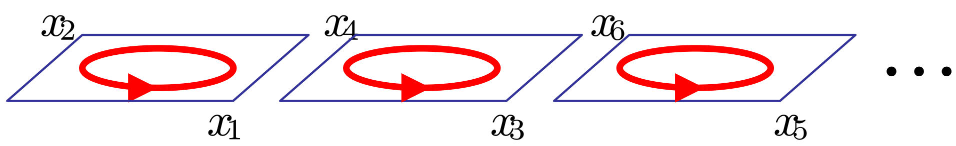
Black Hole Solutions of

$$R_{\mu\nu} = 0$$

in $D > 4$

- Why expect richer black hole physics in D>4?
  - More degrees of freedom $O(D\text{-}1) \supset U(1)^{\lfloor (D\text{-}1)/2 \rfloor}$
  - Rotation:



    - more rotation planes
    - gravitational attraction ⇔ centrifugal repulsion

$$-\frac{GM}{r^{D-3}} + \frac{J^2}{M^2 r^2}$$

  - ∃ extended black objects: black p-branes

4D black holes

- Static: Schwarzschild
- Stationary: Kerr

$$\mu = 2M$$

$$ds^2 = -dt^2 + \frac{\mu r}{\Sigma}\left(dt + a\sin^2\theta d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + (r^2 + a^2)\sin^2\theta d\phi^2$$

$$\Sigma = r^2 + a^2\cos^2\theta\,, \qquad \Delta = r^2 - \mu r + a^2\,, \qquad a = \frac{J}{M}$$

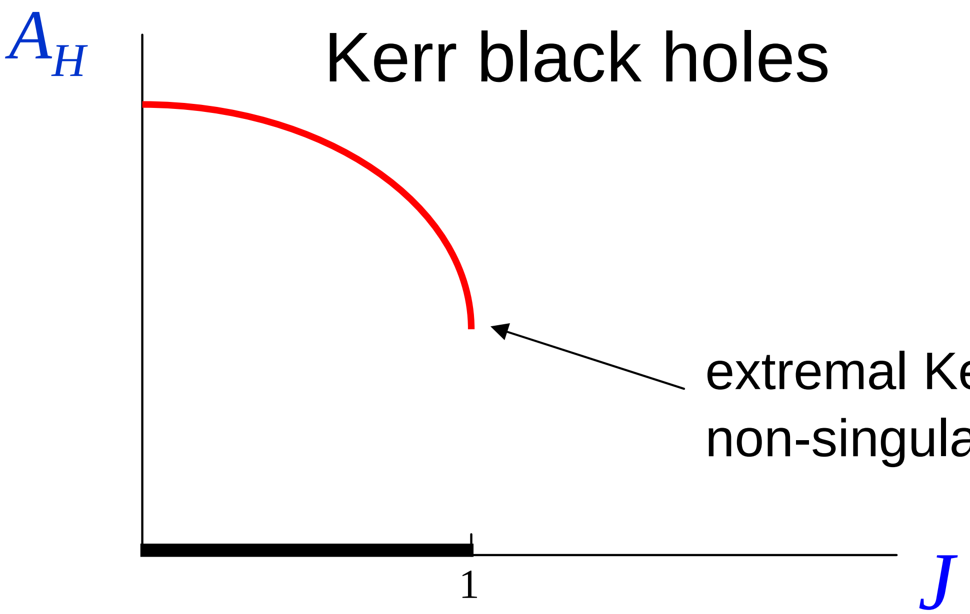
Horizon: $k = \partial_t + \frac{a}{\mu r_+}\partial_\phi\,, \quad |k|^2 = 0 \text{ at } r = r_+\,, \text{where } \Delta = 0$

Δ=0 $\Rightarrow$ $M \geqslant a$: Upper bound on J for given M

$$J \leqslant M^2$$

4D black holes

fix M=1



Uniqueness theorem: *End of the story*

Black holes in *D*>4

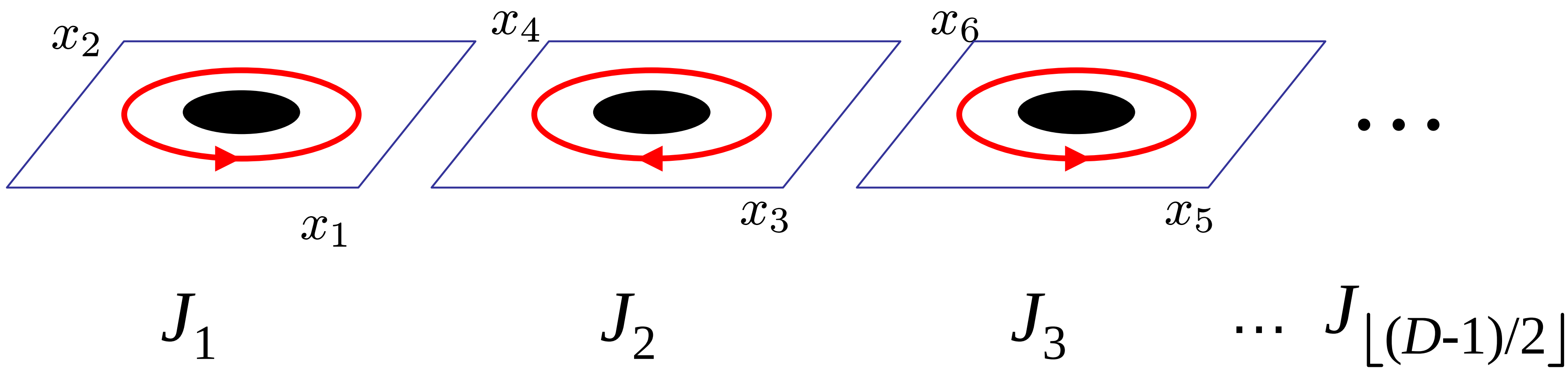
- Schwarzschild is easy: *Tangherlini 1963*

