Applications of holography to strongly coupled media

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Lecture I : Introduction

Hydrodynamics

Lecture II : Finite-T correlators from AdS/CFT

Kinetic coefficients

Lecture III : Nonrelativistic CFTs and holography

AdS/CFT :

theory w/o gravity $\Leftrightarrow$ theory with gravity in higher dimension

$\mathcal{N}=4$ SYM theory type IIB in $AdS_5 \times S_5$

$d = 4$

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\vec{x}^2\right) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$

- $N_c, g \rightarrow$ parameters
  $\lambda = g^2 N_c$
- $g_s \rightarrow$ string coupling
- $l_s = \sqrt{\alpha'}$ string length
  R
  $\Big\}\ R/l_s$

Taking the limits

$\lambda = \left(\frac{R}{l_s}\right)^4 \rightarrow \infty \Leftrightarrow$ Einstein gravity

$N_c \rightarrow \infty \Leftrightarrow$ classical limit

Black holes

- characterized by M, Q
- thermodynamic system: T, μ

$$ds^2 = \frac{r^2}{R^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{R^2}{r^2 f(r)}dr^2 + R^2 d\Omega_5^2 \quad \Longleftrightarrow \quad \text{thermal state of the QFT } (d=4)$$

$$f(r) = 1 - \frac{r_0^4}{r^4}$$

This metric is a solution of

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = \# g_{\mu\nu}$$

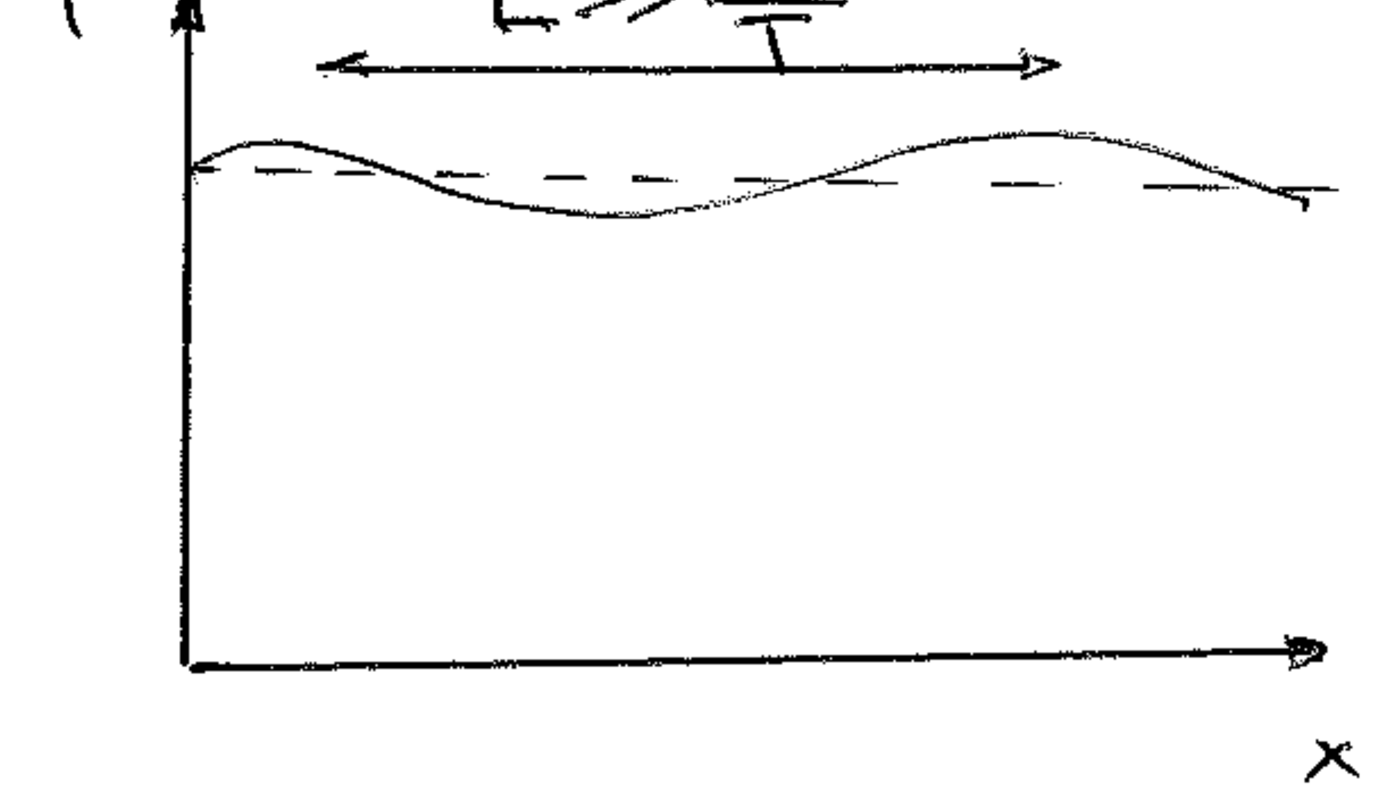
This metric is also called "black brane" which is translationally invariant in $\vec{x}$-direction

Hawking-temperature T_H of the black brane $\Longleftrightarrow$ temperature of the field theory (e.g. temp. of the plasma)

The entropy can be determined and one gets

$$S(\lambda \to \infty) = \tfrac{3}{4}\, S(\lambda = 0) \quad \text{at same } T$$
$$\sim N_c^2 T^3$$

Hydrodynamic regime



$\frac{1}{T}$ typical length scale & time scale

L wavelength perturbation

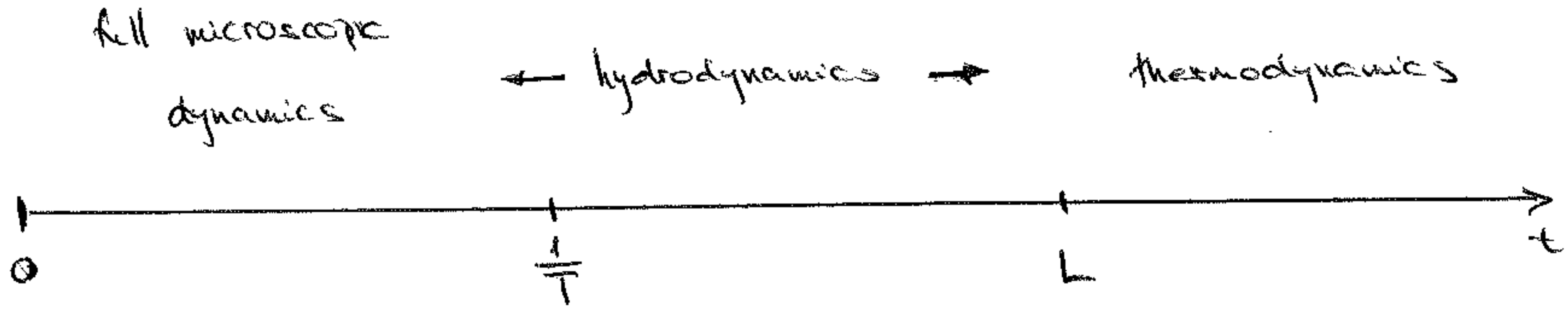
In the regime $L \gg \frac{1}{T}$

$\to$ hydrodynamic regime

during the time $t \sim \frac{1}{T}$ $\rightarrow$ local thermal equilibrium $T = T(t,\vec{x})$

