Cobordism Defects in String Theory

No global symmetries in quantum gravity !

[Banks, Dixon '88, Susskind '95, Banks, Seiberg '11, Ooguri, Harlow '18]

Black hole entropy

AdS / CFT

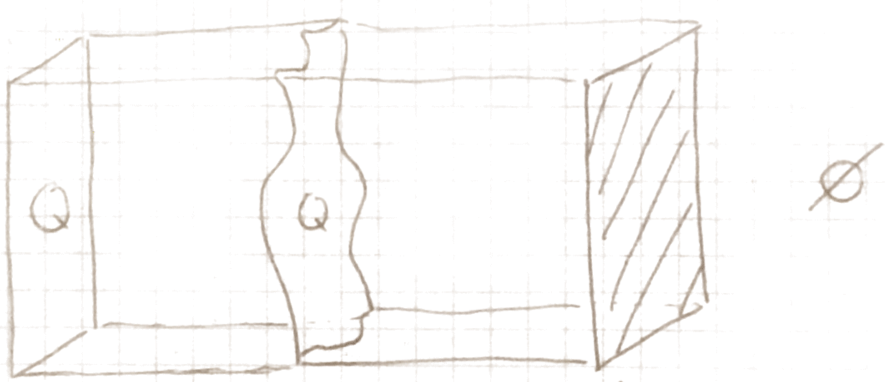
cannot be reconstructed

Global symmetries define conserved charges*
(→ separate theories into sectors labeled by charge)

* · does not cost energy
· cannot be measured at infinity
→ no Gauss' law

These pose obstructions to boundary conditions.

[Here, I work in Euclidean spacetime]



Every quantum gravity must allow for boundary conditions (deformable to nothing)
→ transition to sector with trivial charges
→ no global symmetries

→ <u>Cobordism Conjecture</u>

[McNamara, Vafa '19]

Cobordism theory : $\Omega^{\xi}_{k}(X)$

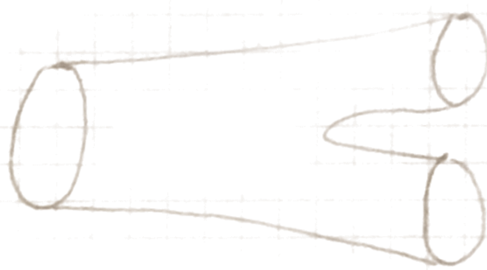
tangential structure

gauge bundles and scalar

dimension

specifies deformation classes of manifolds

$M \sim N$ if there is Y such that $\partial Y \sim M - N$



with all relevant structures (orientation, spin structure, gauge bundles, ...) extended from M, N to Y.

Defines an Abelian group under disjoint sum

$M + N$

M is trivial if it is a boundary $M = \partial Y$
$M \sim \emptyset$

this is precisely what we want !

Thus, to avoid global symmetries we demand

$$\Omega_k^{QG} = \emptyset$$

for $k < D$ (number of spacetime dimensions)

If not the generators of cobordism groups carry global charge*** that cannot be deformed away.

Strategy:
- set up a structure approximating QG
- find cobordism groups*
- include objects that "kill" non-trivial generators

(these are needed to get rid of symmetries that are not taken care of by semi-classical gravity)**

Warm-up examples

$$\Omega_1^{SO}(pt) = \emptyset$$

$$\Omega_1^{Spin}(pt) = \mathbb{Z}_2$$

one needs to define a spin structure, here boundary conditions of fermions around circle

$$\Psi(x + 2\pi R) = \pm \Psi(x)$$

*** either as compactification

non-compact

or connected sum

* this is often the hardest step involving heavy maths in algebraic topology

** We can ask what are the obstructions to deform to nothing and how to get rid of them.

Typically
- topological features of spacetime ($\mathrm{tr} R^2, \omega_i, \ldots$)
- topological features of gauge bundles ($\mathrm{tr} F^2, F_{p+1}, \ldots$)

$\psi \to -\psi$ by 2π rotation on disk

not a boundary of spin manifold

but

$\sim \emptyset$ $\qquad \mathbb{Z}/2\mathbb{Z} = \mathbb{Z}_2$

if this was a quantum gravity we would need to include an object that flips the fermion boundary condition, a spin defect