$$ds^2 = -\left(1 - \frac{\mu}{r^{D-3}}\right)dt^2 + \frac{dr^2}{1 - \mu/r^{D-3}} + r^2 d\Omega_{(D-2)}$$

$$\mu \propto M$$

Rotation

- *Myers+Perry (1986)*: rotating black hole solutions with angular momentum in an arbitrary number of planes



e.g. $D=5,6$: J_1, J_2

$D=7,8$: J_1, J_2, J_3 etc

- They all have spherical topology S^{D-2}

- Consider a **single spin**:

$$ds^2 = -dt^2 + \frac{\mu}{r^{D-5}\Sigma}\left(dt + a\sin^2\theta\, d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + (r^2 + a^2)\sin^2\theta\, d\phi^2$$

$$+ r^2\cos^2\theta\, d\Omega^2_{(D-4)}$$

$$\Sigma = r^2 + a^2\cos^2\theta\,, \qquad \Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}\,,$$

$$\mu \propto M$$

$$a \propto \frac{J}{M}$$

$$\frac{\Delta}{r^2} - 1 = -\frac{\mu}{r^{D-3}} + \frac{a^2}{r^2}$$

gravitational

centrifugal

- Consider a **single spin**:

Horizon: $\Delta=0$

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$

***D*=5:** $r_h^2 + a^2 - \mu = 0 \quad \Rightarrow \quad r_h = \sqrt{\mu - a^2}$

$\Rightarrow \; a^2 \leqslant \mu \quad \Rightarrow$ upper bound on J for given M

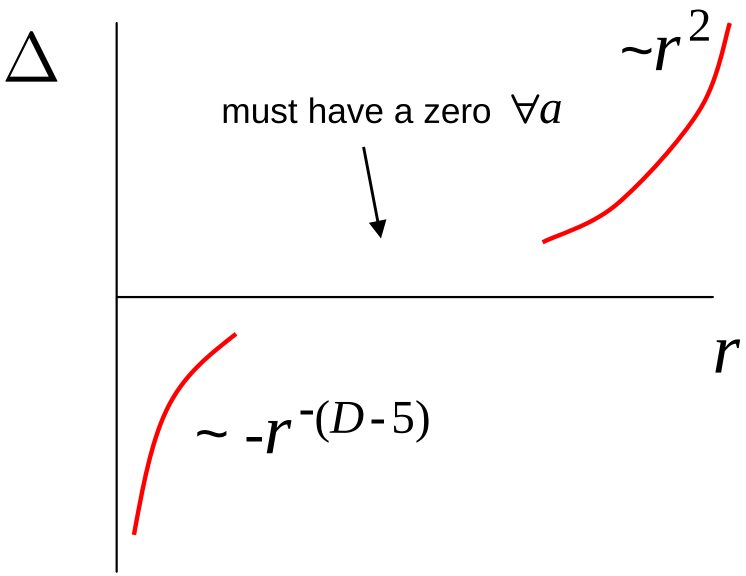
- similar to 4D
- but extremal limit $a^2 = \mu \;\Rightarrow\; r_h=0$

  this is *singular, zero-area*

$D \geqslant 6$:

