

INTRO TO
THE HIGHER
STRUCTURE
OF SYMMETRY
CATEGORIES

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DEL
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References:

Many lecture notes are
available on these
topics

McGreevy 2204...

Schüfer-Numbe 2305...

Gomes 2303...

Bhardwaj et al 2307...

Shao 2308...

Brennan et al 2306...

~~1961~~ 2407...

Carpeneville, DT, Runkel
"Topological defects"
2311...

Sivers lectures

Vol I 2411...

Vol II to appear

Thanks to my
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INTRO TO HIGHER
STRUCTURES OF SYMMETRY
CATEGORIES

~~INTRO & MOTIVATION~~

Symmetries play a key role
in our understanding of QFT.

I assume most of you have
been exposed already to
the most basic results on
the topic. For instance
I assume that most of
you can distinguish

global
symmetry

(vs)

gauge
symmetry

organizes
the spectrum
of the QFT
& constrains
correlators

does not
act on
anything, but
it is a
useful tool
to write
local lagrangians
(gauge redundancy)

there are various features:
• Coleman-Mandula theorem
(symmetries of S-matrix)

~~on the other hand~~

My main interest in
symmetries comes from
their interplay w/ RG

UV $\rightsquigarrow$ IR
Microscopic Macroscopic

Abreity w/ ordinary symmetries

$G_{UV} \longrightarrow G_{IR}$
group morphism

- not ~~surjective~~ surjective
(decouple charged massive
states)
- not ~~surjective~~ surjective
(drop higher order interactions,
 $\rightarrow$ emergent symmetries)

We are after two applications:

UV $\rightarrow$ IR: start microscopic
& use symmetry to constrain
what happens @ macroscopic
scale

IR $\rightarrow$ UV: given macroscopic
effect, use symmetry to
infer microscopic origin.

RMK

(locality): defining feature of
QFT^{symmetries} is that they preserve
locality.

Naively:

$\varphi(x) \mapsto F(\varphi(x))$ (★)
map that depends
only on φ and
its derivatives at
 x .

$x' \neq x$: spacetime
symmetry

$x' = x$: internal
symmetry

PMK: sometimes gauge
symmetries are called local:
do not capture locality
w/ gauge redundancy.

All textbook exmples you
might have encountered
so far preserve locality.

But not all local symmetries
have the form $(\star)$.
Transformations of this
kind are often referred
to as classical symmetries
of the field. In
Witten's lectures you might

have seen some aspects
of these. But there
are other types of local
symmetries of QFTs,
known in the literature
as "quantum symmetries"
or "topological symmetries"
or "analogic symmetries".

These symmetries emerge
from quantum #'s associated
to topologically non-trivial
field configurations.

exmples:

winding symmetries of compact
bosons, monopole symmetries
in $2+1$ d Abelian gauge
theory, chiral-Weyl
symmetries of various
kinds (we will see
examples)

In many instances
dualities map
topological symmetries
to classical symmetries
of dual degrees of
freedom and viceversa.

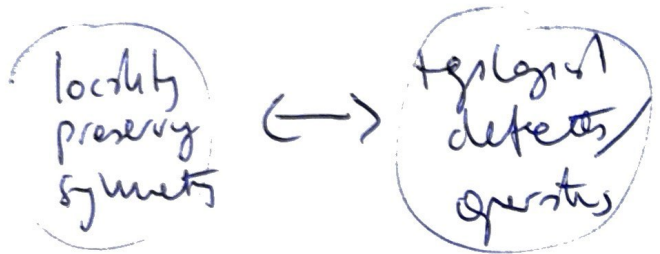
Therefore:

1-) this distinction
should not be
regarded as
fundamental

2-) ~~there are also~~
~~we should return~~
~~the manipulations~~
of the quantum
field that preserve
locality

~~Following the lecture we~~
~~should note that therefore we~~
~~must distinguish between~~
~~local and non-local symmetries~~
~~and the manipulations of the QFT~~

From a contemporary
standpoint a universal
characterization of locality
preserving symmetries is
via topological operators/defects ¹⁻²



The RHS is the definition
of the LHS.