$\Omega_1^{Spin}(BU(1))$ same as above for spin but what about $U(1)$ gauge field

gauge background defined by $e^{i\oint_{S^1} A}$ the holonomy but all of these can be bounded using Stokes' theorem

non-trivial F

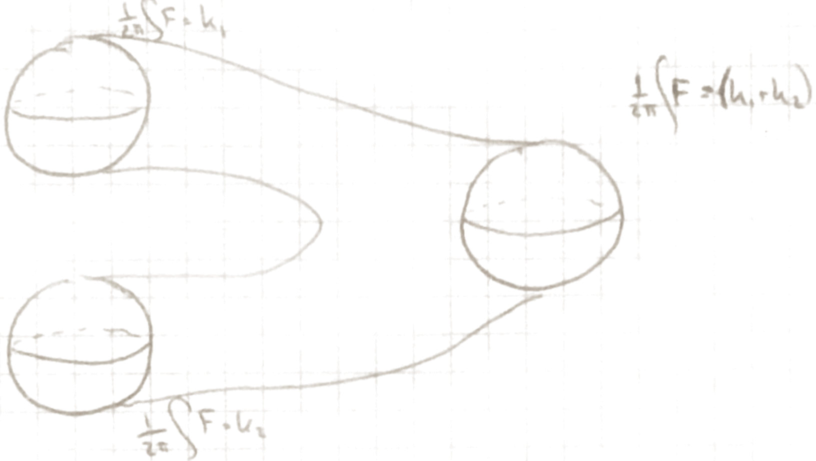
with $\int_{D^2} F = \int_{S^1} A$

$\rightsquigarrow$ no new obstructions

$\Omega_2^{Spin}(BU(1))$ (spin part $\mathbb{Z}_2$ generated by $S^1_+ \times S^1_+$)

more interestingly a new generator

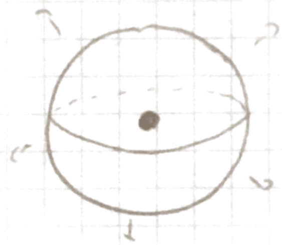
S^2 with $\frac{1}{2\pi}\int F = 1$ generating $\mathbb{Z}$



magnetic flux is conserved since for smooth $U(1)$ bundles $dF = 0$

$\rightsquigarrow$ to get rid of $\mathbb{Z} \subset \Omega_2^{\mathrm{Spin}}(BU(1))$ we need to include objects violating that

$\rightsquigarrow$ magnetic monopoles



$$\frac{1}{2\pi}\int_{\partial Y} F = n$$

characteristic classes often appear in the analysis of bordism groups

In string theory

We have RR-fields $U(1)$ p-form fields C_p

$$\Omega_n^{\mathrm{Spin}}(U(1)\ \text{p-form}) = \Omega_n^{\mathrm{Spin}}(B^pU(1))$$

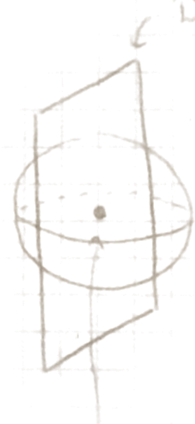
one will always find $\Omega_{p+1}^{\mathrm{Spin}}(B^pU(1)) \supset \mathbb{Z}$

with generator S^{p+1} with $\frac{1}{2\pi}\int_{S^{p+1}} F_{p+1} \in \mathbb{Z}$

we need to include "monopoles" of RR fields

→ <u>D-branes</u>

Of course this is only an approximation only capturing RR charges.
But D-branes have an inner structure (gauge bundle) captured by K-theory



looks like D(p-3)-brane

$$\int_{D(p-1)} F \wedge C_{p-2}$$

$$F = 2\pi\, \delta_{p-3}$$

→ so top D-brane should be enough, which would be captured by a refined version

$$\Omega_k^{K\text{-theory}}$$

⌜ This should be a caveat that we are dealing with approximations ⌟

<u>Now something new:</u>

<u>Heterotic string</u> [Kaidi, Ohmori, Tachikawa, Yonekura '23] [Debray '23]

there is a $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ gauge group with $\mathbb{Z}_2$ exchanging the two E_8 factors