Horizon: $\Delta = 0$

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$

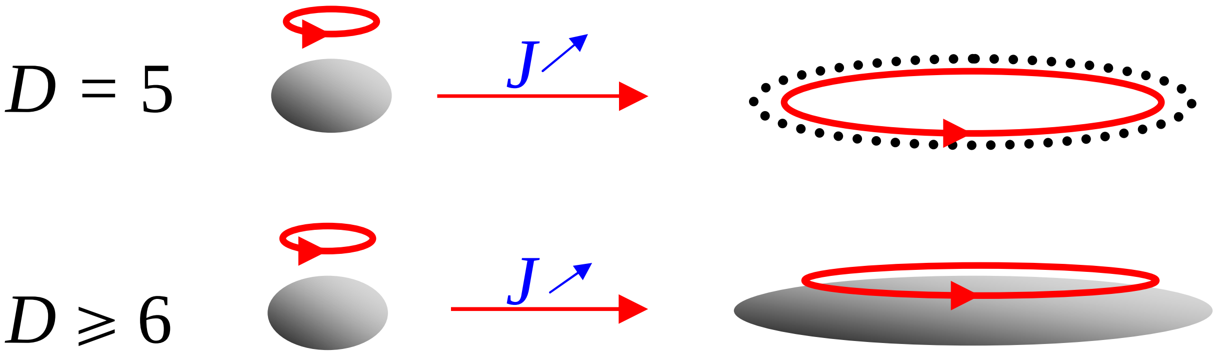


For fixed μ there is an outer event horizon for *any* value of a

$\Rightarrow$ No upper bound on J for given M

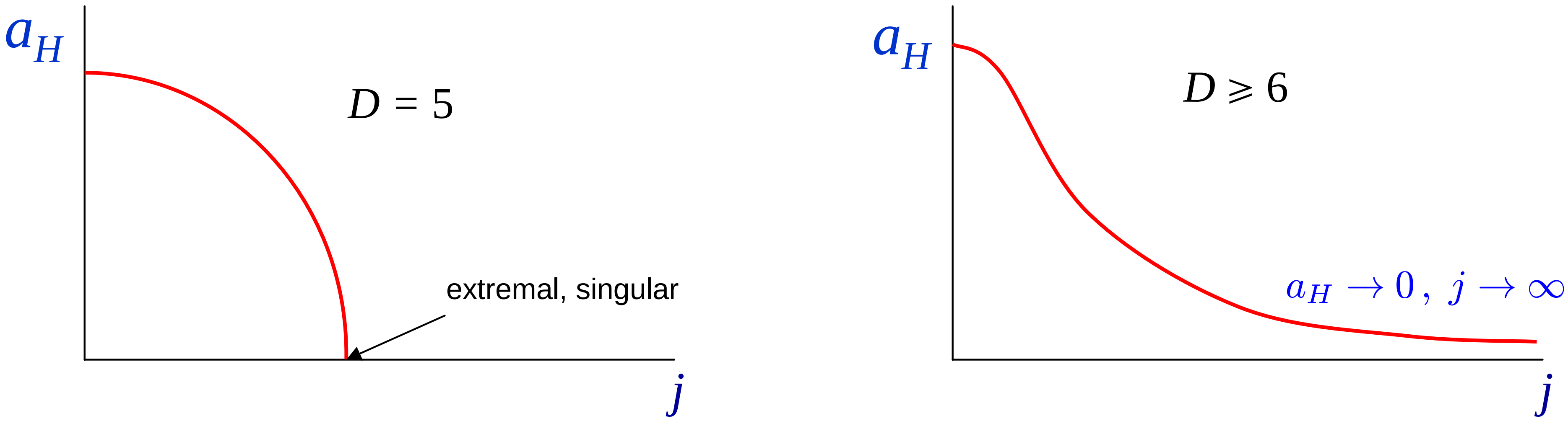
$\Rightarrow \exists$ ***ultra-spinning black holes***

- **Single spin MP black holes**:



upper (extremal) limit on J is singular

no upper limit on J for fixed M



Is this all there is in D>4?

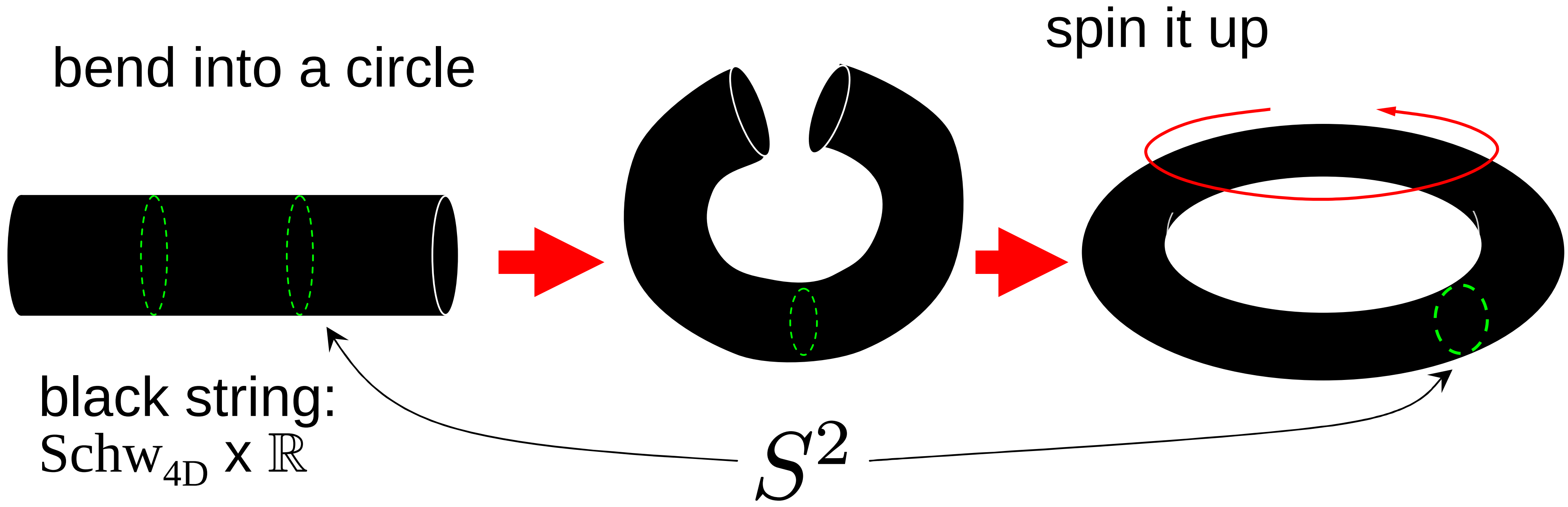
Not at all

Combine black branes & rotation:

$\Rightarrow$ Black Rings + other blackfolds in $D \geqslant 5$

$\Rightarrow$ *Pinched* black holes in $D \geqslant 6$

The forging of the ring (in D=5)



Horizon topology S^1x S^2

Exact solution available -- and fairly simple

RE+Reall 2001

Metric

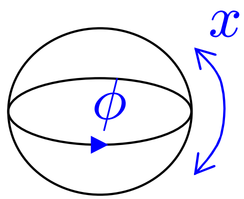
ψ-rotation

$$ds^2 = -\frac{F(y)}{F(x)}\left(dt - C\,R\,\frac{1+y}{F(y)}\,d\psi\right)^2$$

$$+ \frac{R^2}{(x-y)^2}\,F(x)\left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right]$$