$\mu = \mu(t,\vec{x})$

during time $t \gtrsim L$ $\rightarrow$ global thermal equilibrium



for a quantum field theory

$\lambda \ll 1$: $\ell_{mfp} \sim \dfrac{1}{\lambda^2 T \ln 1/\lambda}$

hydrodynamics $\rightarrow$ $L \gg \ell_{mfp}$

<u>Hydrodynamics</u> (Landau & Lifshitz, vol. 6)

- $T(t,\vec{x})$, $\mu(t,\vec{x}) \sim n(t,\vec{x})$
  density
- Goldstone bosons $\Leftarrow$ superfluid hydrodynamics (^{4}He)
- unbroken U(1) gauge sym. (massless photons) $\Leftarrow$ Magnetohydrodynamics

In the following the to be considered degrees of freedom are
$T(t,\vec{x})$ & $\mu(t,\vec{x})$

Neutral plasma:

$$\nabla_\mu T^{\mu\nu} = 0$$

expressible in terms of local variables characterizing local thermal equations

$$T = T(x) \ , \ u^\mu = u^\mu(x) \quad \text{with} \quad u^\mu u_\mu = -1$$

$$T^{\mu\nu} = \underbrace{(\epsilon + P) u^\mu u^\nu + P g^{\mu\nu}}_{\text{ideal hydrodynamics}} + \mathcal{O}(\partial)$$

deviation from ideal hydrodynamics

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

$$u^\mu = (1, \vec{0})$$

Ideal hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$
$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu}$$

ϵ & P are not independent ; $\epsilon = \epsilon(T)$

$$p = p(T) \rightarrow s = \frac{\partial p}{\partial T}$$
$$\epsilon = Ts - p$$

There is no dissipation from ideal hydrodynamics

$$\partial_\mu (s u^\mu) = 0$$

In order to see dissipations, let's consider

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

- constructed from $u^\mu, \partial_\mu T$
- has one derivative

$$\tau^{\mu\nu} \ll T^{\mu\nu}_{\text{ideal}}$$

$$u^\mu \rightarrow u^\mu + \delta u^\mu \ ; \quad \tau^{\mu\nu} \rightarrow \tau^{\mu\nu} - (\epsilon + P)(u^\mu \delta u^\nu + u^\nu \delta u^\mu)$$

→ doing the same for $T \to T + \delta T$

& impose : $\boxed{u^\mu \tau_{\mu\nu} = 0}$

→ using the "Landau-Lifshitz frame" $u^\mu = (1, \vec{0})$

$\Leftrightarrow \quad \tau_{00} = \tau_{0i} = 0$

$$\tau^{\mu\nu} = p^{\mu\alpha} p^{\nu\beta} \left(-\eta \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \tfrac{2}{3} g_{\alpha\beta}\, \partial \cdot u \right) - \zeta\, g_{\alpha\beta}\, \partial \cdot u \right)$$

shear viscosity bulk viscosity

with

$$p^{\mu\alpha} = g^{\mu\alpha} + u^\mu u^\alpha$$

<u>Correlation fcts. of the stress-energy tensor</u>

$$\langle T^{\mu\nu}(x)\, T^{\alpha\beta}(y) \rangle = ? \qquad |x-y| \gg \frac{1}{T}$$

Linear response theory

$$\mathcal{L} \to \mathcal{L} + J\phi \qquad J \to \text{small}$$

$$\langle \partial \phi(x) \rangle = \int dy\, \underbrace{\langle \phi(x)\, \phi(y) \rangle \Big|_{J=0}}_{\text{retarded Green's fct.}} J(y) + \mathcal{O}(J^2)$$

$$G_R(x-y) = \Theta(x^0 - y^0) \langle [\phi(x), \phi(y)] \rangle$$

Thinking of

$J \sim h_{\alpha\beta}$ metric perturbation

$\phi \sim T^{\alpha\beta}$

<u>perturbations:</u>

$$h_{xy} = h_{xy}(t) \ll 1 \qquad ; \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

SO(3) tensor
(x, y, z)

$$T = T_0 + \cancel{\delta T(t)}$$
$$u^\mu = \left(1 + \mathcal{O}(\vec{v}^2), \cancel{\vec{v}}\right)$$

$$\delta T^{xy} = -\eta\left(\nabla_x u_y + \nabla_y u_x\right) + \cdots \qquad (\text{projection} \to 1)$$
$$\approx -\eta\left(\underbrace{\partial_x u_y}_{0} + \underbrace{\partial_y u_x}_{0} - 2\Gamma^\lambda_{xy}\, u_\lambda\right)$$
$$= 2\eta\Gamma^0_{xy}\, u_0 = \eta\partial_0 h_{xy}$$

linear part in h_{xy}

$$T^{xy} = P g^{xy} + \delta T^{xy} \approx \eta\partial_0 h_{xy} + P h_{xy}$$
$$= \int dy\; G_{xy,xy}(x-y)\, h_{xy}(y)$$

$$G_{xy,xy}(\omega, \vec{q}=0) = P - i\eta\omega + \cdots$$

Kubo's formula

$$\eta = -\lim_{\omega\to 0} \frac{1}{\omega}\, \mathrm{Im}\, G_R^{xy,xy}(\omega, \vec{q}=0)$$

Lecture 2 :

Correlators from AdS/CFT

- $T = 0$
- $T \neq 0$

Shear viscosity

$T = 0$:

$$ds^2 = \frac{R^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right) \qquad r\text{-}z = \frac{R^2}{r}$$

boundary at $z \to 0$

horizon at $z \to \infty$

operator in CFT $\longleftrightarrow$ field

$\hat{\theta}$ $\qquad\qquad$ ϕ , $m_\phi = 0$

AdS/CFT

$$\boxed{Z_{CFT}[J] = \int \mathcal{D}\phi \, e^{-S[\phi] + \int J\hat{\theta}} = \exp\left(-S_{cl}[\phi]\right)}$$

ϕ = classical solution to EoM

$\phi|_{z \to 0} = J$

Correlation fct.

$$\langle \theta(x)\,\theta(y)\rangle = \left.\frac{\delta^2 \ln Z[J]}{\delta J(x)\,\delta J(y)}\right|_{J \to 0} = \frac{\delta S_{cl}[\phi]}{\delta\phi(z=0,x)\,\delta\phi(z=0,y)}$$

Let's consider the following action for a scalar field

$$S[\phi] = -\frac{K}{2}\int d^5x \sqrt{-g}\, g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi$$

$$m_\phi \longrightarrow \Delta\hat{\theta} = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 + m^2R^2}$$

$0 \qquad\qquad 4$

EoM

$$\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi\right)=0 \qquad \text{with BC} \quad \phi(z=0,x)=J(x)$$

