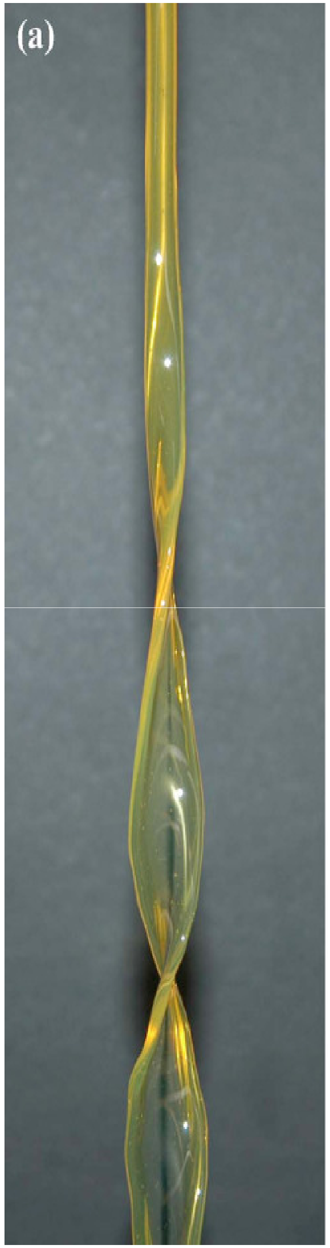
Time Evolution





Choptuik, Lehner, Olabarrieta, Petryk, Pretorius, Villegas, gr-qc/0304085

Rayleigh-Plateau Time evolution on a rotating tube



Kubitschek & Weidman (2007)

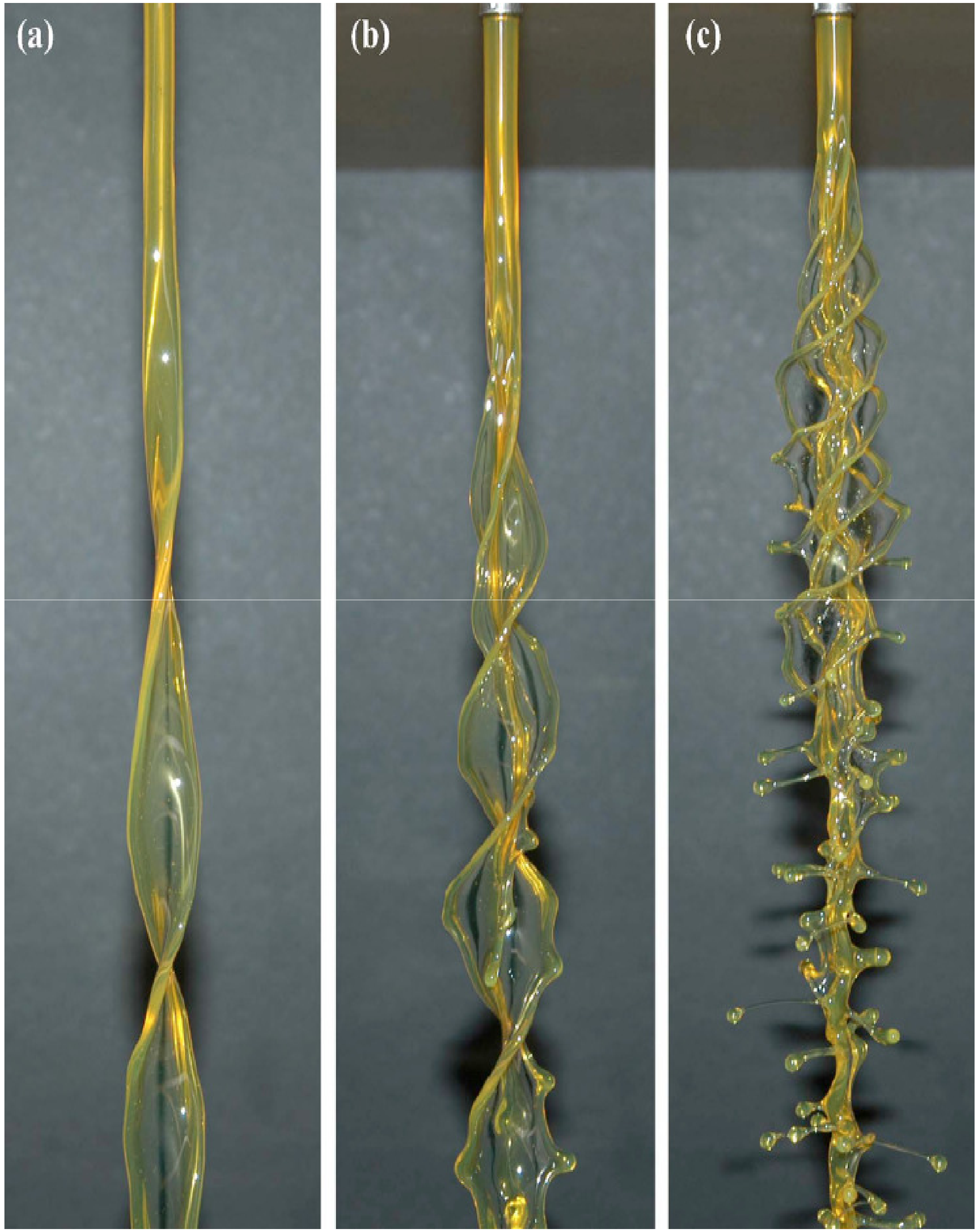
Thank You!