S^1

S^2



$$F(\xi) = 1 + \lambda\xi$$

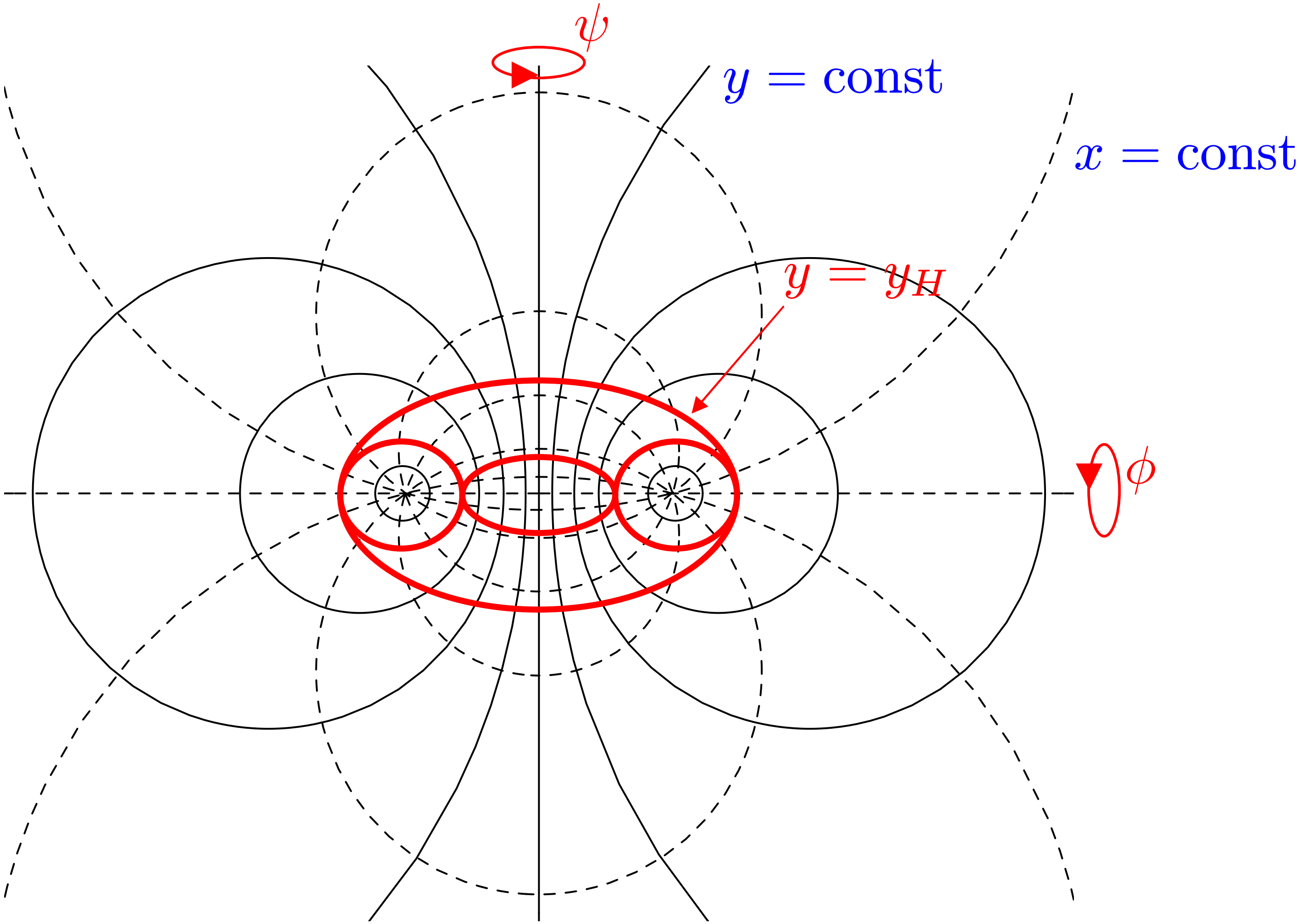
$$G(\xi) = (1-\xi^2)(1+\nu\xi)$$

Parameters λ, ν, R

$\nu \sim R_2/R_1 \rightarrow$ shape, $\lambda/\nu \rightarrow$ velocity

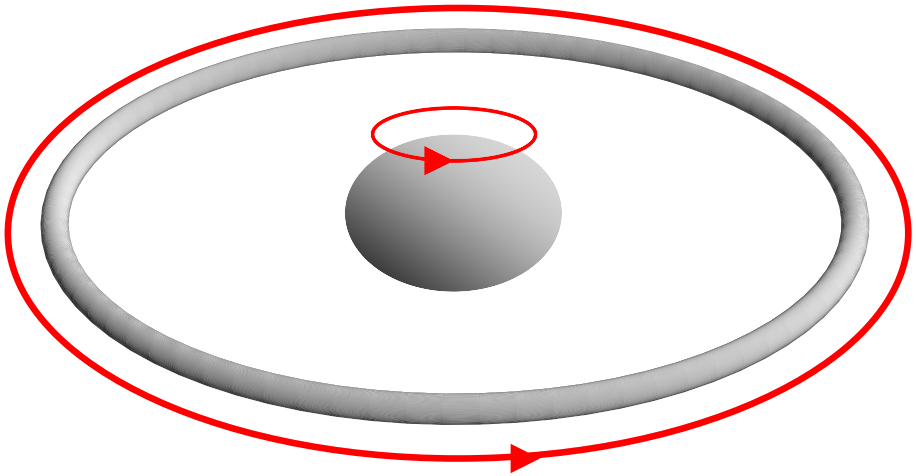