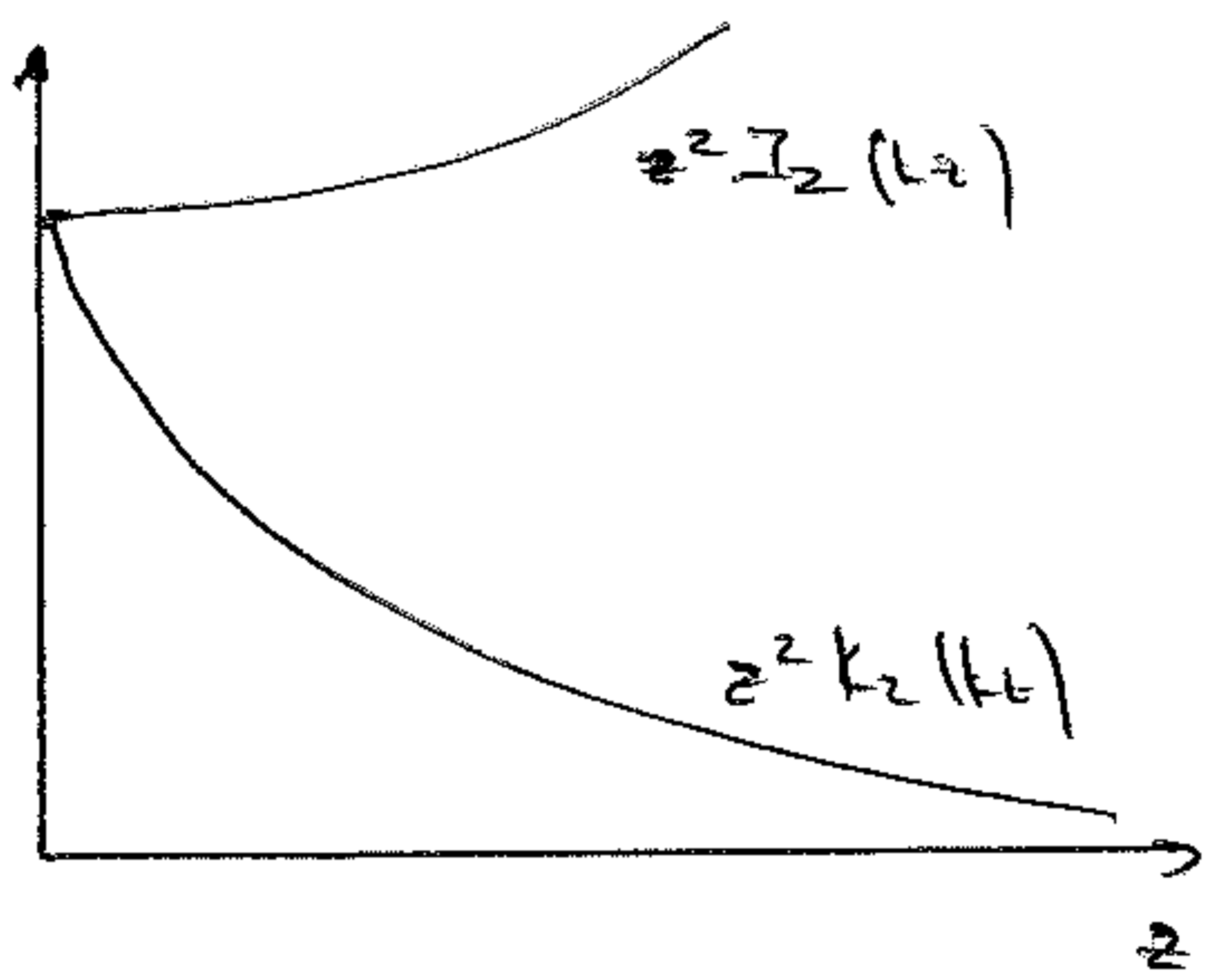
$$\partial_z\left(\frac{1}{z^3}\partial_z\phi\right)+\frac{1}{z^3}\partial_\mu^2\phi=0$$

$$\left(\frac{\phi'}{z^3}\right)'-\frac{k^2}{z^3}\phi=0 \qquad ; \quad k^2=|\vec{k}|^2-k_0^2>0$$

$$\left(\text{Fourier decomposition} \quad \phi(x,z)=\int\frac{d^4k}{(2\pi)^4}e^{ikx}\phi(z,k)\right)$$

$$\phi(k,z)=\#\, z^2K_2(kz)+\#\, z^2I_2(kz)$$

- I_2 & K_2 → modified Bessel fcts.
- using the BC, $I_2(kz)$ will be excluded
- BC $\lim_{z\to\infty}\phi(k,z)\to 0$



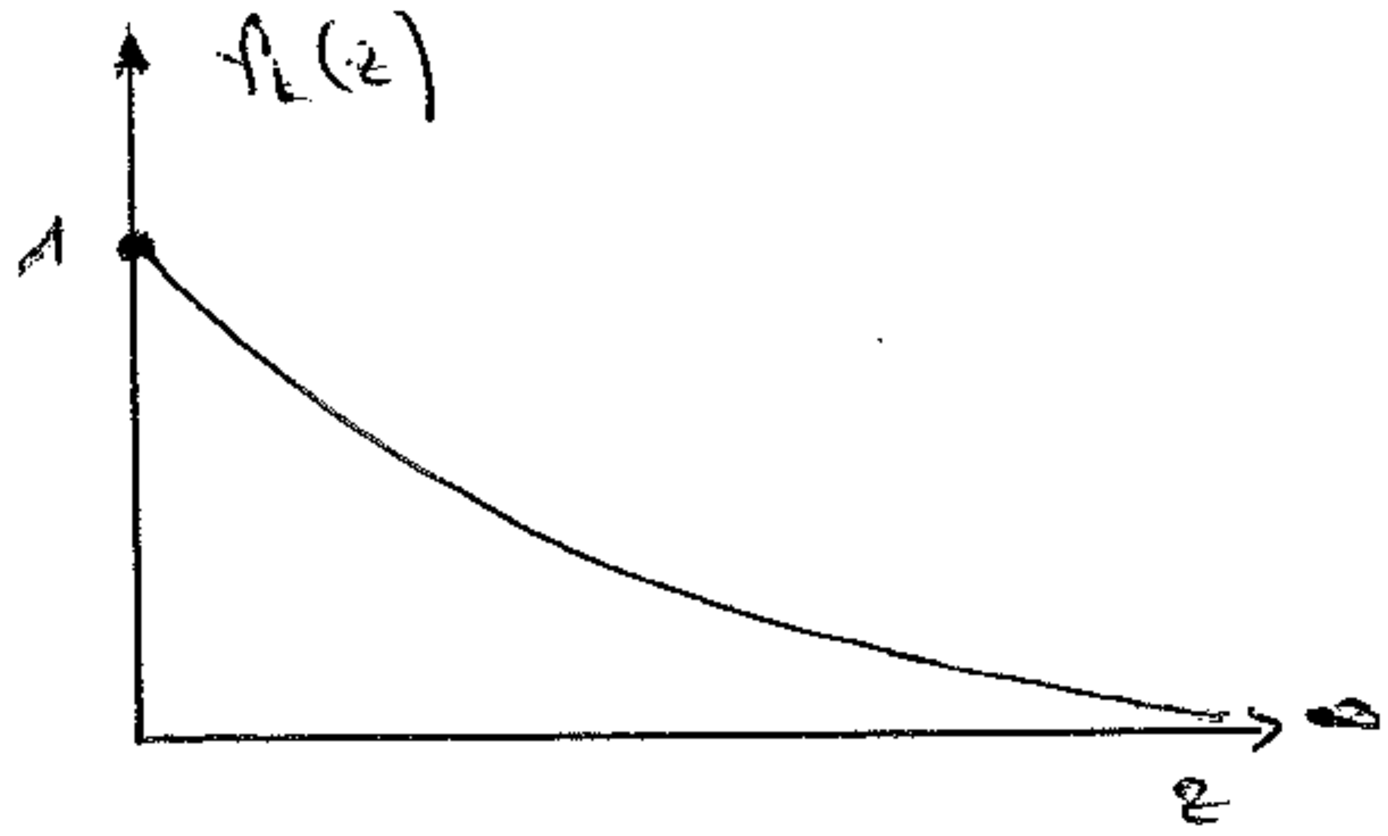
Final solution for ϕ

$$\phi(k,z)=\frac{k^2z^2}{2}K_2(kz)\cdot J(k)$$

$$\phi(k,z)=J(k)\, f_k(z)$$

"mode fct."