Moreover we need to satisfy Bianchi identity

$$dH = \mathrm{tr}\, R^2 - (\mathrm{tr}\, F_1^2 + \mathrm{tr}\, F_2^2)$$

defining a twisted string structure

⌜ this is actually great news, since we do not have to deal with K3 as generator of $\Omega_4^{Spin}(pt)$. ⌟

$$\Omega_1^{het} = \mathbb{Z}_2 \times \mathbb{Z}_2 \quad \leftarrow \text{new}$$

same as Ω_1^{Spin}

one needs a <u>new</u> 7-brane in the heterotic string

M-theory on Moebius strip

it breaks supersymmetry

<u>type IIB</u> [Debray, MD, Heckman, Montero '23]

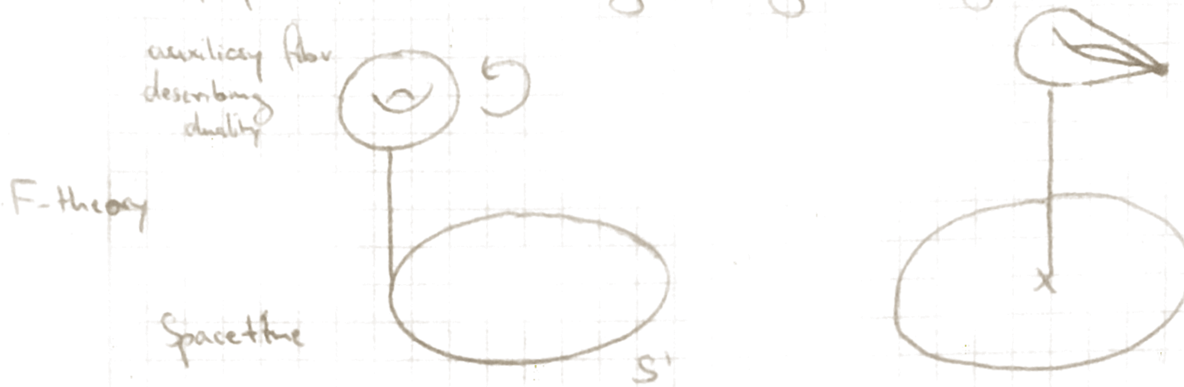
type IIB has duality

$$\begin{array}{ccc} SL(2,\mathbb{Z}) & \xrightarrow[\text{reversal}]{\text{worldsheet orientation}} & GL(2,\mathbb{Z}) \\ \text{fermions} \downarrow & & \downarrow \\ Mp(2,\mathbb{Z}) & \longrightarrow & Pin^+(GL(2,\mathbb{Z})) \end{array}$$

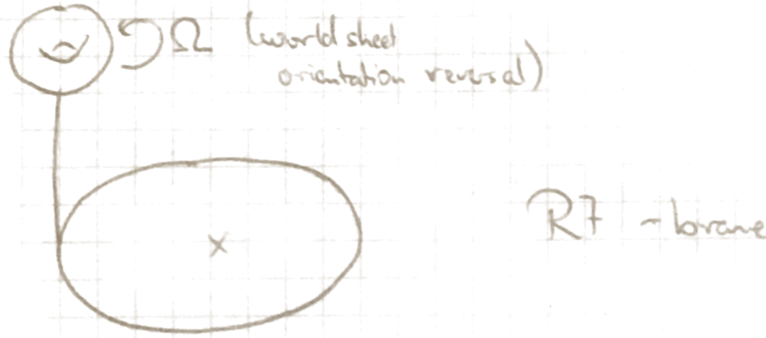
approximation $\Omega_k^{Spin-Pin^+(GL(2,\mathbb{Z}))}(pt)$

$$\Omega_1^{Spin-Pin^+(SL(2,\mathbb{Z}))} = \mathbb{Z}_2 \times \mathbb{Z}_2$$

First factor taken care of by stack of $[p,q]$-7-branes generating E_7 gauge dynamics.



The second factor needs a new 7-brane



Also many more (new) interesting configurations (NHC, S-folds, topological twists, ...)

Have you noticed: $\Omega^{Spin}_{1}(pt)$ is gone
in type IIB we know what it is

but also smooth

$\to$ many things to discover!