equilibrium $\rightarrow \lambda(\nu)$

"Ring coordinates" x, y



Multi-black holes

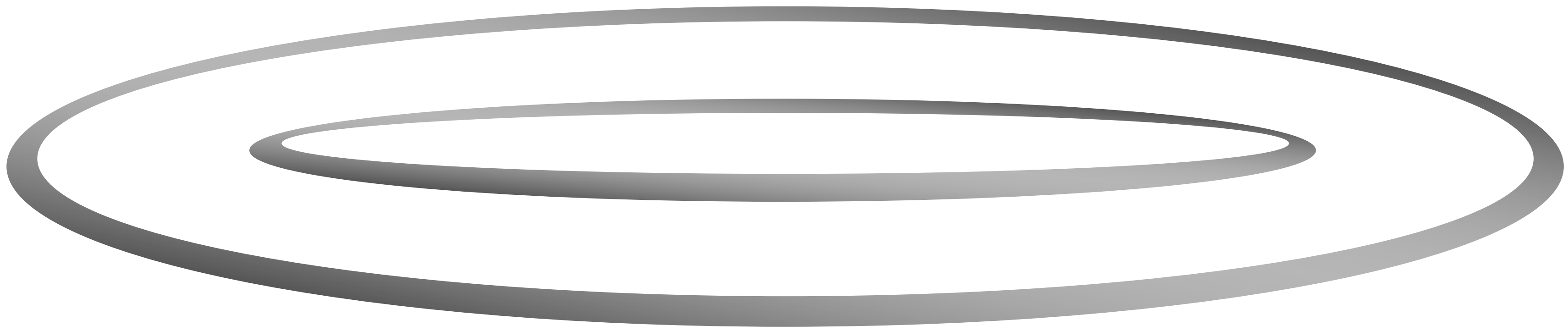
- *Black Saturn:*



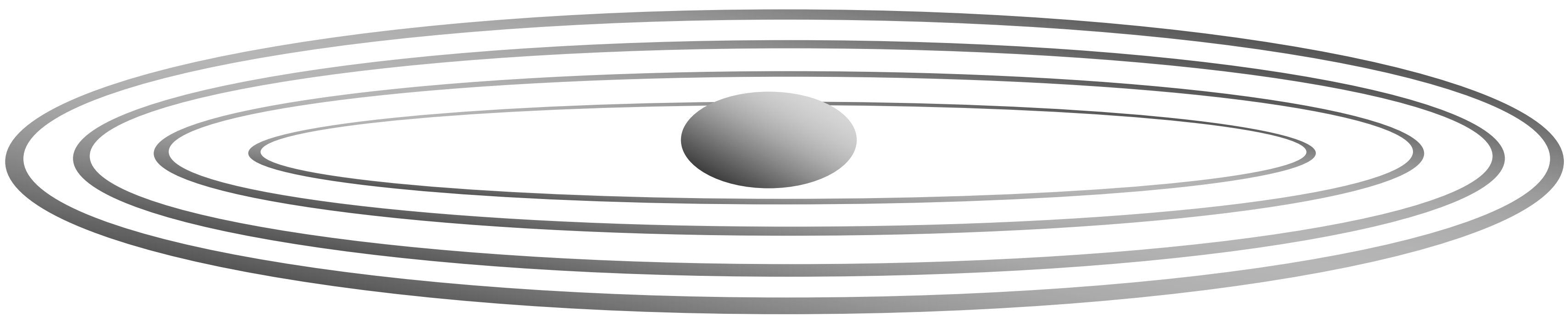
- Co- & counter-rotating, rotational dragging…