$$\phi(z,k)\xrightarrow[z\to 0]{} J(k)$$



At zero-temperature we have seen

$$f_k(z)=\frac{k^2z^2}{2}K_2(kz)$$

The next step to obtain the two-points correlation fct. is to plug the solution $\phi(k,z)$ into the classical action & differentiate the action twice with respect to the sources $J(k)$

$$S_{cl} = -\frac{\kappa}{2}\int d^5x \sqrt{-g}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$= -\frac{\kappa}{2}\int d^5x \left(\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\phi\partial_\nu\phi\right) - \phi\, \cancel{\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi\right)}\right)$$

$\left(\int d^5x\, \partial_\mu c^\mu = \int d^4x\, c_{\text{normal}}\right)$ (total derivative)

$$= -\frac{\kappa}{2}\int_\varepsilon d^4x \sqrt{-g}\, g^{zz}\phi\,\partial_z\phi$$

$$= \frac{\kappa}{2}\int d^4x\, \frac{R^3}{z^3}\phi\phi'$$

$$= \frac{\kappa}{2}\int \frac{d^4k}{(2\pi)^4}\frac{R^3}{z^3} J(k)\, J(-k)\, f_k(z)\, f_k'(z)$$

$$\Rightarrow \langle OO\rangle_k = \frac{\kappa R^3}{z^3} f_k(z)\, f_k'(z)\Big|_{z\to 0}$$

$$f_k(x) = \frac{k^2z^2}{2}K_2(kz) = 1 - \frac{(kz)^3}{4} - \frac{(kz)^4\ln(kz)}{16} + \dots$$

$$\langle OO\rangle_k \sim \#\; k^4\ln(k\varepsilon) + (\text{polynomial terms in } k, \varepsilon)$$

$$\sim \#\;\frac{1}{(x-y)^8} + \dots$$

$$\sim \#\;\frac{1}{(x-y)^{2\Delta}} + \dots$$

$4 = \Delta \to$ conformal dim. for ϕ

$T \neq 0$

$$ds^2 = \frac{R^2}{z^2}\left(-f\,dt^2 + d\vec{x}^2 + \frac{dz^2}{f}\right) \quad ; \quad f(z) = 1 - \frac{z^4}{z_0^4}$$

$$z_0 \sim T_H^{-1}$$

$$\phi(k,z) = J(k)\, f_k(z)$$

← "mode function" $\neq$ "f" in the metric

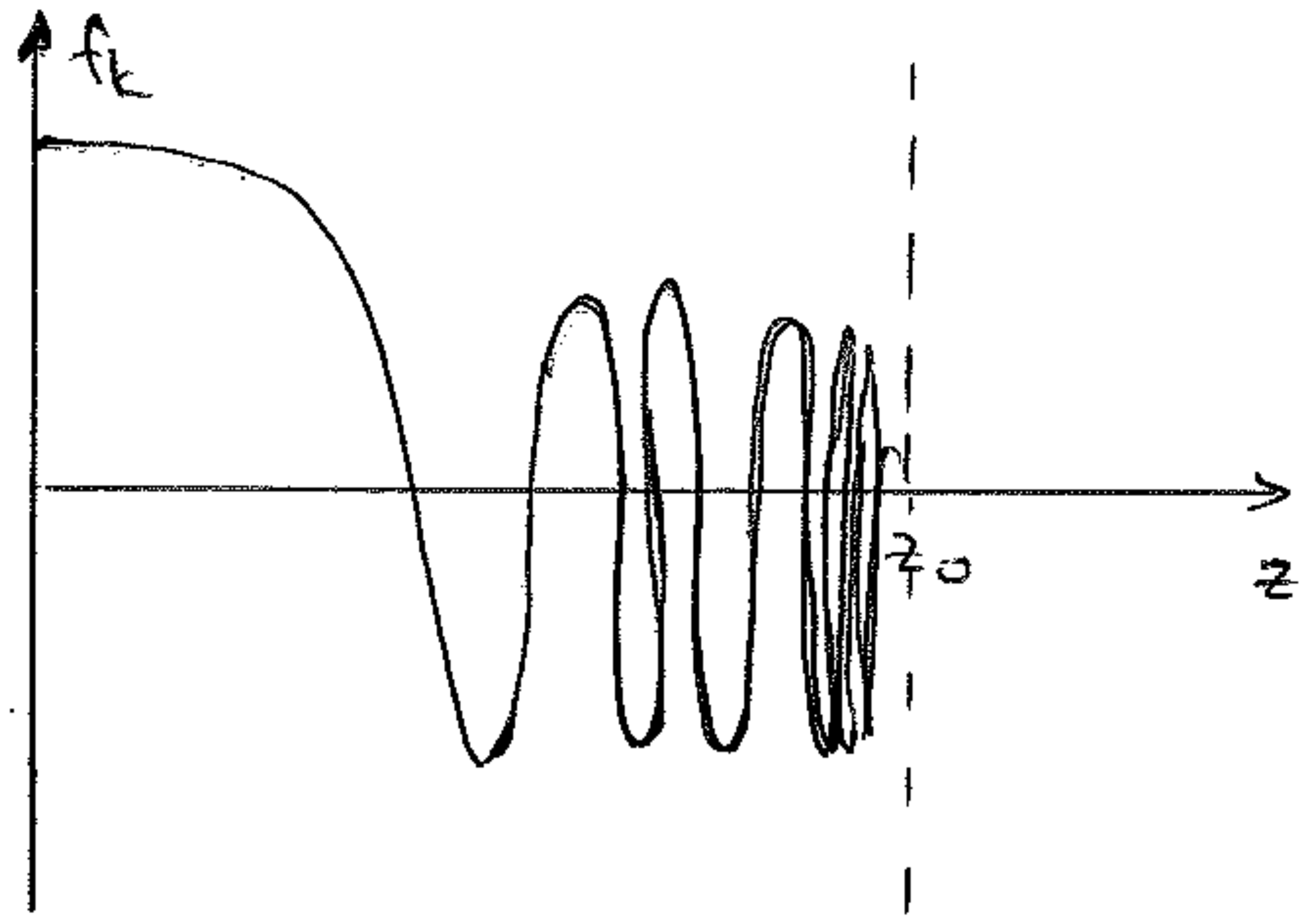
EoM

$$\left(\frac{f(z)}{z^3} f_k'\right)' + \left(\frac{\omega^2}{z^3 f} - \frac{q^2}{z^3}\right) f_k(z) = 0 \quad ; \quad k = (\omega, \vec{q})$$