Rayleigh-Plateau Time evolution on a rotating tube

Kubitschek & Weidman (2007)

Thank You!

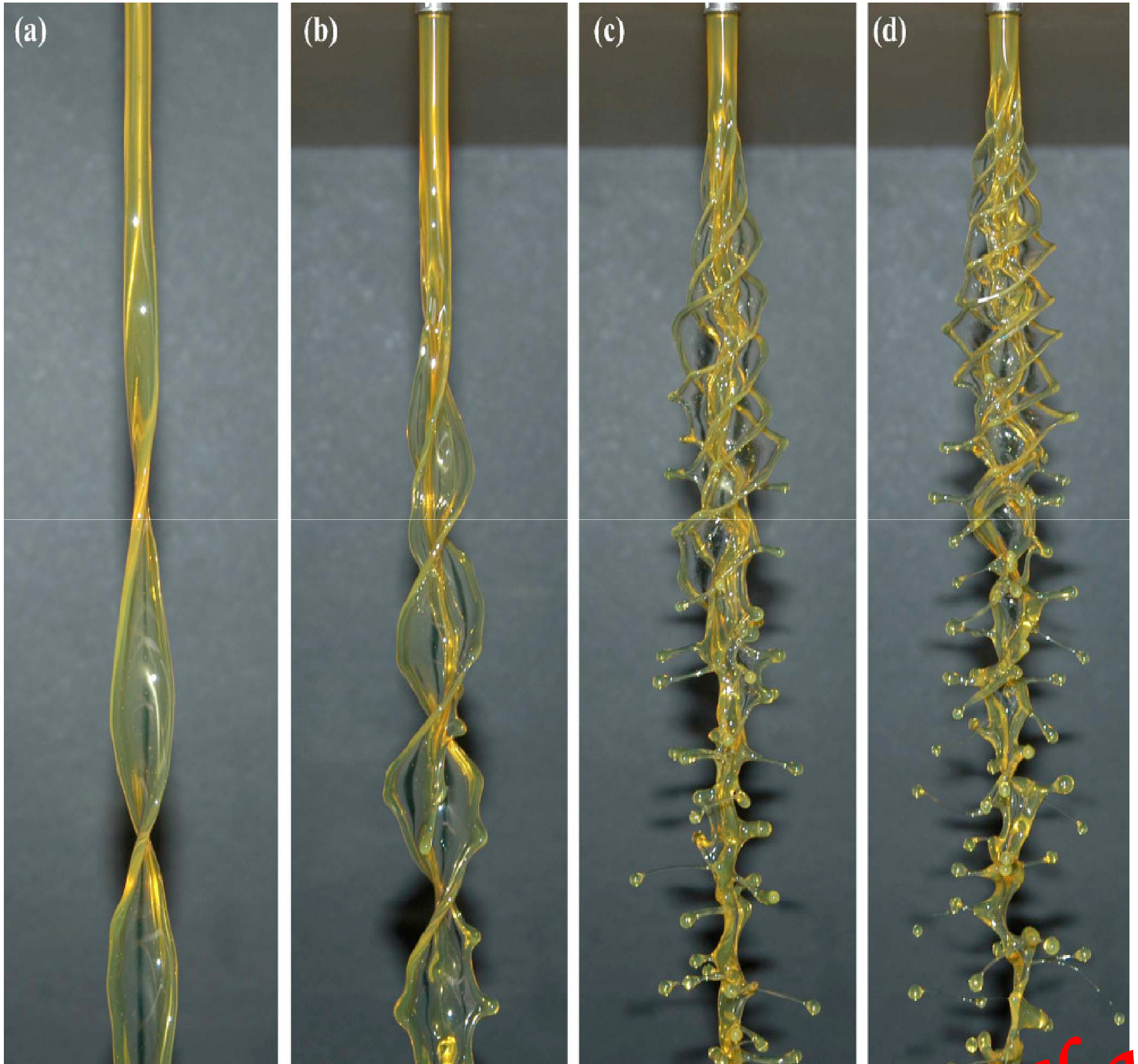
Rayleigh-Plateau Time evolution on a rotating tube



Kubitschek & Weidman (2007)

Thank You!

Rayleigh-Plateau Time evolution on a rotating tube



Kubitschek & Weidman (2007)

Thank You!