Multi-rings are also possible

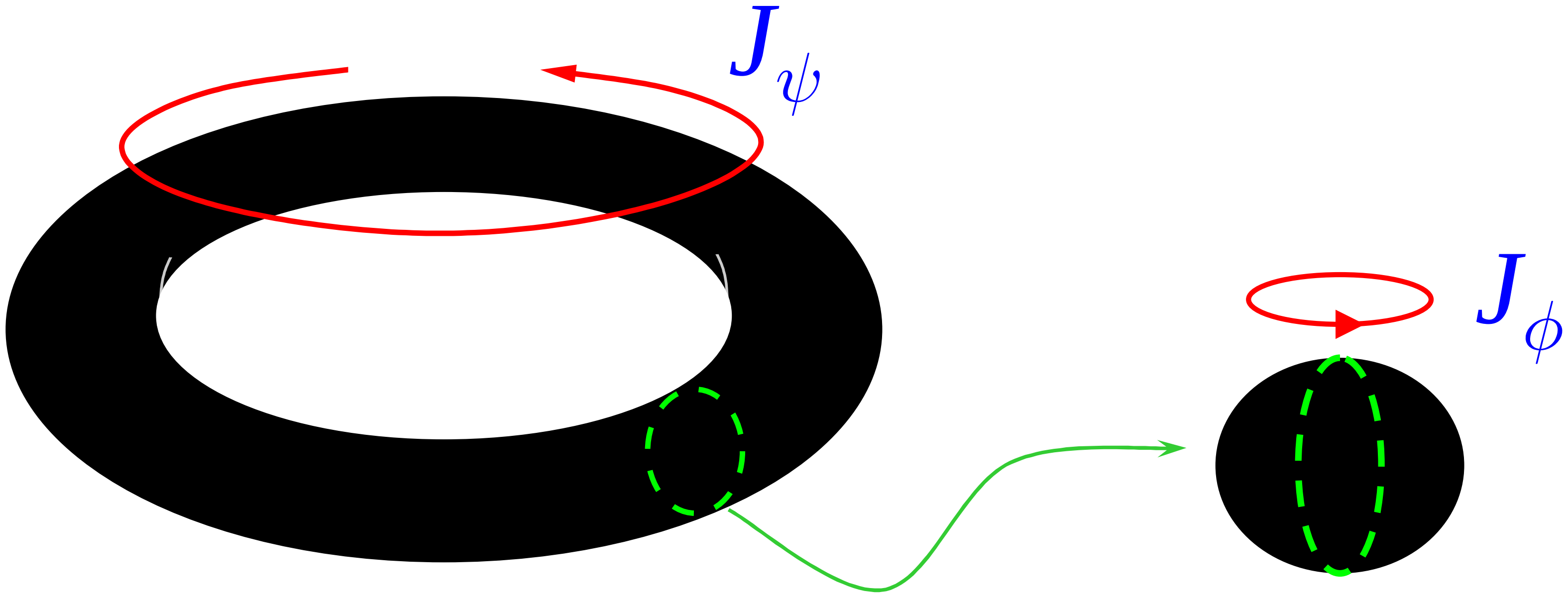
- Di-rings explicitly constructed



- Systematic, increasingly messy construction, with arbitrary number of rings

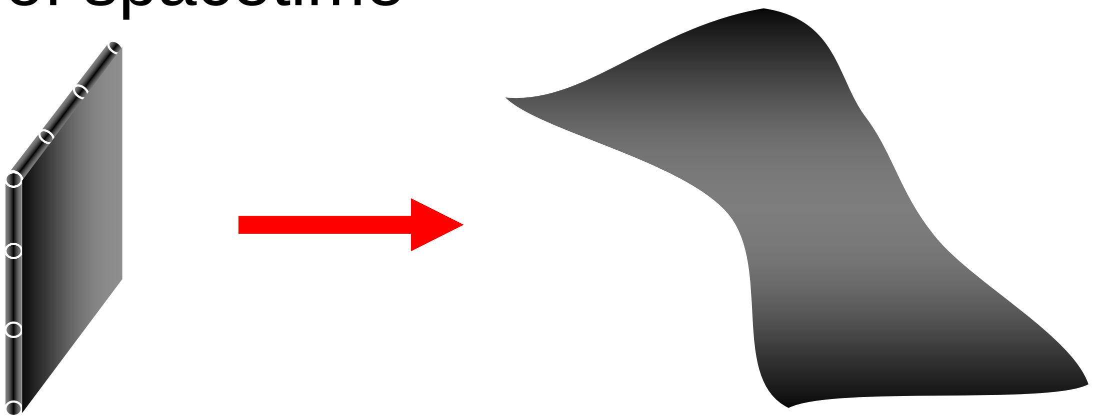


Black rings w/ two spins

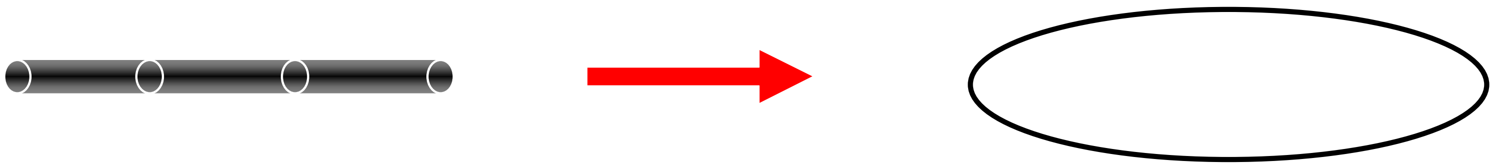


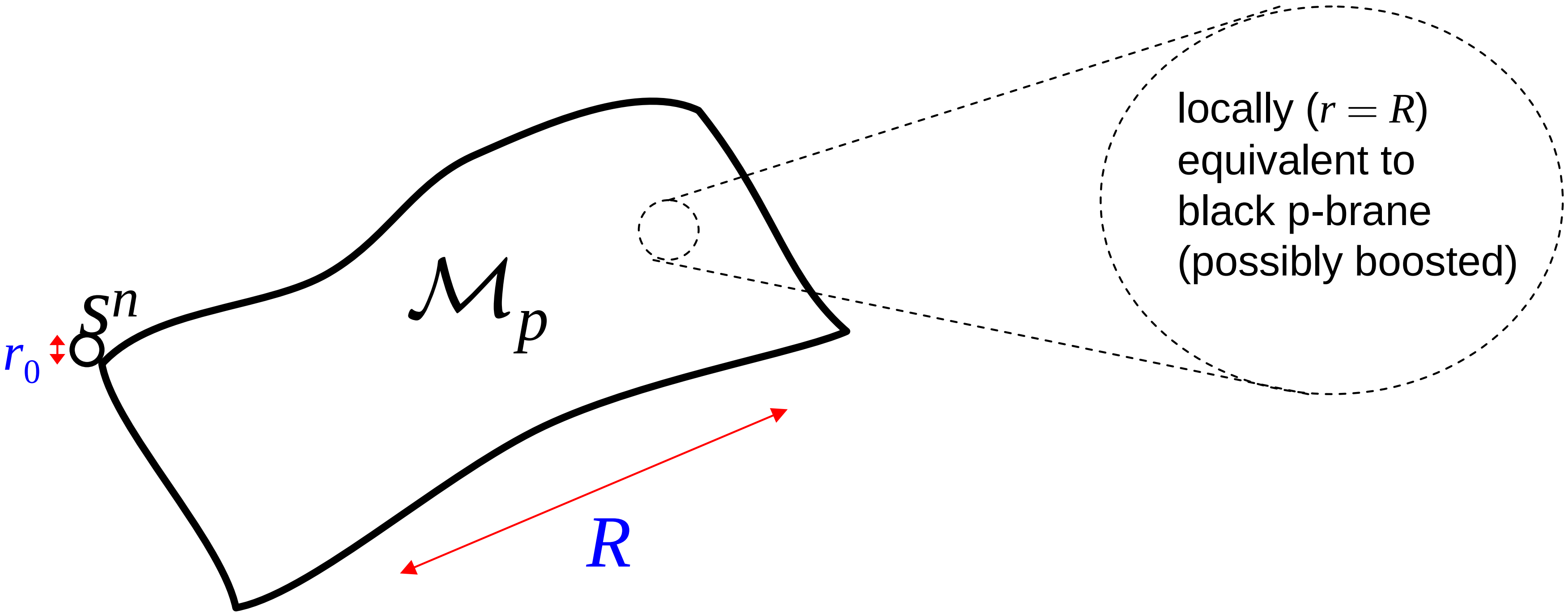
Black holes from *blackfolds*

- **Blackfold**: **Black** p-brane w/ worldvolume = curved submani**fold** of spacetime