↓

dominant term at $z \to z_0$, since $f(z_0) = 0$

$$f_k(z) \sim \left(1 - \frac{z}{z_0}\right)^{\pm \frac{i\omega}{4\pi T}}$$



For the $T \neq 0$, the computation of the two points fct is more complicated. To obtain the retarded Green function, following expression will be used

$$\langle \mathcal{O}\mathcal{O} \rangle_k^{\text{retarded}} = \frac{\kappa R^3}{z^3} f_k^*(z)\, f_k'(z)$$

$f_k(z)$: incoming-wave at the horizon

advanced Green fct ↔ out-going-wave

Next, we will apply the above prescription for computing the shear viscosity

$$\eta = -\frac{1}{\omega}\, \mathrm{Im}\, G_{ret}^{xy,xy}(\omega, \vec{q}=0)\Big|_{\omega \to 0}$$

In order to write down the EoM for the metric fluctuation h_{xy} $S[h_{xy}]$, it is more convenient to define

$$\phi = g^{xx} h_{xy}$$

We know

$$S = \int \frac{d^{10}x}{2\kappa_{10}^2} \sqrt{-g_{(0)}}\,(R - 2\Lambda) \longrightarrow \frac{V(S^5)}{4\kappa_{10}^2} \int d^5x \sqrt{g}\, g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

If $f_k(z)$ is a solution

$$\frac{f}{z^3}\left(f_k^* f_k' - f_k f_k^{*\prime}\right) = \text{const} \qquad (\underline{\text{conserved flux}})$$

$$\Big\downarrow{\scriptstyle\text{near horizon}}$$

$$= \frac{i\omega}{2\pi T} |f_k|^2$$

$\nwarrow 1$

near the boundary $z \to 0$, the expression above is mapped to $\operatorname{Im} G_R(\omega)$, and the result is

$$\operatorname{Im} G_{ret} = \underbrace{\frac{-V(S^5)}{2\kappa_{10}^2} \frac{R^3}{z_0^3}}_{\eta}\ \omega$$

The entropy density

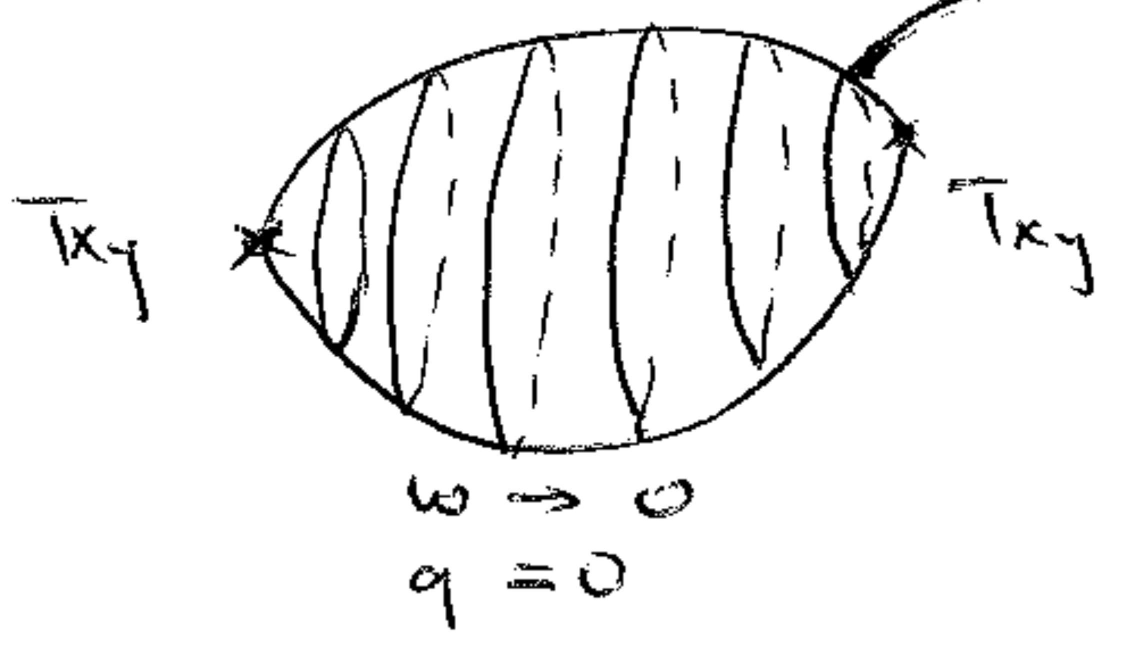
$$s = \frac{S}{V} = V(S^5) \frac{R^3}{z_0^3} \frac{2\pi}{\kappa_{10}^2}$$

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

$\longleftarrow$ in the same regime with the results obtained by measurements at RHIC

__How to calculate η/s in field theory?__

$\to$ __weak coupling__



$\overset{\text{AdS/CFT}}{\longrightarrow}$

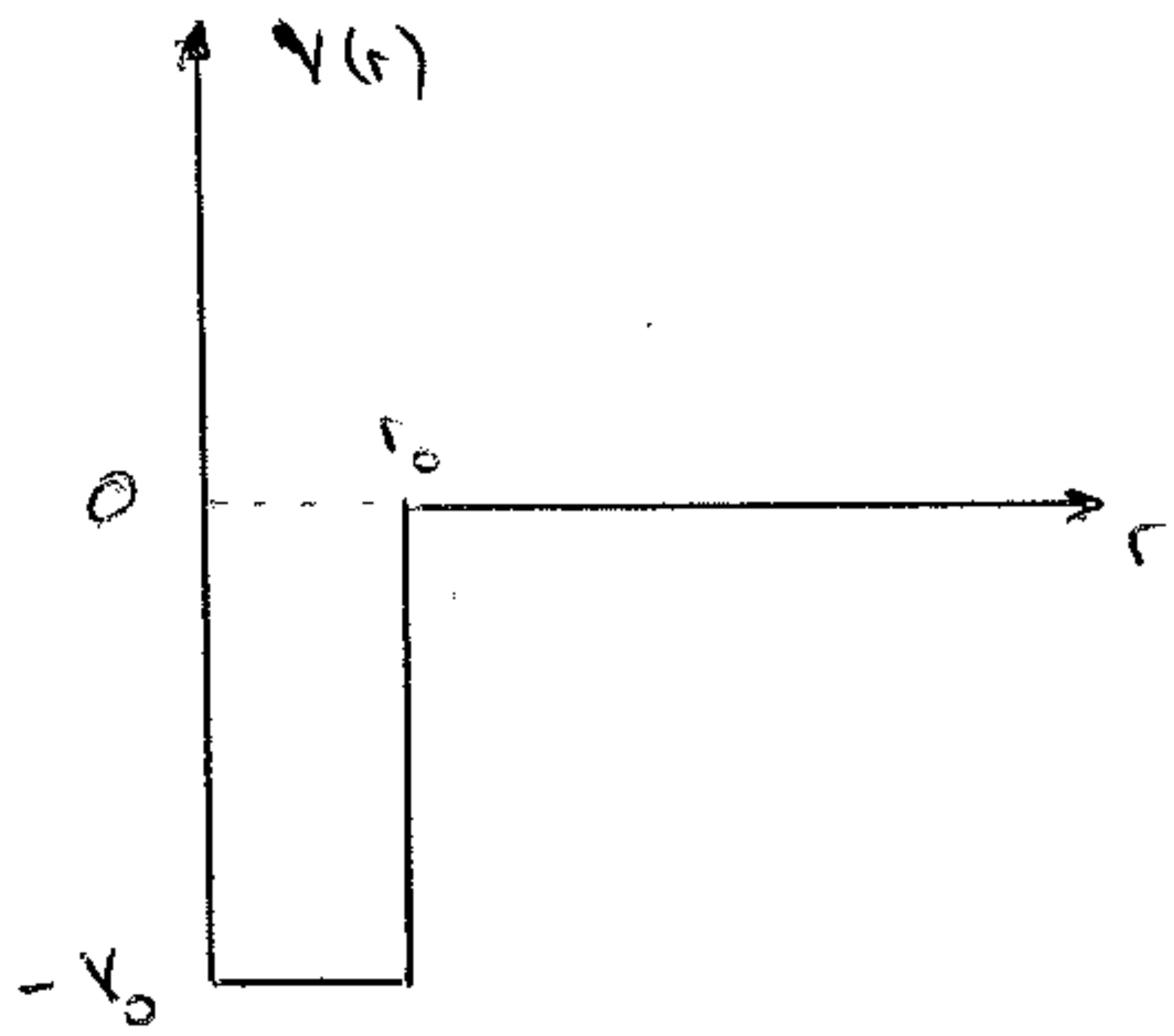
$T^{\mu\nu} \sim (\partial\phi)(\partial\phi)$