Topological operators/defects, etc:
Extended operators & defects
in the field theory, defined
over regions of spacetime
(their support) that can
be deformed and displaced
at no energy cost and
are such that when
inserted in correlators the
latter depend on their support.

in a gauge-topological way
i.e.

$$\langle D(\Sigma) \dots \rangle$$

~~depends~~ depends only
on the homotopy type
of Σ (i.e. up to
isotopy) up to contact
terms between the topological
defect/operator and
other operator insertions.

ultimately these
contact terms encode
the charges, the representation
of the other operators and
defects in the QFT spectrum
with respect to the
topological operator/defect
inserted.

REQUIRE: FAITHFUL ACTION

RMK / CAVEAT: there
is an infinite
discrepancy in
terminology: outside
of the generalized
symmetry community
"topological defect," still
dynamical objects

~~What are these good for?~~

What are these
good for?

FACT: QFTs can
have extended
configurations

Wilson 74
t Hooft 74
Polyakov 75

These also have a support so we can insert them in our path integrals.

In general these field configurations are not quasi-topological.

Geometric global symmetries: framework to describe the conserved quantum numbers of extended field configurations

Kapustin, Thorngren 2013

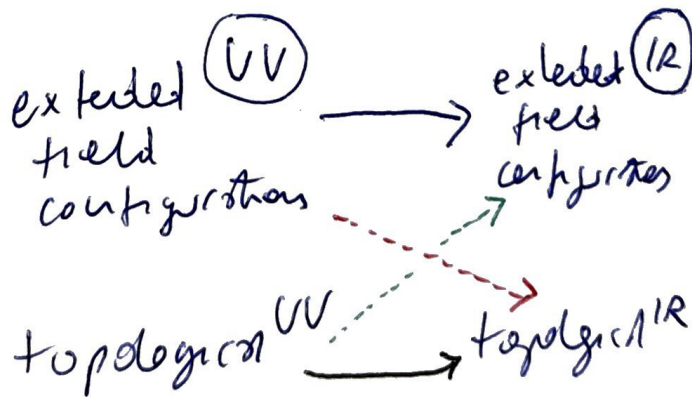
Gaiotto, Kapustin, Seiberg, Willett 2014

Topological operators/defects provide generalized quantum #s for extended field configurations.

Hope: obtain a finer resolution on the structure of invariants of a QFT.

In particular we expect to obtain

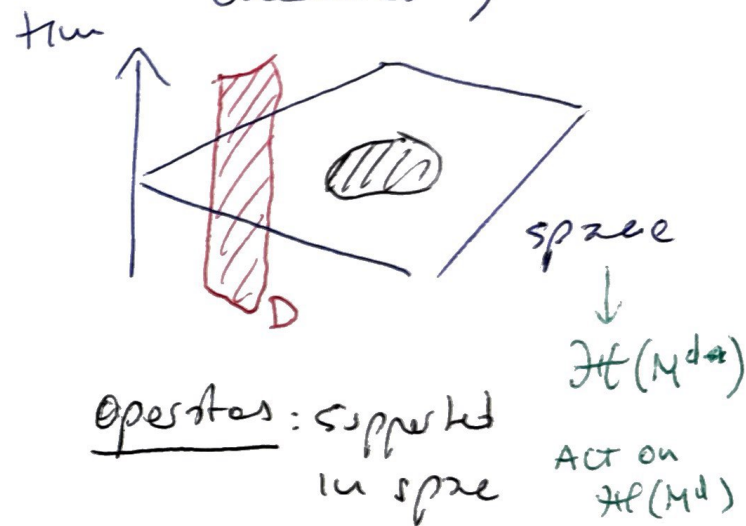
RICHER CONSTRAINTS ON RG FLOWS.