- If blackfold worldvolume is spatially compact, then it describes a black hole
  - Eg, black ring as circular black string:

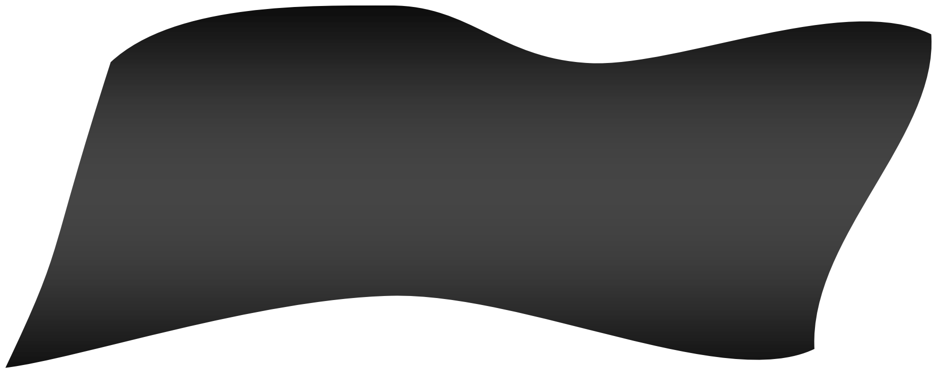



S^n
r_0
$\mathcal{M}_p$
R
locally ($r = R$)
equivalent to
black p-brane
(possibly boosted)

General Classical Brane Dynamics

- Equations for any worldvolume source of energy-momentum, in probe (test brane) approx,

$T_{\mu\nu}$



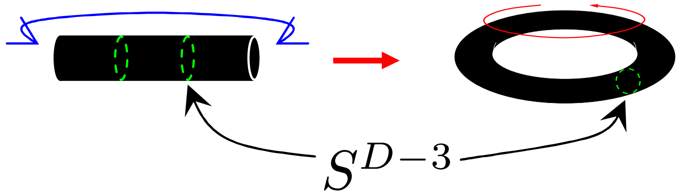
$$\nabla_\mu T^{\mu\nu} = 0$$

For a black p-brane, $T_{\mu\nu}$ is the one computed in earlier exercise

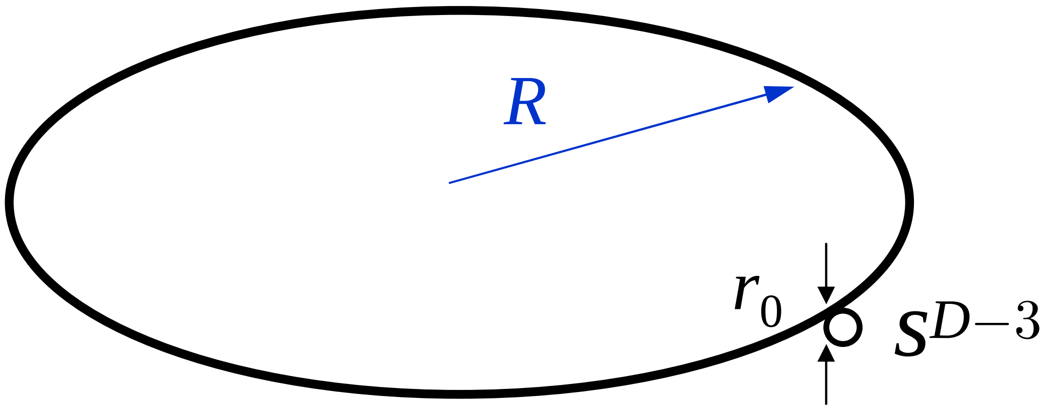
The Blackfold Gallery

Artist: M Rasmussen. "Blackfolded form", hand-built stoneware, height 22cm 20x25cm wide - **£325.00**

Thin black rings in D>5

- Heuristic:  seems plausible

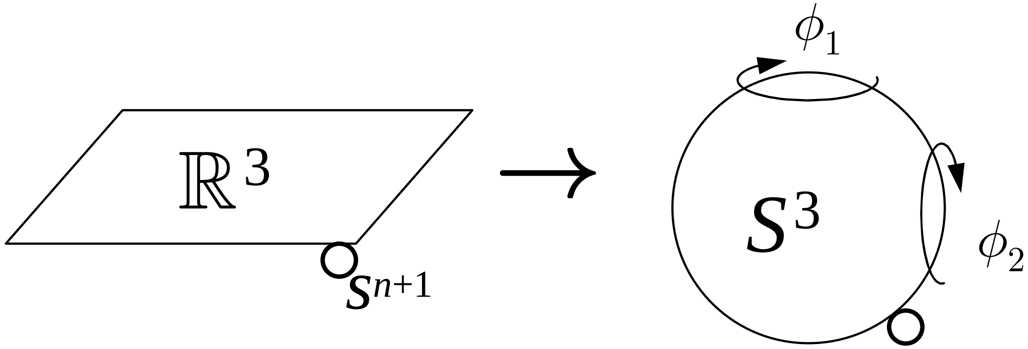
- Thin black rings ≃ circular boosted black strings
- Can easily solve the equations to construct