$\Longleftrightarrow$ $\boxed{\text{Boltzmann's kinetic eq.}}$ $\longrightarrow$ hydrodynamics $\eta, \ldots$

$\searrow$ $\eta \simeq e \cdot \lambda_{mfp} \longleftarrow \mathcal{O}\left(\frac{1}{T}\right)$

Lecture 3:

- non-relativistic fermions at unitarity
- non-relativistic conformal invariance
- non-relativistic CFTs
- holography?



$\exists \quad V_0^{crit} = \frac{\#}{m r_0^2}$

Taking the limit $r_0 \to 0$ and keeping $V_0 r_0^2$ critical

This type of interaction is called zero-range interaction

$\to$ infinite scattering length

$\to$ resonance

Considering fermions with spin ½ & trying to find out some universal aspects of the interactions among them at unitary.

$m = 1 \to \psi(x,y)$

$$-\frac{1}{2}\left(\nabla_x^2 + \nabla_y^2\right)\psi + V(x-y)\psi = E\psi$$

$$-\nabla_r^2\psi + \cancel{V(r)}^{\text{zero-range}}\psi = E\psi$$

for $r \gg r_0 \to 0$

$r \to$ small $\qquad \nabla_r^2\psi = 0 \quad \to \quad \psi = a + \frac{b}{r}$

$$\to \psi(r) = \frac{\text{const}}{r} + 0 + O(r)$$

Using the boundary condition, so that $\psi(r)$ takes the form

$$\psi(r) = \text{const}\left(\frac{1}{r} - \frac{1}{a}\right) + \mathcal{O}(r)$$

scattering length

Problem of unitary fermion is the problem of solving the Schrödinger Eq.

$$\left(-\frac{1}{2}\sum_i \nabla_{x_i}^2 - \frac{1}{2}\sum_j \nabla_{y_j}^2\right)\psi = E\psi$$

$$\psi(\underbrace{x_1, \dots x_N}_{\text{Spin}\uparrow}, \underbrace{y_1, \dots y_M}_{\text{Spin}\downarrow})$$

$$\psi(x_i, y_j) \xrightarrow[x_i \to y_j]{} \frac{C\,(\text{other coord.})}{|x_i - y_j|} + \mathcal{O}(|x_i - y_j|)$$

Boundary condition

$$\psi(x_i \to x_j) \sim \mathcal{O}(x_i - x_j)$$

Consider the <u>harmonic potential</u>

$\omega = 1$

$$H = \text{kinetic} + \frac{x^2}{2} + \frac{y^2}{2}$$

non-interacting $\rightarrow$ $\psi(x,y) = e^{-\frac{|x|^2 + |y|^2}{2}}$

with the boundary cond. (interacting) $\psi(x,y) = \dfrac{e^{-\frac{|x|^2 + |y|^2}{2}}}{|\vec{x} - \vec{y}|}$

<u>Ground state energy</u>: non-interacting $E = 3 = \frac{3}{2} + \frac{3}{2}$; interacting $E = 2 = \frac{3}{2} + \frac{3}{2} - 1$

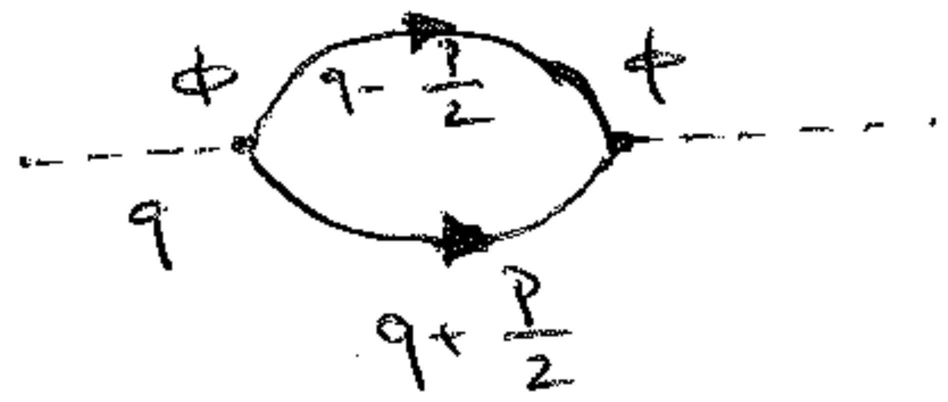
Fermions

$$\mathcal{L} = \underbrace{\sum_\sigma \left(\psi_\sigma^\dagger i\partial_t \psi_\sigma - \frac{|\nabla\psi|^2}{2}\right)}_{\text{free theory}} + \underbrace{c_0\, \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1}_{\text{interacting term}}$$

Introducing a auxiliary field ϕ

(*) $$\boxed{\mathcal{L} = \sum_\sigma \left(\psi_\sigma^\dagger i\partial_t \psi_\sigma - \frac{|\nabla\psi|^2}{2}\right) + \phi^* \psi_2 \psi_1 + \phi\, \psi_1^\dagger \psi_2^\dagger - \frac{\phi^*\phi}{c_0}}$$

$$G^{-1} = \underbrace{G_0^{-1}}_{c_0^{-1}} - \sum$$



self-energy $\sim \int^{\Lambda} \frac{d^4 p}{(2\pi)^4} \frac{1}{p_0 - \frac{q_0}{2} - \frac{1}{2}\left(\vec{q} - \frac{\vec{p}}{2}\right)^2} \cdot \frac{1}{p_0 + \frac{q_0}{2} - \frac{1}{2}\left(\vec{q} + \frac{\vec{p}}{2}\right)^2}$

(~~after renormalization~~) divergence $\simeq \Lambda + \# \sqrt{\frac{\vec{q}^{\,2}}{4} - q_0}$

UV-cut-off

<u>fine-tuning</u> $\rightarrow$ $c_0^{-1} + \Lambda = 0$

c_0 is called the fine-tuning parameter

$[c_0] = -1$

Galilean symmetry

$t \rightarrow t + a$ $\quad H \quad\quad \psi \rightarrow e^{i\alpha}\psi \quad N$