RMK: operators VS defects:

in QFTs with a time evolution

(e.g. Lorentzian QFTs dH or Hamiltonian lattice models)



defect: supported along time

CHANGE $\mathcal{H}(M^d)$

- topological defect $\leadsto$ twisted Hilbert space
- non-topological defect $\leadsto$ bulk-defect excitations

topological defects/operators^(1.3)
are naturally organized
in terms of a "higher
fusion category".

idea: $\mathcal{Z} \in \text{QFT}_{d+1}$
are objects

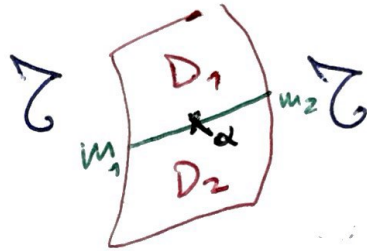
topological interfaces
of codimension 1
are morphisms

$\mathcal{Z}_1$  $\mathcal{Z}_2$
 $I \in \text{Hom}(\mathcal{Z}_1, \mathcal{Z}_2)$

topological defects/operators
of $\mathcal{Z}$ form the category

$$\mathcal{C} := \text{End}(\mathcal{Z})$$

objects of $\mathcal{C}$ are
topological interfaces
(defects/operators)
of codimension 1:



morphisms of $\mathcal{C}$ are
topological interfaces
within the topological
interface itself.

$$m \in \text{Hom}(D_1, D_2)$$

we always have a
trivial interface of
 $\mathcal{Z}$ with itself, $1_{\mathcal{Z}}$

$$m \in \text{End}(1_{\mathcal{Z}})$$

are genuine topological
defects/operators of
codimension 2.

~~Always then we have~~
~~a trivial interface~~
~~yes, we do~~

~~Always~~

There can also be
interfaces may
themselves, there are
higher morphisms

$$\alpha \in \text{Hom}(m_1, m_2)$$

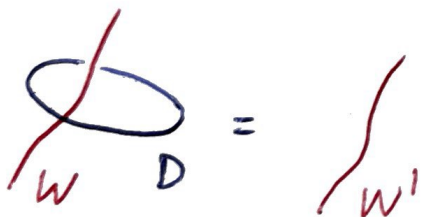
Always then we always
have the trivial interface
of the trivial interface
 $1_{1_{\mathcal{Z}}}$.

$$\alpha \in \text{End}(1_{1_{\mathcal{Z}}})$$

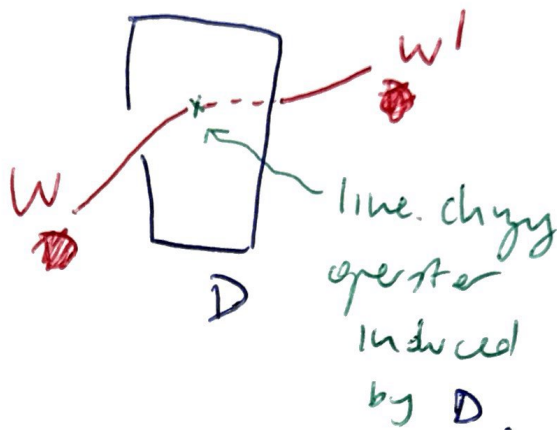
we genuine topological
defects/operators of
codimension 3.

And we can go on
till we exhaust the
available dimensions
of spacetime.

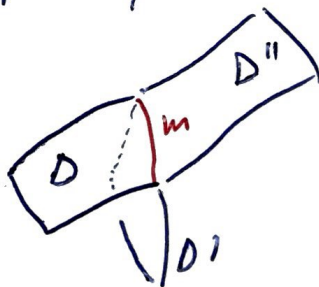
We can have contact
terms arising from a
linking action



and also from
crossing actions



monoidal structure
from parallel fusion of
1d branes



$$m \in \text{Hom}(D \otimes D', D'')$$

RMK: p-form
symmetries:

$$G^{(0)} \subseteq \text{End}(\mathbb{Z})$$

$$G^{(1)} \subseteq \text{End}(\mathbb{Z})$$

$$G^{(2)} \subseteq \text{End}(\mathbb{Z})$$

$\vdots$

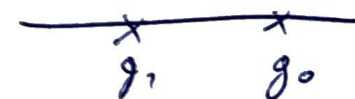
in particular

~~RMK~~ $G^{(0)}$ can be
viewed as

configuration space:



$\neq$



$G^{(p)}$ has configuration space
-- must be
abelian