Horizon S^1 x s^{D-3}

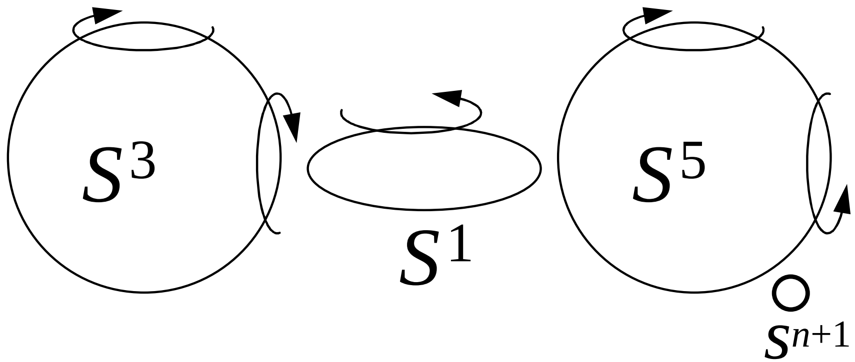
Products of spheres

- $\mathbb{R}^3 \text{x} s^{n+1} \to S^3 \text{x} s^{n+1}$



- Can do it for any product of odd-spheres

$$\prod_{p_a \in \text{odd}} S^{p_a} \times s^{n+1}$$

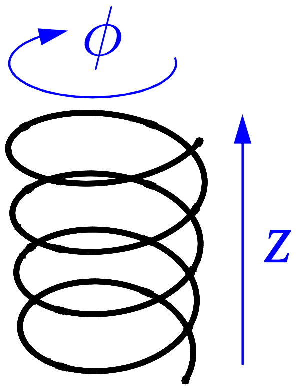


Helical black strings and rings

- Place a black string along an isometry ζ of background

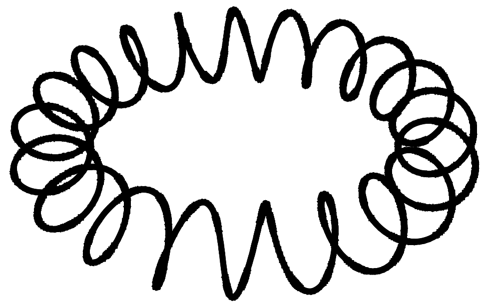
$$\zeta = k\partial_z + \partial_\phi$$

Helical black string



$$\zeta = n\partial_{\phi_1} + m\partial_{\phi_2}$$

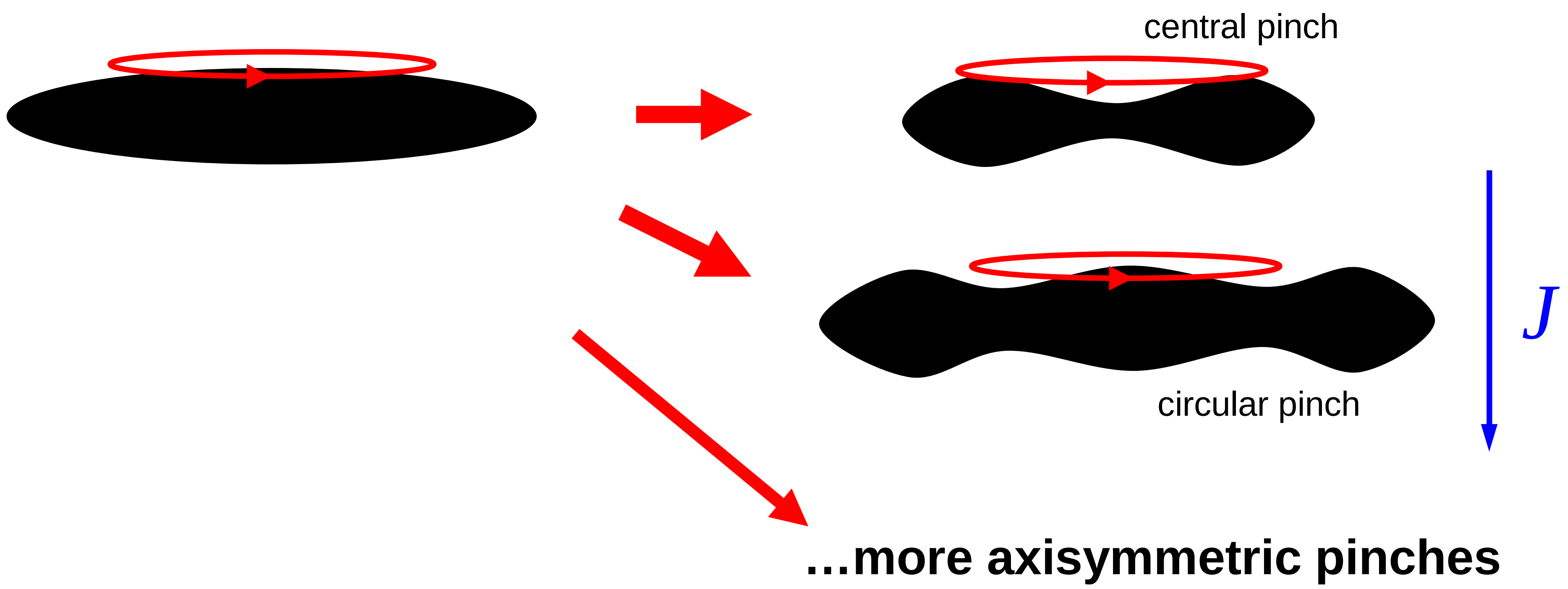
Helical black ring (*slinky*)



(n.b: profile is static!)

Pinched black holes

- Constructed recently by perturbing the rotating black hole



Conclusion: *More is different*

Vacuum gravity $R_{\mu\nu} = 0$ in

- D=3 has no black holes
  - GM is dimensionless → can't construct a length scale

    (Λ, or h, provide length scale)
- D=4 has **one** black hole
  - but no 3D bh → no 4D black strings → no 4D black rings
- D=5 has **four** black holes (two topologies); black strings → black rings, helical rings, infinitely many multi-bhs…
- $D\geqslant 6$ has **infinitely many** black holes (many topologies, lumpy horizons…); black branes→ rings, toroids…, infinitely many multi-bhs…