$\vec{x} \rightarrow \vec{x} + a$ $\quad \vec{P}$

$\vec{x} \rightarrow \Lambda \vec{x}$ $\quad M_{ij}$

$\begin{cases} t \rightarrow t \\ \vec{x} \rightarrow \vec{x} - vt \end{cases}$ $\quad K_i \quad , \; [K_i, H] \sim P_i$

$\begin{cases} t \rightarrow \lambda^2 t \\ \vec{x} \rightarrow \lambda \vec{x} \end{cases}$ $\quad D \quad\quad \begin{cases} t \rightarrow \frac{t}{1 - \lambda t} \\ \vec{x} \rightarrow \frac{\vec{x}}{1 - \lambda t} \end{cases} \quad C$

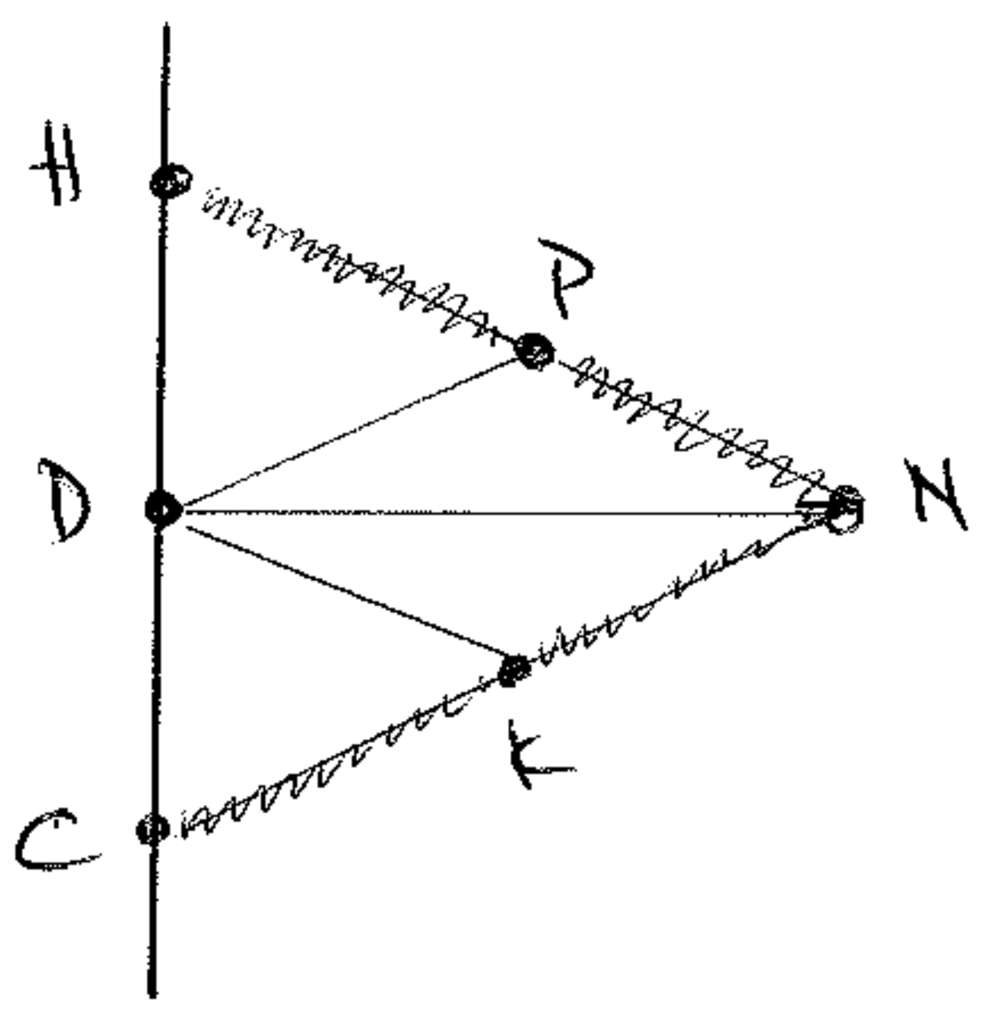
The Lagrangian (*) is invariant under the Gallilean symmetry

$$\psi(t,\vec{x}) \to \tilde{\psi}(t,\vec{x}) = e^{im\vec{v}\cdot\vec{x} + i\frac{mv^2}{2}t}\,\psi(t,\vec{x}-\vec{v}t)$$

Since D is also a symmetry of the Lagrangian, another sol. for $\psi(t,\vec{x})$ is

$$\tilde{\psi}(t,\vec{x}) = \lambda^{3/2}\,\psi(\lambda^2 t, \lambda\vec{x})$$

The Gallilean group is larger than the symmetry group for non-relativistic conformal theories, since it contains D- & C- symmetry.



$$[P_i, K_j] = -i\delta_{ij} N$$

$$[N, \text{any}] = 0$$

$\{C, D, H\}$ form a subgroup $SO(2,1)$

Non-relativistic conformal field theories

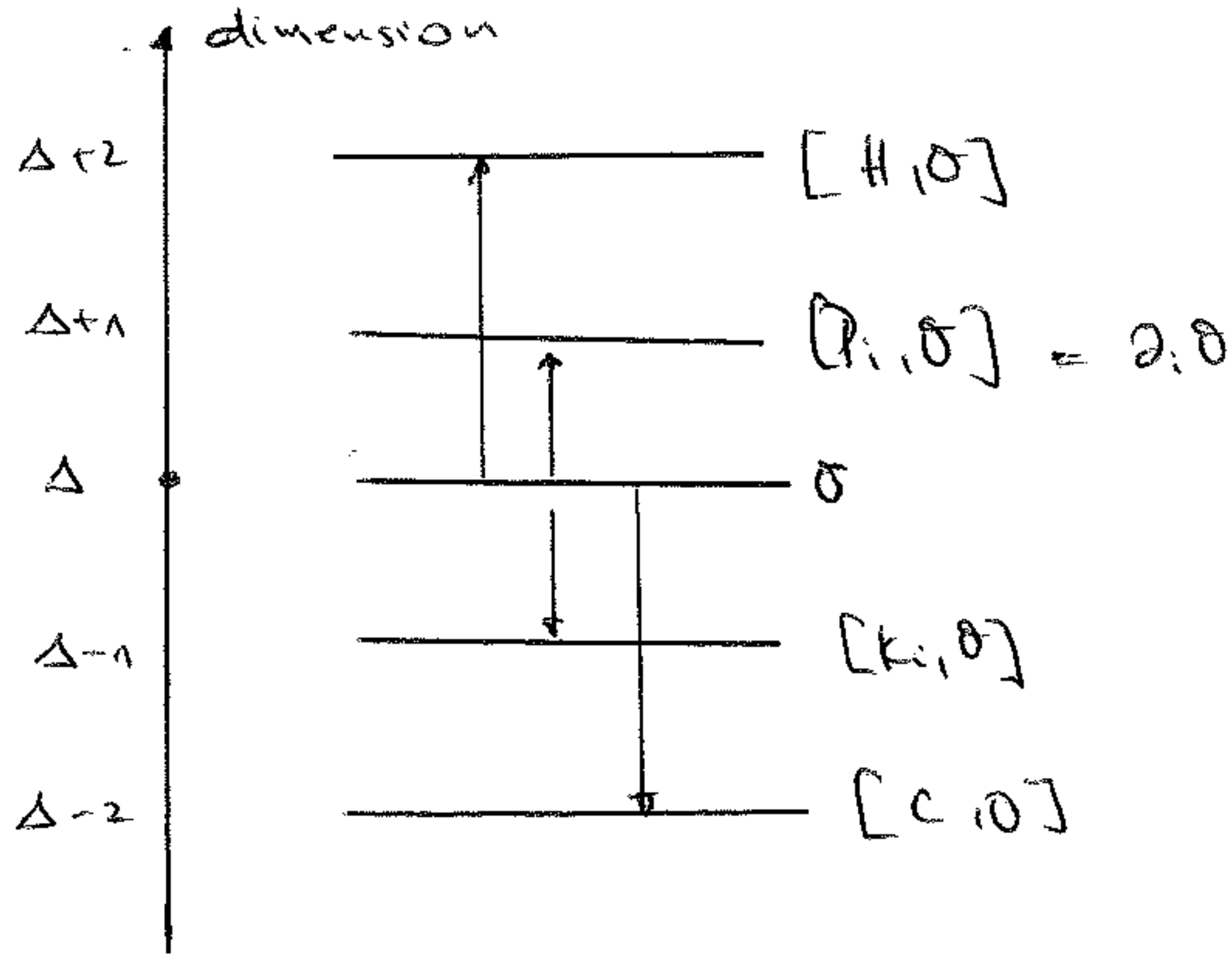
Local operators $\mathcal{O}(t,\vec{x})$

