Exercise Sheet for Hirosi Ooguri

04.08.2010

Problem 1 (Thursday)

Show that the twisting of the worldsheet action

$$S \to S + \int A\bar{J} + \bar{A}J, \quad \text{where } A = \frac{i}{2}\omega \ ,$$

(and ω is the spin connection), deforms the energy momentum tensor as

$$T \to T + \frac{1}{2}\partial J \ .$$

Problem 2 (Thursday)

Assuming that gauge fields are not turned on, show that A-branes satisfying

$$G^{\pm} = \bar{G}^{\pm} \quad (\text{or } G^{\pm} = -\bar{G}^{\pm})$$

are Lagrangian submanifolds.
Similarly, show that B-branes satisfying

$$G^{\pm} = \bar{G}^{\mp} \quad (\text{or } G^{\pm} = -\bar{G}^{\mp})$$

are holomorphic submanifolds.

Problem 3 (Friday)

Find the toric diagram for $\mathbb{C}^4/U(1)$ with $Q = (-1, -1, +1, +1)$. This gives the resolved conifold. Describe the geometry using the toric diagram. Where is the small S^2?

Problem 4 (Friday)

Show that the mirror of the resolved conifold becomes the deformed conifold in the limit of the Kähler moduli $t \to 0$. Verify that the topological string partition function $F_g(t)$ of the former approaches that of the latter in the limit $t \to 0$.

Problem 5 (extra problem)

Let us study the Gaussian matrix integral

$$\int dM e^{-\frac{1}{2g_s}\hbar M^2}$$

Write this as the integral over eigenvalues:

$$\int \prod_{i=1}^{N} \mathrm{d}\lambda_i \prod_{i<j} (\lambda_i - \lambda_j)^2 e^{-\frac{1}{2g_s}\sum_i \lambda_i^2}$$

Look for the stationary points of this integral:

$$\frac{1}{g_s}\lambda_i = 2\sum_{i\neq j} \frac{1}{\lambda_i - \lambda_j}$$

In the large N limit, we can write this as

$$\frac{1}{t}\lambda = 2P\int \frac{\rho(\lambda')}{\lambda - \lambda'}\mathrm{d}\lambda', \tag{1}$$

where P means the principal value and $\rho(\lambda)$ is the eigenvalue density

$$\rho(\lambda) = \lim_{N\to\infty} \frac{1}{N}\sum_i \delta(\lambda - \lambda_i) \ .$$

Show that (**??**) is solved by

$$\rho(\lambda) = \frac{1}{2\pi t}\sqrt{4t - \lambda^2} \ .$$

This is known as the Wigner semi-circle law.