Dimension of the op. $[D, \mathcal{O}(0)] = i\Delta_{\mathcal{O}}(0)$

for $\psi \to \Delta_\psi = \frac{3}{2}$

$$[D, K_i] = -iK_i$$

$$[D, P_i] = iP_i$$



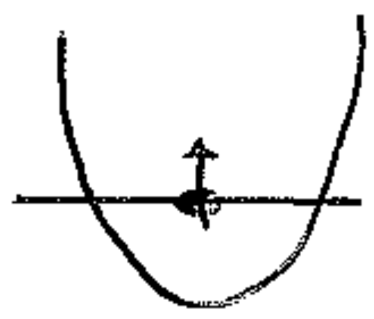
$$[D, K_i] = -i K_i$$

$$[D, P_i] = +i P_i$$

$\mathcal{O}$ = primary operator $\Longleftrightarrow$ eigenstate of a system of particles in harmonic potential

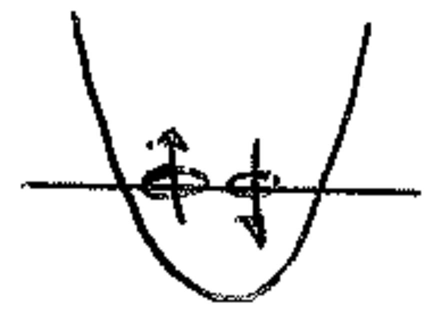
$[C, \mathcal{O}(0)] = [K_i, \mathcal{O}(0)] = 0$

$[\psi_0] = \frac{3}{2}$



$\rightsquigarrow \frac{3}{2} = E$

$[\phi] = 2$



$2 = E$

$$\langle \phi(x)\phi(0)\rangle = \int d^4p \, e^{ipx}\left(\sqrt{\frac{p^2}{4m}} - p_0\right)^{-1} \sim \frac{1}{|x|^4}$$

$$C = \int d\vec{x}\, \frac{x^2}{2} \psi^\dagger \psi$$

$$H_{osc} = H_{\text{free space}} + C$$

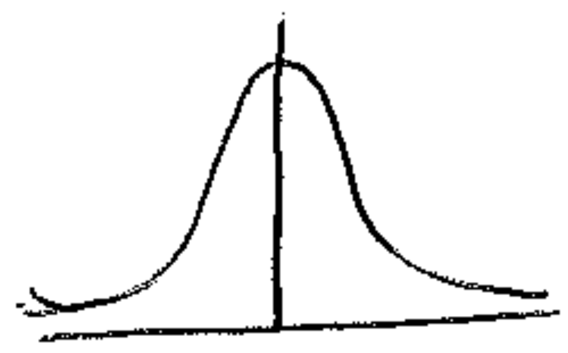
for this case, the correspondence can be proven algebraically

$\mathcal{O}$ $\quad \psi, \phi \to$ primary operator

$$|\psi_\mathcal{O}\rangle = e^{-H}\, \mathcal{O}^\dagger(0)|0\rangle$$

$$(H + C)|\psi_\mathcal{O}\rangle = \Delta |\psi_\mathcal{O}\rangle$$

harmonic potential



Reference

Y. Nishida, DTS

OPE in NR conformal FT

$$\mathcal{O}_1(t,x)\,\mathcal{O}_2(0,0) = \sum_n \underbrace{C_n(t,\vec{x})}_{\frac{1}{|x|^{\Delta_1+\Delta_2-\Delta_n}} f_n\left(\frac{x^2}{t}\right)} \mathcal{O}_n(0)$$

Characterization of a state, consider the Tan's parameter appearing in the corr.-fct.

$< \psi^+ \psi >$, $< \phi^+ \phi >$

$\Leftrightarrow [\psi^+\psi] = 3$ $\quad$ $\Leftrightarrow [\phi^+\phi] = 4$

Holographical Realization

Is there any spacetime with the symmetry (isometry) of the Schrödinger group?

conformal group $\quad \partial_\mu^2 \phi = 0 \quad$ (Klein-Gordon-Eq)

Schrödinger group $\quad 2im\,\partial_t \psi - \nabla^2 \psi = 0 \quad$ (Schrödinger-Eq)

One can go from the Schrödinger Eq. $\leftrightarrow$ Klein-Gordon-Eq.*
by using light-cone coordinates

$$x^{\pm} = \frac{x^0 \pm x^d}{\sqrt{2}}$$

$$\rightarrow \partial_\mu^2 \phi = 0 \rightarrow -\partial_+\partial_-\phi + \vec{\nabla}^2_{d-1}\phi = 0$$

for $\phi \sim e^{imx^-}\psi \rightarrow$ Schrödinger Eq.

Conformal group in $(d+1)$ spacetime dim. $\subset$ Schrödinger group in $(d-1)$ spatial dim

(pick all generators that commute with P^+)

Constructing the geometry

Starting with AdS_6

$$ds^2 = \frac{1}{z^2}\left(dz^2 - dt^2 + \sum_{i=1}^{4} dx_i^2\right)$$

$$= \frac{1}{z^2}\left(-2dx^+dx^- + dz^2 + d\vec{x}^2\right)$$

this geometry allows more symmetries than the Schrödinger group
$\Rightarrow$ adding one more term $\Rightarrow$ the same sym. as the Schrödinger gr.

$$ds^2 = \frac{1}{z^2}\left(-2dx^+dx^- + dz^2 + d\vec{x}^2\right) - \underbrace{\frac{(dx^+)^2}{z^4}}$$

breaks the relativistic scaling invariance

$x^+ \to \lambda x^+$

$x^- \to \lambda x^-$

$x^i \to \lambda x^i$

$z \to \lambda z$

or

$\left.\begin{array}{l} x^+ \to \lambda x^+ \\ x^- \to \lambda^{-} x^- \end{array}\right\} \to \begin{array}{l} x^+ \to \lambda^2 x^+ \\ x^- \\ x^i \to \lambda x^i \end{array}$

$$\boxed{x^+ \hat{=} \text{time}}$$

non-relativistic